Beta-Conversion and the Being Constraint

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Modal contingentists face a dilemma: there are two attractive principles of which they can only accept one. In this paper I show that the most natural way of resolving the dilemma leads to expressive limitations. I then develop an alternative resolution. In addition to overcoming the expressive limitations, the alternative picture allows for an attractive account of arithmetic and for a style of semantic theorizing that can be helpful to contingentists.

1 Beta-Equivalence

The lambda-operator, "λ", is a device for turning formulas into predicates. For instance:

Formula	**Predicate**
Runs(x)	λx(Runs(x))
[*Read:* it runs]	[*Read:* is such that it runs]

We shall say that the following formulas are related by β-conversion:

Substitution into "Runs(x)"	**Application of "λx(Runs(x))"**
Runs(a)	λx(Runs(x))a
[*Read:* a runs]	[*Read:* a is such that it runs]

More generally, when ϕ is a formula and t is a singular term, we shall say that the formulas $\phi[t/x]$ and $\ulcorner\lambda x(\phi)t\urcorner$ are related by *β-conversion*.[1] And we shall

[1] *Notation*: $\phi[t/x]$ is the result of substituting t for every free occurrence of x in ϕ; $\ulcorner\lambda x(\phi)t\urcorner$ is the result of applying the predicate $\ulcorner\lambda x(\phi)\urcorner$ to t.

say that sentences are β-*equivalent* when one can get from one to the other by finitely many steps of β-conversion. (I'm ignoring some subtleties here; see Appendix A for the full story.[2])

2 The Puzzle

It is natural to think that β-equivalent sentences are necessarily equivalent. Alas, this natural thought leads to a puzzle. For the following theses, which are all initially plausible, turn out to be jointly inconsistent:[3]

Contingentism: $\neg\Box\forall x\Box\exists y(y = x)$.

[*Read:* it's not the case that, necessarily, everything necessarily exists.[4]]

The Being Constraint: $\Box\forall x\Box(Fx \rightarrow \exists y(x = y))$.[5]

[*Read:* necessarily, everything is such that, necessarily, it is only F if it exists.]

***β*-Necessity:** β-equivalent sentences are necessarily equivalent.

Any two of the theses are mutually consistent, so the puzzle can be defused by rejecting any one of the three. But which one?

Since all three theses are theoretical claims, it would be a mistake to proceed merely by consulting intuition. The right methodology is to defend a coherent philosophical picture of the underlying terrain and to motivate the rejection of one of the puzzle's premises on the basis of that picture. Williamson (2013) defends a picture on which contingentism fails. Stalnaker (1977, 2012) defends a picture on which β-necessity fails. Here I will instead defend a picture on which the Being Constraint fails.[6] My proposal has a lot in common with (Fine 2005). (In slogan form, I'll defend Fine, without the metaphysics.)

[2] Appendices are available at `web.mit.edu/arayo/www/bcbc-proofs.pdf`

[3] For illuminating discussion, see Dorr 2017. As Dorr points out, one can prove the inconsistency as follows: the Being Constraint gives us $\Box\forall x\Box((\lambda z\neg\exists y(z = y))x \rightarrow \exists y(x = y))$, which is β-equivalent to $\Box\forall x\Box(\neg\exists y(x = y) \rightarrow \exists y(x = y))$, which is logically equivalent to $\Box\forall x\Box\exists y(y = x)$.

[4] Here and throughout I use "exists" as a stylistic variant of "is something".

[5] Here F is an arbitrary one-place predicate. As set forth in Williamson 2013, §4.1, the Being Cosntraint is the result of generalizing the principle above to n-place predicates.

[6] For a defense of the Being Constraint, see Stephanou 2007 and Jacinto 2019*b*.

3 The Property View

I won't say much about Williamson's picture here. Instead, I'll start by considering Stalnaker's (2012) picture:

The Property View

> **P1:** A one-place predicate expresses a property.[7]
>
> **P2:** A sentence $\ulcorner Fa \urcorner$ is true at a world w iff: at w, the property expressed by F is exemplified by the referent of a.
>
> **P3:** For an object to exemplify a property at a world w, the object must exist at w.
>
> And similarly for n-place predicates.

P1 and P2 are naturally thought of as semantic truisms. And although Stalnaker has sophisticated reasons for accepting P3, the claim can also be motivated on intuitive grounds. For it is natural to think that a property is a *way* for a thing to be and therefore that exemplifying a property is a matter of *being a certain way*. But how could an object be a certain way without being something? Doesn't an object have to *be* in order to be *somehow*?

The Property View provides an answer to the puzzle of the preceding section. On the one hand, it entails the Being Constraint, given reasonable assumptions. (See proposition 1, Appendix B). On the other, a contingentist can use the Property View to build a barrage of counterexamples to β-Necessity. Consider, for instance, the sentences A and A^{λ}:

A	**A$^{\boldsymbol{\lambda}}$**
Wise(Alice) $\vee$ $\neg$Wise(Alice)	$\lambda x(\text{Wise}(x) \vee \neg\text{Wise}(x))$ Alice
[*Read:* Either Alice is wise or it is not the case that Alice is wise.]	[*Read:* Alice is such that that either she is wise or it is not the case that she is wise.]

[7] Here I assume that the predicate in question contains no free variables. But it is easy to generalize the proposal: an (open or closed) one-place predicate expresses a property relative to a variable assignment. This approach to free variables departs from Stalnaker, who only takes the semantic value of a predicate to be a property in the special case predicates with no free variables. Rather than taking an open formula to express a proposition relative to a variable assignment, he takes it to express a propositional function *simpliciter*. And, more generally, rather than taking an open predicate to express a property relative to a variable assignment, he takes it to express a property-function *simpliciter*.

A and A$^\lambda$ are β-equivalent. But suppose the contingentist thinks there is a world w at which Alice fails to exist. Since A is a tautology, it is true at w. But by P1, the complex predicate "$\lambda x(\text{Wise}(x) \vee \neg\text{Wise}(x))$" expresses a property. And by P3, Alice can only exemplify that property at w if she exists at w. So, by P2, A$^\lambda$ is not true at w. So A and A$^\lambda$ are not necessarily equivalent.

4 Expressive Limitations

Attractive as it is, the Property View gives rise to potentially problematic expressive limitations.

Start with a warm-up case. On the Property View, there is no such thing as a Kripkean "external" identity predicate "$=^e$", which is such that, for any terms t and t', the formula $\ulcorner t =^e t' \urcorner$ is true at a world w iff the individual denoted by t is identical to the individual denoted by t', *regardless of whether the individuals in question exist at* w. As Fine (2005) points out, there are contexts in which this expressive limitation can be overcome. For although the Property View does not admit of an external identity *predicate* "$=^e$", it allows for the *formula* $\ulcorner \Diamond(t = t') \urcorner$, which is model-theoretically equivalent to $\ulcorner t =^e t' \urcorner$ in the relevant contexts.[8] On the other hand, this strategy presupposes that every model takes the denotations of t and t' to each exist at some world. And although this assumption is plausible when the denotations of t and t' are concrete individuals, it is potentially problematic when they are sets of individuals drawn from different worlds. (We will consider examples of such sets in sections 13 and 14.)

Here is a second, more tenacious expressive limitation. The Property View entails that there is no such thing as an "external" one-one correspondence predicate "$\approx^e$", which is such that, for any plural terms tt and tt', the formula $\ulcorner tt \approx^e tt' \urcorner$ is true at a world w iff the individuals denoted by tt are just as many as the individuals denoted by tt', *regardless of whether the individuals in question exist at* w. This time, however, there is no analogue of Fine's trick. The Property View entails that there is no such thing as a *formula* model-theoretically equivalent to $\ulcorner tt \approx^e tt' \urcorner$.[9] For instance, Corollary 1, Appendix E entails that

[8] For ϕ to be *model-theoretically equivalent* to ψ is for ϕ and ψ to be true at the same worlds with respect to any model and variable assignment.

[9] A friend of the Property View might attempt to get around the problem by paraphrasing occurrences of "$\approx^e$" in arithmetical terms. For instance, she might paraphrase "$tt \approx tt'$" as "$\exists n(\text{Num}(tt, n) = \text{Num}(tt', n))$". In order for this proposal to deliver the right results, though,

the Property View is incompatible with a sentence model-theoretically equivalent to the following:[10]

Comparing Counterpossibles

$$\forall xx \Diamond \exists yy (xx \approx^e yy \wedge \text{CounterPoss}(xx, yy))$$

> [*Read:* for any individuals xx, it's possible for there to be some individuals yy, which are just as many as the xx but couldn't have co-existed with them.]

> $\ulcorner\text{CounterPoss}(xx, yy)\urcorner$ abbreviates $\ulcorner\neg\Diamond\exists x\exists y(\Diamond(x \prec xx) \wedge \Diamond(y \prec yy))\urcorner$.

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one must assume that the domain of each world contains as many numbers as there might have been individuals (since otherwise "$\Box\forall xx\Box(xx \approx^e xx)$" would count as false). Such an assumption entails that there is a definite answer to the question of how many individuals there might have been—a claim that is presupposed by Williamson (2013) but is unlikely to be accepted by contingentists such as Stalnaker (2012). For further discussion, see Rayo (2020).

[10] In proving the result, I assume the modal language in question includes n-place second-order variables and quantifiers for any $n \geq 0$. To make the result as strong as possible, I assume that the second-order vocabulary is interpreted so as to reflect higher-order necessitism: the view that, necessarily, every property exists necessarily. (Interpreting second-order quantifiers so as to reflect higher-order contingentism would not make the language any more expressive.) It is worth emphasizing, however, that if the language was further enriched with third-order quantifiers, and if the higher-order quantifiers were interpreted so as to reflect higher-order necessitism, then a proponent of the Being Constraint could use the following characterization of "$xx \approx^e yy$":

$$\exists \mathscr{R}\, (\forall X \exists! Y\, (M(X, xx) \wedge M(Y, yy) \wedge \mathscr{R}(X, Y)) \wedge \forall Y \exists! X\, (M(X, xx) \wedge M(Y, yy) \wedge \mathscr{R}(X, Y)))$$

where

$$\exists! Z\phi = \exists V \forall Z\, (\phi \leftrightarrow \Box\forall z\Box\, (Vz \leftrightarrow Zz))$$

$$M(Z, zz) = \Diamond\exists z\, (z \in zz \wedge \Box\forall v\Box\, (Zv \rightarrow v = z))$$

On the other hand, higher-order necessitism goes against the spirit of contingentism, as defended in, e.g. Stalnaker 2012. Higher-order contingentism requires a non-trivial model-theory, which has been studied by Fine 1977, Stalnaker 2012, Fritz & Goodman 2016, Fritz 2018*a*,*b*. But Fritz & Goodman 2016 show that there is a certain sense in which the most natural way of spelling out that model-theory is in tension with the Being Constraint, when generalized so as to apply to all types. I do not know whether external one-one correspondence can be expressed in a contingentist higher-order language that is interpreted using the relevant models (assuming reasonable auxiliary assumptions in place). But it is worth noting that the resulting languages suffer from non-trivial expressive limitations. For instance, Fritz 2018*b* shows that, given certain assumptions, they cannot express claims of the form "There are κ possible Fs" when κ is an uncountable cardinality. (For philosophical discussion, see Fritz & Goodman 2017.)

Since Comparing Counterpossibles would appear to express an intelligible claim, its inexpressibility is an awkwardness for the Property View.

The Property View's expressive limitations are also a potential liability when it comes to the project of giving a semantics for modal languages that the contingentist is in a position to take at face value. On the most straightforward construal of Kripke's possible-words semantics for modal sentences, one's semantic theory carries commitment to "merely possible" entities, which the contingentist is not in a position to take at face value. As we will see below, it is possible to make some progress on this problem by rejecting the Being Constraint. But, on the most plausible contingentist assumptions I am aware of, there is no way of overcoming the problem when the Being Constraint is in place. More specifically, Stalnaker (2012) hoped to give a compositional semantics for modal languages in which "all intermediate values in the composition are actual things." (125), but Jacinto (2019*a*) showed that the most natural development of Stalnaker's proposal cannot be made to work when the Being Constraint is in place.[11]

I hasten to add that not every expressive limitation is a liability, since we want to steer clear of a language that allows us to make nonsense claims.[12] In this paper I will defend an alternative to the Property View, which overcomes some its expressive limitations, and—I hope—does so without lapsing into nonsense.

5 Transcendental Properties

Fine (2005) develops a picture of properties that is inconsistent with the Property View. His proposal is based on a distinction between "the *necessary truths proper*, those that hold whatever the circumstances, and the *transcendental truths*, those that hold regardless of the circumstances." (My emphasis.) This distinction is spelled out using a non-standard conception of possible world:

> A possible world, under this alternative conception, is constituted, not by the totality of facts, or of how things might *be*, but by the totality of circumstances, or of how things might *turn out*. [...]

[11]Stalnaker (2016) ultimately concludes from Jacinto's observation that "virtual propositional functions [which are artifacts of the model that do not correspond to anything in reality] seem to play an ineliminable role in the compositional process by which complex predicates and quantificational constructions are interpreted" (ft. 14).

[12]For further discussion, see Rayo 2020.

> A necessary truth will then be a worldly proposition whose truth-value always turns favorably on how things turn out, while a transcendental truth will be a true proposition whose truth-value does not turn on how things turn out.

For example, Fine takes the proposition that Socrates is self-identical to be a transcendental truth: he thinks it is true regardless of circumstance—and so, in particular, regardless of whether Socrates exists.

And, of course, Socrates is not a special case. What's really doing the work is the property of self-identity, which Fine takes to be a *transcendental property*: a property that "will be exemplified by objects regardless of how things turn out and so should be taken to be exemplified by objects regardless of whether or not they exist."

Transcendental properties are inconsistent with thesis P3 of the Property View. For whether an object exemplifies a transcendental property at a world is independent of whether the object exists at that world.[13] For instance, Socrates exemplifies the property of self-identity at worlds in which he fails to exist.

Fine goes on to suggest that formal properties like self-identity are not the only transcendental properties. He thinks that "the transcendental essence of an object constitutes a kind of skeletal 'core' from which the rest of the essence can be derived". He thinks, for example, that the property of being a man, which is part of Socrates's essence, might count as transcendental. If this is right, Socrates exemplifies the property of being a man at a world whether or not he exists at that world. Thesis P3 of the Property View will therefore be subject to a broad range of counterexamples.

I think Fine is on to something in distinguishing between holding "whatever the circumstances" and holding "regardless of the circumstances". But I find the specifics of his distinction somewhat elusive. One of the main objectives of this paper is to characterize a distinction that bears a family resemblance to Fine's but allows for a more perspicuous characterization. Whereas Fine distinguishes between transcendental and non-transcendental *properties*, I will propose a distinction between transcendental and non-transcendental *predicates*. Although I cannot promise that the new distinction will do justice to Fine's intent, we will see that it has some interesting philosophical applications.

[13] Fine thinks it is, strictly speaking, incorrect to think of a transcendental truth as true in a possible world. But he also thinks there is an "extended" sense of truth on which transcendental truths are true at every world (and therefore necessary). It is the extended sense I have in mind here.

6 Trancendental Predicates

Consider the following characterization of a transcendental predicate, where W is one's space of possible words:

> An n-place predicate ψ of L is *transcendental* (by the lights of one's semantic theory M for L) iff the following is a theorem of M:
>
> > $\ulcorner\psi(x_1, \ldots, x_n)\urcorner$ is true at world $w \in W$ (relative to a given variable assignment) iff: Φ.
>
> where Φ is a formula in which neither "w" nor "W" occur free.

Here is an example. The standard semantic clauses for negation and disjunction deliver the following result:[14]

> $\ulcorner\psi \vee \neg\psi\urcorner$ is true at world $w \in W$ iff: either ψ is true at w or it is not the case that ψ is true at w.

Since the right hand side of this biconditional is itself a tautology, we can use a classical metatheory to derive the result that $\ulcorner\psi \vee \neg\psi\urcorner$ is true at w iff $\top$, for $\top$ an arbitrary tautology. This entails that $\ulcorner\psi \vee \neg\psi\urcorner$ is a transcendental formula and that its truth at w imposes no demands on w. (All of this by the lights of a classical metatheory. An intuitionist metatheory might disagree.)

The resulting picture suggests an intuitive characterization of transcendental truth: *to be transcendentally true is for one's truth to impose no demands on the world.*[15] It also gives us a nice way of connecting our notion of a transcendental

[14]The clauses I have in mind are these:

- $\ulcorner\phi \vee \psi\urcorner$ is true at w iff: either ϕ is true at w or ψ is true at w;
- $\ulcorner\neg\phi\urcorner$ is true at w iff: it is not the case that ϕ is true at w.

[15]What if our metatheory proves not Φ but its negation? In that case our metatheory is able to prove the negation of $\ulcorner\psi(x_1, \ldots, x_n)$ is true at $w\urcorner$ without making any assumptions about w. So, in the context of our metatheory, we can think of $\ulcorner\psi(x_1, \ldots, x_n)\urcorner$'s *falsehood* as imposing no demands on w. Alternatively, we can think of its truth as imposing an *absurd* demand on w—a demand that can be shown to be unsatisfiable independently of any assumptions about w. This suggests an intuitive conception of transcendental falsehood: to be transcendentally false is for one's falsehood to impose no demands on the world (or for one's truth to impose an absurd demand).

predicate with Fine's distinction between having one's truth-value "regardless of the circumstances" and having it "whatever the circumstances". For example, we can cash out the idea that ϕ is true "regardless of circumstance", as opposed to "whatever the circumstance", as the claim that ϕ's truth at a world imposes no demands on that world.

There are, however, some important differences between my notion of transcendentality and Fine's. As we have seen, when Fine distinguishes between holding "whatever the circumstances" and holding "regardless of the circumstances", he is concerned with propositions rather than sentences, and when he distinguishes between being exemplified by an object "however things turn out" and "regardless of how things turn out", he is concerned with properties rather than predicates. As a result, he must rely on highly fine-grained conceptions of propositions and properties. In particular, he must rely on a conception of proposition fine-grained enough to allow for a distinction between different kinds of necessarily true propositions: those that are true "whatever the circumstances" and those that are true "regardless of the circumstances". On our treatment of the issue, in contrast, it is *formulas*—and, more generally, *predicates*—that are said to be transcendental. As a result, there is no need for fine-grained conceptions of propositions and properties. In particular, there is no need to distinguish between two kinds of necessarily true propositions. What we have instead is two different ways of expressing such propositions. A transcendental formula does so by imposing no demands on the world. A non-trascendental formula does so by imposing a non-trivial demand on the world that turns out to be satisfied at every world. (A certain kind of theist might think that this is true of "there is a god".)

It is also worth emphasizing that a predicate that counts as transcendental in my sense needn't correspond to a property (or proposition) that counts as transcendental in Fine's. For example, a logically true formula like "$\exists x(x = s) \vee \neg\exists x(x = s)$" counts as transcendentally true in my sense. But Fine thinks that the proposition that Socrates exists or does not exist is true "whatever the circumstances", on the grounds that "its truth-value turns on whether or not Socrates exists, which is a matter of how things turn out, and its truth-value, as so determined, is always the Truth."

It is, of course, possible for one's metatheory to prove the above biconditional, while proving neither Φ nor its negation. In that case we should say that, from the perspective of our metatheory, the truth of $\ulcorner\psi(x_1, \ldots, x_n)\urcorner$ either imposes no demands on w or imposes a trivial demand, but that we don't know which.

7 Why care about transcendence?

In the previous section we saw that $\ulcorner \phi \vee \neg\phi \urcorner$ can be shown to be transcendentally true, from the perspective of a classical metatheory with standard semantic clauses. Now consider an interlocutor who uses a different metatheory to reach the conclusion that $\ulcorner \phi \vee \neg\phi \urcorner$ is necessarily true without being transcendentally true: she thinks its truth imposes a demand on the world that nonetheless turns out to be satisfied necessarily. Would it make any real difference?

I think it would. Suppose, for example, that you are a classical logician and are pressed by an intuitionist to justify your claim that $\ulcorner \phi \vee \neg\phi \urcorner$ is a necessary truth for any ϕ. The range of possible answers at your disposal will depend on whether you think of $\ulcorner \phi \vee \neg\phi \urcorner$ as transcendental.

Consider, first, a scenario in which you don't take $\ulcorner \phi \vee \neg\phi \urcorner$ to be transcendentally true. Instead, you think its truth imposes a demand D, wich turns out to be satisfied necessarily. Then the project of explaining why $\ulcorner \phi \vee \neg\phi \urcorner$ is necessarily true calls for two distinct tasks. First, you need to explain why the truth of $\ulcorner \phi \vee \neg\phi \urcorner$ demands D; second, you need to explain why D is satisfied necessarily. But how might you address the second of these tasks? You could try an abductive argument and claim that the necessary satisfaction of D is the simplest explanation for the truth of the many known instances of excluded middle. Or you could claim that D is entailed by a "metaphysical law" and argue that such laws are in place as a matter of necessity.[16] Neither of these strategies strikes me as especially promising, and I'm not aware of anything better.

Now consider a scenario in which you take $\ulcorner \phi \vee \neg\phi \urcorner$ to be transcendentally true. Here we only need to worry about the first of the two tasks mentioned above: that of explaining why $\ulcorner \phi \vee \neg\phi \urcorner$ has the truth conditions that it has. There is no need to explain why such truth-conditions are satisfied, because nothing is required to satisfy them.

The lesson is that transcendence matters in at least this sense: whether or not one takes a formula to be transcendentally true can shape one's grounds for accepting it. I would like to suggest, moreover, that there is a special case in which this point is especially significant: the case in which the language under study is a formal language, whose semantic clauses are in place *by stipulation*. Let me explain.

[16] Why a metaphysical law and not a logical law? Because logical laws, as standardly understood, are laws concerning the truth of *sentences*, or connections between the truth of sentences, and D is meant to describe a demand on the world.

The problem of ascertaining the truth-conditions of an English sentence like "either Jones is tall or it's not the case that Jones is tall" is a problem in natural language semantics, which cannot be resolved without taking a stand on the semantics of English expressions like "or" and "not", or on the difficult question of whether English is governed by, e.g. a negative free logic. But suppose that our focus is not on the English sentences but on a formal analogue: "$T(j) \vee \neg T(j)$". Suppose, moreover, that we think of formalized sentences not as *translations* of their natural language counterparts but as their *regimentations* (Quine 1960). There is no presumption of synonymy or sameness of logical form. We require only that whatever we hoped the given English sentence would achieve in the context of a particular project can be achieved by its formalization, to our own satisfaction. This means, in particular, that regimentation is not a device for shedding light on the original sentences: we simply choose to replace the original sentences with their regimentations for the purposes of the relevant project.

When we are working with a formalized language, and when the language is thought of as an instrument of regimentation, it becomes much easier to determine whether a given sentence is transcendental. For one is free to *stipulate* that the langauge's logical connectives are to be governed by the standard clauses. And with such clauses in place, one can use a classical metatheory to show that $\ulcorner \phi \vee \neg\phi \urcorner$ is transcendentally true for any ϕ. So one will be in a position to justify the claim that $\ulcorner \phi \vee \neg\phi \urcorner$ is a necessary truth without having to resort to abductive arguments or "metaphysical laws".

It goes without saying that such a justification is based on assumptions that one's interlocutor might dispute. Firstly, there is no guarantee that one's interlocutor will see one's stipulations as conferring legitimate meanings on the expressions of one's formalized language. (Think about Prior's "TONK".[17]) Secondly, one's interlocutor might complain that the use of a classical metatheory begs the question in the present context. (Consider, for example, the point of view of an intuitionist.)

But the fact that our argument can't be used to win over every interlocutor doesn't render it pointless. At the very least, we can use it to reassure ourselves that the necessary truth of $\ulcorner \phi \vee \neg\phi \urcorner$ can be explained in a way that is satisfactory by our own lights. (Compare: the fact that an eye-exam can't be used to win over a skeptic about the external world doesn't render it pointless. At the

[17]TONK is a supposed connective with the introduction rule of disjunction but the elimination rule negation. See Belnap 1962, Prior 1967.

very least, it can be used to reassure a non-skeptic that her eyesight is working properly.)

8 The Propositional Function View

The Property View allows for transcendental formulas. (For example, it allows tautologies to be counted as transcendental.) But it does not allow for transcendental n-place predicates ($n > 0$), since it entails that $\ulcorner F(x_1, \ldots, x_n)\urcorner$ can only be true at w if the referents of $x_1, \ldots, x_n$ exist at w, which is a non-trivial demand on w.

In this section we will consider an alternative to the Property View, one that is able to overcome some of the expressive limitations I mentioned in section 4 by allowing for transcendental n-place predicates ($n > 0$). On this alternative view, the semantic values of predicates are *propositional functions* rather than properties.[18]

A (one-place) propositional function is a function that takes an individual as input and delivers a proposition as output.[19] One might be tempted to model a propositional function as a collection of tuples $\langle x, p\rangle$, where x is an individual and p is a proposition. In the present context, however, such modeling will not do. Generally speaking, the best way to think of a function is as a *condition* that delivers an output given an appropriate input. In the special case in which one's domain of inputs can be characterized in advance, there is little harm in representing a condition of this kind as the collection of pairs $\langle x, y\rangle$, where x is a member of the relevant domain and y is the condition's output given input x. In the case at hand, however, it is not clear that the domain of the function can be characterized in advance. The problem arises because we want to be able to apply a propositional function *within the context of a modal clause*. And the contingentist thinks that, within such contexts, a propositional function might be given an input that does not, in fact, exist. Suppose, for example, that f

[18]One could get similar results by working with a conception of property that vindicates P1 and P2 but not P3. I use propositional functions because they carry less philosophical baggage than properties. Unlike propositional functions, properties are used widely across philosophy (and, to some extent, outside philosophy). So there is a temptation to think of them as constrained independently of their role within semantics. For example, a critic might wish to argue that properties—the "real" properties that are "out there"—satisfy P3, and object to the proposal on those grounds alone. I am anxious to steer clear of such debates.

[19]An n-place propositional function is a function that takes a sequence of n individuals as input and yields a proposition as output.

is the propositional function that takes an individual as input and yields the proposition that that individual is a plumber as output. We want the following to be true:

> I might have had a sister x such that the proposition $f(x)$ is true at all and only worlds where x is a plumber".

Since I do not, in fact, have sister, contingentists can be expected to think that nothing is a possible sister of mine. So they are not in a position to characterize an ordered pair $\langle x, f(x) \rangle$ that captures the result of applying f in the modal context above. Fortunately, they can still think of f as a condition, poised to deliver outputs given suitable inputs, regardless of whether a domain of possible inputs can be characterized in advance.[20] (Compare: A machine might be constructed so as to take a sheet of paper as input and deliver the result of printing a gold star on that sheet of paper as output. The machine is poised to deliver its outputs given suitable inputs, and would remain so poised even if there were sheets of paper that do not, in fact, exist.)

With this as our background, I can state the proposal:

The Propositional Function View

F1: An n-place predicate expresses an n-place propositional function.[21]

F2: The sentence $\ulcorner Ft_1 \ldots t_n \urcorner$ expresses the proposition $f(z_1, \ldots, z_n)$, whenever the n-place predicate F expresses the n-place propositional function f and the singular terms $t_1, \ldots, t_n$ refer, respectively, to $z_1, \ldots, z_n$.

(Here and throughout we will think of propositions as 0-place propositional functions. Accordingly, F1 entails that sentences express propositions.)

One way to motivate the Propositional Function View is to adopt a theory of semantic value that mirrors a standard syntax of functional types. It's natural to think of a one-place predicate as a function that takes as input a singular

[20]Stalnaker 2012 has a different conception of propositional functions. He thinks of a propositional function as a property that relates individuals to propositions. A consequence of this view is that propositional functions cannot satisfy F2 unless one thinks that a property can apply to an individual at a world even if the individual fails to exist of that world.

[21]Here I assume that the predicate in question contains no free variables. But it is easy to generalize the proposal: an (open or closed) n-place predicate expresses an n-place propositional function relative to a variable assignment.

term and delivers as output a sentence. Correspondingly, it's natural take the semantic value of a one-place predicate to be a function that takes as input the semantic value of a singular term (i.e. an object) and delivers as output the semantic value of a sentence (i.e. a proposition); in other words: a propositional function.

As we have seen, the Property View entails that there is no such thing as the external identity predicate "$=^e$" or the external one-one correspondence predicate "$\approx^e$". The Propositional Function View, in contrast, faces no such expressive limitations, since there is no reason "$=^e$" and "$\approx^e$" couldn't express propositional functions. For instance, "$=^e$" can express the function that takes a pair of individuals as input and delivers as output: (a) a necessary proposition, if the objects are identical, and (b) an impossible proposition otherwise.[22]

The Propositional Function View provides an answer to the puzzle of section 2. On the one hand, it entails β-necessity, given reasonable assumptions (proposition 15, Appendix F). On the other, a contingentist can use the Propositional Function View to construct easy counterexamples to the Being Constraint. Consider, for instance:

Proposition	**Propositional Function**	**Predicate**
$\top$	$f(x) = \top$	F
[A proposition true at every possible world]	[A propositional function that takes any object to $\top$]	[A one-place predicate that expresses f]

Now let a refer to Alice. By F2, $\ulcorner F(a) \urcorner$ expresses $\top$ and is therefore true at every world. But a contingentist can be expected to think that there is a world w at which Alice fails to exist. So "$F(a) \wedge \exists y(a = y)$" is false at w. So the Being Constraint fails.[23]

[22]Note that although a contingentist would not be in a position to model such a function as a collection of ordered-pairs, she can still think that it corresponds to a well-defined condition. And there is no reason she can't deploy that condition it in the context of a modal clause, within which its inputs might be specified to be individuals that do not, in fact, exist, or individuals that could not exist together.

[23]Are there reasons to favor the Propositional Function View over a variant of the Property View that keeps P1 and P2 in place but allows for a property to be true of an individual at a world even if the individual fails to exist at the world? Although the theoretical work that will occupy us here could be carried out using either of these views, I think there is a dialectical ad-

9 Model Theory

In this section I will outline a model theory for each of the Property and Propositional Function Views. (I shall assume, for simplicity, that we are working within a necessitist metatheory, while remaining neutral about whether the object-language is interpreted so as to vindicate necessitism or contingentism. In section 14, I will return to the question of what can be achieved when one works within a contingentist metatheory.)

Object Language We focus on a second-order modal language, $L^{\diamond}$, which has been enriched with a lambda operator. (See Appendix B for details.)

One can simulate the behavior of plural quantification in $L^{\diamond}$, so we'll help ourselves to plurals whenever convenient. (See Appendix C.)

Models A *model* is a cuadruple $\langle W, D, I, @\rangle$.

W is a set of worlds. Each world $w \in W$ has a *domain*, $\bar{w}$, which is assumed to be a subset of D, the model's "outer domain". I is function that assigns a semantic value to each atomic predicate P^n_i $(i, n \in \mathbb{N})$. $@ \in W$ is the actualized world.

A *contingentist model* is a model $\langle W, D, I, @\rangle$ such that $\bar{w} \neq \bar{v}$ for some $w, v \in W$.

Semantic Values We let $[\![\phi]\!]^{\sigma}_{W,D,I}$ be the *semantic value* of the predicate ϕ at the model $\langle W, D, I, @\rangle$ with respect to a variable assignment σ. (See Appendices B and F for details.)

When ϕ is an n-place predicate, $[\![\phi]\!]^{\sigma}_{W,D,I}$ is a set of $(n+1)$-tuples $\langle w, z_1, \ldots, z_n\rangle$ $(z_1, \ldots, z_n \in D$, $w \in W)$.

In the special case in which ψ is a formula, $[\![\phi]\!]^{\sigma}_{W,D,I}$ is a set of possible worlds, which we shall think of as modeling a *proposition*.[24]

vantage to working with propositional functions rather than properties. Unlike propositional functions, properties are used widely across philosophy (and, to some extent, outside philosophy). So there is a temptation to think of them as constrained independently of their role within semantics. For instance, a critic might think there is an independent fact of the matter about whether properties—the "real" properties that are "out there"—satisfy P3. I am anxious to steer clear of such debates in the present context. Propositional functions help me do so because it is easier to think of them as theoretical postulates, whose features are to be stipulated rather than discovered. (I am indebted to Bruno Jacinto for useful discussion of this point.)

[24]Strictly, $[\![\phi]\!]^{\sigma}_{W,D,I}$ is a set of 1-tuples of worlds. But we identify 1-tuples with their sole elements for ease of exposition.

The semantic value function $[\![\ ]\!]^{\sigma}_{W,D,I}$ will be interpreted differently by the Property and Propositional Function Views.

A friend of the Property View will think of $[\![\phi]\!]^{\sigma}_{W,D,I}$ as representing a property, and will take $\langle w, z_1, \ldots, z_n\rangle \in [\![\phi]\!]^{\sigma}_{W,D,I}$ to represent the fact that $z_1, \ldots, z_n$ exemplify that property at w.

In contrast, a Friend of the Propositional Function View will think of $[\![\phi]\!]^{\sigma}_{W,D,I}$ as representing a propositional function, and will take $\langle w, z_1, \ldots, z_n\rangle \in [\![\phi]\!]^{\sigma}_{W,D,I}$ to represent the fact that the propositional function assigns to $z_1, \ldots, z_n$ a proposition that is true at w. Accordingly, $[\![\phi]\!]^{\sigma}_{W,D,I}$ represents the propositional function that maps z^1, ... z^n to the proposition represented by $\left\{w \in W : \langle w, z^1, \ldots, z^n\rangle \in [\![\phi]\!]^{\sigma}_{W,D,I}\right\}$.

Our model theory represents this disagreement as a difference in the *predication clause*, which determines the semantic value of $[\![\ulcorner \psi x \urcorner]\!]^{\sigma}_{W,D,I}$ on the basis of: (a) the semantic value of $[\![\psi]\!]^{\sigma}_{W,D,I}$ and (b) the referent of "x" (relative to variable assignment σ).

Consider the special case in which ψ is a one-place predicate. On the Property View, $[\![\psi]\!]^{\sigma}_{W,D,I}$ is a one-place property, which can only be exemplified by an individual at a world if the individual exists at the world. So the following is an adequate characterization of the predication clause. (The label is meant to evoque negative free logic.)

PC^{-}: $[\![\ulcorner \psi\, x \urcorner]\!]^{\sigma}_{W,D,I} = \left\{w \in W : \langle w, \sigma(x)\rangle \in [\![\psi]\!]^{\sigma}_{W,D,I} \wedge \sigma(x) \in \bar{w}\right\}$

Intuitively, $\ulcorner \psi x \urcorner$ is true at world w (relative to a variable assignment σ) iff: the property expressed by ψ is exemplified by the referent of "x" and (therefore) the referent of "x" is in the domain of w.

On the Propositional Function View, in contrast, $[\![\psi]\!]^{\sigma}_{W,D,I}$ is a one-place propositional function f and the proposition $f(z)$ can be true at a world even if z fails to exist at the world. So friends of the Propositional Function View would characterize the predication clause by weakening PC^{-} so as to leave out the requirement that $\sigma(x) \in \bar{w}$. What one gets is the following. (The label is meant to evoque positive free logic.)

PC^{+}: $[\![\ulcorner \psi\, x \urcorner]\!]^{\sigma}_{W,D,I} = \left\{w \in W : \langle w, \sigma(x)\rangle \in [\![\psi]\!]^{\sigma}_{W,D,I}\right\}$

Intuitively, $\ulcorner \psi x \urcorner$ is true at world w (relative to a variable assignment σ) iff: the proposition that results from applying the propositional function expressed by ψ to the referent of "x" is true at w.

By adopting PC$^+$, friends of the Propositional Function View can make room for both transcendental and non-transcendental n-place predicates ($n > 0$). For example, they can count "$=^e$" and "$\approx^e$" as transcendental predicates by selecting semantic values such that:

($=^e$) $\ulcorner x =^e y \urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $\sigma(x) = \sigma(y)$.

($\approx^e$) $\ulcorner xx \approx^e yy \urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $|\sigma(xx)| = |\sigma(yy)|$.

And they can treat a predicate like "x is happy" as non-transcendental by selecting a semantic value such that:

(☺) $\ulcorner x$ is happy$\urcorner$ is true at world $w \in W$ (relative to a variable assignment σ) iff: $\sigma(x)$ is happy at w.

In addition, PC$^+$ delivers β-equivalence for arbitrary models and a failure of the Being Constraint for contingentist models. And it allows $L^\diamond$ to express external one-one correspondence.

PC$^-$, in contrast, delivers the Being Constraint for arbitrary models and a failure of β-equivalence for contingentist models. It also delivers the familiar expressive limitations. In particular, it entails that $L^\diamond$ is unable to express external one-one correspondence in spite of containing full second-order variables (Appendix E.5, Theorem 3).

10 Arithmetic

In his discussion of transcendental truths, Fine mentions an intriguing possibility: that arithmetical truths might be counted as transcendental. Here I will defend a related view. I will argue that every sentence of the language of (pure) arithmetic can be regimented as a transcendental formula.

Before turning to the proposal itself, I would like to say a few words about why the view would be attractive if true.

Benacerraf (1973) famously set forth a dilemma. We must either reject a standard semantics for mathematical discourse, according to which mathematical terms refer to mathematical objects, or explain how one could come to know that the world turns out to contain mathematical objects, given that such objects are causally inert.

Benacerraf's Dilemma rests on a crucial presupposition. It assumes that the truth of a sentence like "$2+2=4$", standardly interpreted, imposes non-trivial demands on the world—and, in particular, the non-trivial demand that the world contain numbers. Notice, for example, that the causal inertness of mathematical objects can only be an obstacle to mathematical knowledge if their existence is a non-trivial affair. If "$2+2=4$" is transcendentally true—if its truth imposes no demands on the world—then all it takes to verify its truth is to identify its truth conditions. There is no need to verify that any requirements are satisfied by causally inert portions of the world.

A transcendental arithmetic—a version of arithmetic on which all pure sentences are transcendental—could therefore provide us with an attractive way out of Benacerraf's Dilemma. But how might one defend such a view?

Consider the cardinality operator $\#$, which takes one-place predicates as arguments. On the Propositional Function View, one can set forth a metatheory that takes $\ulcorner \#(F) \urcorner$ to refer to the number of the Fs and delivers the following semantic clauses:[25]

> $\ulcorner \#(F) =^{e} \#(G) \urcorner$ is true at w iff:
>
> (the number of possible individuals z such that: at w, z is F) = (the number of possible individuals z such that: at w, z is G).
>
> $\ulcorner \#(F) =^{i} \#(G) \urcorner$ is true at w iff:
>
> (the number of possible individuals z such that, at w, z is F) = (the number of possible individuals z such that, at w, z is G); and that number is in the domain of w.

On the resulting view, the truth of $\ulcorner \#(F) =^{e} \#(G) \urcorner$ imposes a fairly modest demand on w: roughly speaking, the demand that, at w, there be just as many Fs as G. In particular, one's metatheory will prove the following, given reasonable assumptions:[26]

> $\ulcorner \#(F) =^{e} \#(F) \urcorner$ is true at w iff: $\top$

[25] As noted above, I am assuming, for simplicity, that we are working within a necessitist metatheory. Rayo 2008 develops a version of transcendental arithmetic that is compatible with a contingentist metatheory, albeit at the cost of complicating the semantics.

[26] It is enough to assume, for example, that one's metatheory proves Hume's Principle, and therefore: "(the number of possible individuals z such that: at w, z is F) = (the number of possible individuals z such that: at w, z is F)".

So one will count $\ulcorner \#(F) =^e \#(F) \urcorner$ as transcendentally true. In other words: one will see its truth as imposing no demands on w—not even the demand, concerning the referent of "$\#(F)$", that it be in the domain of w.

The truth of $\ulcorner \#(F) =^i \#(G) \urcorner$, in contrast, imposes much more stringent requirements on w. To the demand imposed by the truth of $\ulcorner \#(F) =^e \#(G) \urcorner$, it adds the requirement, concerning the referents of "$\#(F)$" and "$\#(G)$", that they exist at w. So, $\ulcorner \#(F) =^i \#(F) \urcorner$ won't count as transcendental.

We have seen that friends of the Propositional Function View are in a position to make sense of arithmetical identities of two different kinds: $\ulcorner \#(F) =^e \#(G) \urcorner$ and $\ulcorner \#(F) =^i \#(G) \urcorner$. Which ones should they use?

It all depends on what they hope to achieve. If the aim is to capture the meanings of English sentences like "the number of the forks equals the number of the spoons", the answer should presumably be guided by natural language semantics.[27] But suppose instead that they are interested in a project of regimentation. More specifically, they wish to set forth a language that is rich enough to allow for scientific applications of arithmetic while remaining neutral on its relationship to natural language, and they wish to develop an epistemology for the subject-matter of the new language. In this scenario, there is a definite advantage to regimenting arithmetical identities using $\ulcorner \#(F) =^e \#(G) \urcorner$ rather than $\ulcorner \#(F) =^i \#(G) \urcorner$. For whereas the latter gives rise Benacerraf's Dilemma, the former does not. Recall, for example, that the truth $\ulcorner \#(F) =^e \#(F) \urcorner$ requires nothing of the world. Since there can be no mystery about how a subject could to know that the world is such as to satisfy such a requirement, explaining how a subject is in a position to know that $\#(F) =^e \#(F)$ would be akin to explaining how she is in a position to know that $\phi \vee \neg\phi$.[28]

It is worth emphasizing that the claim that $\ulcorner \#(F) =^e \#(F) \urcorner$ is transcendentally true is made from the point of view of a metatheory that takes substantial arithmetical principles for granted. So the project I have just described is unlikely to win over an interlocutor who is unwilling to engage in mathematical practice. But the fact that our proposal won't win over every interlocutor doesn't mean that it is pointless. Suppose that—by presupposing arithmetic—one is able to characterize a metatheory for a regimented arithmetical language on which arithmetical truths turn out to be transcendental. One can use that result to understand how ordinary subjects are in a position to know arithmeti-

[27]For this sort of approach, see Hofweber 2016. For linguistically-informed criticism, see Balcerak Jackson 2013 and Moltmann 2013. For philosophical discussion, see Rayo 2017*a*.

[28]For further discussion, see Rayo 2013, ch. 4.

cal truths, as expressed by the regimented language. (And if one likes, one can throw away the ladder, by reformulating one's metatheory in the regimented language.)

11 Transcendental Quantifiers

If the project of characterizing a transcendental arithmetic is to succeed, transcendental identity statements are not enough. We also need *transcendental quantifiers*.[29]

To see how such quantifiers might work, it is useful to start by considering the modal sentence "$\exists n \diamond (\text{Prime}(n))$". On a standard possible-worlds semantics, its truth imposes a two-part requirement on the relevant space of possible worlds. First, the existential quantifier demands that there be a suitable value $\mathfrak{n}$ for the variable "n" in the domain of the actual world; second, the occurrence of "$\diamond$Prime(n)" demands that some world witness the truth of "Prime(n)" with respect to $\mathfrak{n}$, and therefore include $\mathfrak{n}$ in its domain.

These two requirements are importantly different: the former arises from the semantics' treatment of quantification; the latter from its treatment of predication. Since the Property View and the Propositional Function View are both claims about the nature of predication, they each take a stand on the second requirement: the Property View sees it as appropriate, the Propositional Function View as excessive. But neither of the views addresses quantification, so neither of them takes a direct stand on the first requirement.

To see how the first requirement might be addressed, consider two contrasting ways of specifying the truth-conditions of arithmetical quantifiers:

(D$^-$) $\ulcorner \exists n\ \phi \urcorner$ is true at w (relative to variable assignment σ) iff:

> there is a number $\mathfrak{n}$ *in the domain of* w such that ϕ is true at w relative to $\sigma[\mathfrak{n}/n]$.[30]

(D$^+$) $\ulcorner \exists n\ \phi \urcorner$ is true at w (relative to variable assignment σ) iff:

> there is a number $\mathfrak{n}$ such that ϕ is true at w relative to $\sigma[\mathfrak{n}/n]$.

According to (D$^+$), the truth of $\ulcorner \exists n\ \phi \urcorner$ imposes a modest demand on w; namely: that it be such as to verify ϕ with respect to some assignment of value to "n",

[29]For details, see Rayo 2015.

[30]As usual, $\sigma[\mathfrak{n}/n]$ is the variable assignment that assigns $\mathfrak{n}$ to n and is otherwise like σ.

regardless of whether that value is a member of w's domain. So, for instance, (D$^+$) entails that "$\exists n\,(n =^e n)$" is transcendental: its truth imposes no demands on the world.

(D$^-$) characterizes a more stringent demand. To the demand specified by (D$^+$), it adds the requirement that the value assigned to "n" be in the domain of w. So, for instance, (D$^+$) entails that "$\exists n\,(n =^e n)$" is *not* transcendental: its truth demands of w that it have a non-empty domain.

Although the Propositional Function View does not entail (D$^+$), its proponents have reason to prefer it over (D$^-$). For, as I noted in section 10, they are in a position to count $\ulcorner \#(F) =^e \#(F) \urcorner$ as transcendentally true. But one would expect $\ulcorner \#(F) =^e \#(F) \urcorner$ to entail $\ulcorner \exists n\,(n =^e n) \urcorner$ (assuming "$\#(F)$" has a referent). So on the plausible assumption that truth conditions respect logical entailments,[31] friends of the Propositional Function View should count $\ulcorner \exists n\,(n =^e n) \urcorner$ as transcendentally true. Since this result is compatible with (D$^+$) but not (D$^-$), they have reason to prefer (D$^+$) over (D$^-$).

Our discussion of transcendental quantifiers has so far focused on arithmetical quantifiers. But the idea is easily generalized:

> A (frst-order) existential quantifier $\exists x$ is *transcendental* iff the following is a theorem of one's semantic theory for L:
>
> > $\ulcorner \exists x\, \phi \urcorner$ is true at world $w \in W$ (relative to σ) iff: there is a z such that Θ and such that ϕ is true at w (relative to $\sigma[z/x]$).
>
> where Θ is a formula in which neither "w" nor "W" occur free.

In addition, when t is a term and x is a variable that can take t's place, we shall say that t is transcendental iff $\ulcorner \exists x\,(x = t) \urcorner$ is transcendentally true. Assuming proponents of the Propositional Function View are in a position to count $\ulcorner \#(F) =^e \#(F) \urcorner$ as transcendentally true, and assuming they take $\ulcorner \#(F) =^e \#(F) \urcorner$ to entail $\ulcorner \exists n\,(n =^e \#(F)) \urcorner$ (for "$\#(F)$" nonempty), they will be in a position to count '$\#(F)$" as a transcendental term.

[31]More specifically: the demands imposed on the world by the truth of a sentence are never more stringent than the demands imposed by the truth of a sentence that entails it.

12 Transcendental Objects

We have seen that proponents of the Propositional Function View have reason to count transcendental terms and quantifiers as legitimate. But we have yet to motivate a *conception of object* that makes room for transcendental terms and quantifiers. That will be the aim of the present section.

The issue is urgent because transcendental terms and quantifiers are at odds with a conception of object that is natural and (I suspect) broadly accepted. On an especially simple version of the view, one takes a possible world to consist of a domain of objects together with an assignment of properties to those objects. One then offers the following characterization of objecthood: to be an object is to be a member of the domain of the actual world. (A necessitist might add: "to be a possible object is to be a member of the domain of some possible world".)

On such a view, there is no room for transcendental terms or quantifiers. Take the case of quantifiers. If the arithmetical quantifier "$\exists n$" is transcendental, "$\exists n\, \top$" will be counted as transcendentally true by any metatheory that includes arithmetic. But on pain of failing to treat "$\exists n$" as a genuine quantifier, we should think that "$\exists n\, \top$" can only be true if *there is* a number (and therefore an object). And on the view we are considering, to be an object is to be a member of the domain of the actual world. So "$\exists x\, \top$" can only be true if the actual world has a non-empty domain. So its truth imposes a non-trivial domain on the world. So it cannot be transcendental.

What sort of conception of object would make room for transcendental terms and quantifiers? The view I would like to consider boasts a distinguished lineage, going back to Frege (1884).[32] Developing the view in detail would take us too far afield, but a rough outline should suffice for present purposes. On the version of the proposal I prefer, one starts with a form of realism: the view that there is a definite fact of the matter about how the world is. But one goes on to individuate ways for the world to be in a very coarse-grained way. One assumes, in particular, that a single way for the world can be fully and accurately described using sentences with very different semantic structures. For example, the way for the world to be that is fully and accurately described by saying "Socrates died" might also be fully and accurately described by saying "Socrates's death took place"—or "dying occurred Socratishly". To use a Fregean metaphor:

[32]The list includes Dummett (1981), Wright (1983), Rosen (1993), Stalnaker (1996), Burgess (2005), Sidelle (2002), Glanzberg (2004), Chalmers (2009), Eklund (2009), Hirsch (2010), Thomasson (2015), and Linnebo (2018). My preferred variant of the view is developed in Rayo (2013), Rayo (2017*b*), and, especially, Rayo (forthcoming).

a single way for the world to be can be "carved up" in different ways by using different sentences to describe it (1884, §64).

Can the view be stated non-metaphorically? The basic point, as I understand it, is that in using different sentences to describe a single way for the world to be, one highlights different *aspects* of that way for the world to be. I can be a little more specific.

For ϕ a sentence, let us use $\ulcorner[\phi]\urcorner$ as a name for the way for the world to be that is described by ϕ, and let say that a *feature of the reality* is a way for the world to be such that the world is, in fact, that way. Now consider a particular feature of reality: [Socrates died]. When we describe it using the sentence "Socrates died", we highlight an aspect of that feature of reality that it has in common with other features of the world that we describe using sentences of the form "Socrates is F" (e.g. [Socrates is wise], [Socrates is snub-nosed]). The highlighted aspect of these features of reality is Socrates.

Now recall that on our coarse-grained individuation of ways for the world to be, we can also choose to describe [Socrates died] using "Socrates's death took place". In doing so, we highlight an aspect of [Socrates died] that it has in common with other features of the world that we describe using sentences of the form"Socrates's death was F" (e.g. [Socrates's death was tragic], [Socrates's death was self-administered]). The highlighted aspect of these features of reality is Socrates's death.

In general, to be an object is to be an aspect of a way for the world to be that is rendered salient by an (open or closed) singular term—and, more specifically, by the collection of ways for the world to be that are described in our language using that singular term.

Note that this conception of object does not presuppose that there is a "metaphysically salient" way of dividing a given feature of reality into aspects, a division that is salient independently of the network of connections between ways for the world to be that is delivered by a compositional language. Think of an object as a *node* in a network of connections between ways for the world to be. Just like there is no sense to be made of the intersection between two roads independently of the roads themselves, so there is no sense to be made of a node in a network of connections independently of the connections themselves.

The analogy with intersections can be carried further. Suppose a construction company is planning to build a road network with plenty of intersecting roads. If the company has enough construction materials to build all the right roads in all the right places, it would be wrong-headed for an executive to worry that the plan might not be completed because of shortages of a further construction material: "intersections". That's not how intersections work. If you build intersecting roads, you *thereby* get intersections.

On the proposed conception of object, something similar is true of objects. Suppose a community sets out to interpret a language, by assigning (coarse-grained) truth-conditions to each of its sentences. As long as their assignment respects logical structure,[33] they need not worry about whether the world will supply the right objects to serve as referents for the language's singular terms. Should they characterize a feature of reality q using a sentence $\ulcorner F(a) \urcorner$, the singular term a will *thereby* refer to whatever aspect of q is rendered salient by the collection of ways for the world to be described by $\ulcorner F(a) \urcorner$, $\ulcorner G(a) \urcorner$, $\ulcorner H(a) \urcorner$, etc. There's no need for the additional requirement that the world supply a suitable stock of objects from which a's reference might be drawn. (I'm glossing over some important details here—see my "Thin Objects" for the full story.)

A nice feature of the resulting conception of object is that it makes room for "transcendental objects": objects whose existence imposes no demands on the world. Transcendental objects are just aspects of a *trivial* feature of reality. For instance, the reference of $\ulcorner \#(F) \urcorner$ is an aspect of $[\#(F) =^e \#(F)]$, which must be a trivial feature of reality if $\ulcorner \#(F) =^e \#(F) \urcorner$ is to count as transcendentally true.

In general, one can think of transcendental terms as referring to transcendental objects and of transcendental quantifiers as ranging over transcendental objects. This allows us to bring together two seemingly inconsistent claims. On the one hand, one can hold onto the view that transcendental quantifiers are genuine quantifiers, ranging over genuine objects. (In particular, one is able to read the arithmetical quantifier "$\exists n$" as saying that *there is* a number, in the ordinary sense of "there is".) On the other hand, one can claim that the truth of a sentence like "$\exists n \top$" demands nothing of the world, since the relevant quantifier ranges over transcendental objects, whose existence demands nothing of the world.

[33]More specifically: if ϕ is a logical consequence of ψ, the truth-conditions assigned to ϕ should be no more demanding than the truth-conditions assigned to ψ. See Rayo forthcoming for further details.

13 Trans-World Objects

In this section I would like to consider the notion of a *trans-world* object: an object that can only be singled out by reference to counterpossibles.[34] For example, in order for the following sentence is to be true, "$\{x, y\}$" must refer to a trans-world object:

Trans-World Sets $\Box\forall x\Box\forall y\,(\text{CounterPoss}(x,y) \rightarrow (\text{Two-membered}\,(\{x,y\})))$

Read: Necessarily, for any individual x, necessarily, if an individual y is such that it impossible for x and y to exist together, then $\{x, y\}$ has two members.

> $\ulcorner$CounterPoss$(x, y)^\urcorner$ abbreviates $\ulcorner\neg\Diamond\,(\exists u \exists t\,(u = x \wedge t = y))^\urcorner$; "Two-membered" is an atomic predicate, which is intended to apply to all and only two-membered sets.

In order to deny that there is sense to be made of trans-world objects, one must deny that a sentence like [Trans-World Sets] could be true. But it seems to me that that would be an awkward result. Suppose there is a knife k whose blade could have been used to make a different knife, k', which couldn't have coexisted with k. Would it really be nonsense to talk about the set $\{k, k'\}$, and describe it as having two members? I can certainly imagine a metaphysical picture that rules out talk of $\{k, k'\}$ as nonsense. But such a picture would come at a cost, since it would restrict our ability to use set-theoretic resources in modal contexts.

On the Property View one is committed to such a cost, since one is committed to the Being Constraint, which entails that [Trans-World Sets] is false (assuming there could be counterpossibles).[35] On the Propositional Function View, in contrast, there is room for accommodating trans-world objects.[36]

[34] For further discussion, see Fine's postscript to Prior & Fine 1977 and Salmon 1987, ft. 55.

[35] *Proof:* the Being Constraint entails that "Two-membered $(\{x, y\})$" can only be true at a world w (relative to a variable assignment σ) if the denotation of "$\{x, y\}$" (relative to σ) exists at w. But the truth of "Counterpossibles(x, y)" guarantees that the values of "x" and "y" (relative to a suitable σ) cannot both exist at w. And—on the assumption that a set can only exist at a world if its members exist at that world—the denotation of "$\{x, y\}$" (relative to σ) cannot exist at w.

[36] The crucial observation is that one can treat "Two-membered" as expressing the propositional function f such that, for any x, $f(x) = \top$ if x is a two-membered set and $f(x) = \bot$ otherwise. (Note that the quantifier "for any x" above be understood as transcendental, and be given a semantic clause along the lines of (D^+) from section 11, as opposed to (D^-).)

I do not claim to have shown that there are trans-world objects, or that the existence of such objects follows from the Propositional Function View. I claim only that the Propositional Function View might be used to develop a broader semantic picture on which trans-world objects might be accommodated. In the next section, we will see that such objects can play an important theoretical role.

14 Semantics

In this final section, I'd like to highlight some connections between the Being Constraint and the project of giving a semantics for modal languages while presupposing a contingentist metatheory.

I'll start by distinguishing between a few different ways in which one might attempt to shed light on the workings of a language, from a semantic point of view. At one end of the spectrum, there is a sentence-level translation from the object language into one's metalanguage. A translation of this kind can be used to state the truth-conditions of object-language sentences in a language one understands. But it doesn't do much more than that. In particular, there is no guarantee that it will shed light on the manner in which the truth-conditions of object-language sentences depend on the meanings of their constituent parts.

Further along the spectrum there is a *compositional* translation-method, which proceeds by translating individual lexical items of the object language into metalinguistic analogues. A translation-method of this kind allows the theorist to ascertain not only the truth-conditions of an object-language sentence but also the meanings of its constituent parts. On the other hand, the translation's ability to shed light on the way in which the truth conditions of object-language sentence depend on the meanings of their constituent parts will be no better than the theorist's understanding of the way in which the truth-conditions of sentences in her own language depend on the meanings of their constituent parts.

At the other end of the spectrum lies the project of setting forth a *proper semantics*, which assigns to each lexical item of the object-language a semantic value: a theoretical entity that is designed to shed light on the way in which an expressions constituents contribute to its meaning. Suppose, for example, that one's object-language is the simple language of the Boolean propositional calculus. One can develop a proper semantics for such a language by assigning to each sentence a set of possible worlds and to each of the connectives "$\wedge$",

"$\vee$", and "$\neg$", the set-theoretic operations of intersection, union, and complementation, respectively. These assignments can then be used to derive a fully explicit account of the way in which a sentence's truth-conditions depend on the meanings of its constituent parts.

Notice, moreover, that our proper semantics creates a framework for generalization. For example, it allows the theorist to generalize her treatment of "$\wedge$", "$\vee$", and "$\neg$", by claiming that, in general, an n-place propositional connective expresses a function that assigns to each n-tuple of sets of possible worlds a set of possible worlds. Such generalizations, in turn, allow the theorist to develop a *model theory*: an account of all possible ways of interpreting the object-language's vocabulary, while respecting its semantic structure. And once a model theory is in place, the theorist might use it to characterize a *logic* for the object language, by identifying sentences that will count as true on any interpretation of its (non-logical) vocabulary.

Let us now return to our main focus: a contingentist treatment of modal languages. To fix ideas, consider:

Sister $\Diamond \exists x\, (\text{Sister}\,(\text{AR}, x) \wedge \lambda y\, (y = x)\, x)$

> [*Read:* it might have been the case that there is an individual who is my sister and is such as to be identical to it.]

Which semantic value should we assign to the predicate "$\lambda y\, (y = x)$" (relative to a variable assignment σ)? Here is one straightforward answer: it is the function f that assigns to each possible individual the set of worlds at which that individual is identical to σ("x"). This function is certainly well-defined from the point of view of a necessitist, who is in a position to quantify over possible individuals. But it is not clear that the contingentist is in a position to characterize f. Firstly, she denies that one can quantify over "merely possible" individuals. So it is not clear that she is in a position to avail herself of a suitable domain for f. Secondly, since I do not, in fact, have a sister, the contingentist can be expected to deny that there is something that might have been my sister and therefore that there is an appropriate value for "x". In the absence of such a value, f will be ill-defined on any domain.

As a result, the most straightforward route to a proper semantics for modal languages is not available to the contingentist. This does not, of course, show that alternative approaches couldn't work. Back in section 4, I mentioned an alternative due to Stalnaker and noted that it does not immediately deliver the result that "all intermediate values in the composition are actual things." But I have done nothing to rule out other approaches, and will not attempt to do so here. I will also set aside the project of developing a contingentist metatheory by setting forth a semantics that is not taken at face value (Stalnaker 2012, Appendix C). Instead, I will highlight an advantage of the Propositional Function View over the Property View.

I will proceed by considering the prospects of developing a "semi-disquotational" semantic theory for modal languages within a contingentist metatheory. A semi-disquotational semantics lies somewhere towards the middle of our spectrum. It is more illuminating than a compositional translation of the object-language into the metalanguage, but not quite as illuminating as a proper semantics. It does, however, have the advantage of being available to a contingentist.

In two important respects, a semi-disquotational semantics works just like a standard Kripke-style semantics for modal languages: (1) modal operators are treated like quantifiers ranging over possibilities, and (2) the semantic value of an n-place predicate ϕ (relative to a variable assignment) is a function that assigns to each sequence of individuals $\langle z_1, \ldots, z_n \rangle$, the set of possibilities at which ϕ applies to $z_1, \ldots, z_n$ (relative to that variable assignment). But there is also a crucial difference. In a standard semantics, semantic value are built from a single stock of individuals (one's "outer domain") and a single stock of possibilities (one's space of possible worlds). In a semi-disquotational semantics, there is no single stock of individuals or possibilities which is available to the theorist to characterize the semantic values of her object-language. Instead, she starts out with a initial stock of individuals (those that, in fact, exist) and an initial space of possibilities (those that can be characterized using only individuals that, in fact, exist). She then introduces talk of merely possible individuals, and possibilities that can only be characterized with respect to such individuals *by working within the scope of suitable modal clauses*. Here is an example:

Sister $\Diamond \exists x\, (\text{Sister}\,(\textsc{ar}, x) \wedge \lambda y\, (y = x)\, x)$

> *Proper Semantics statement of truth-conditions:*
> [Sister] is true iff: there is a possibility π (in one's space of possible worlds) and an individual z (in one's outer domain) such that: (1) z is in the domain of π, (2) according to π, z is my sister, and (3) z is such as to be identical with z.
>
> *Semi-disquotational statement of truth-conditions:*
> [Sister] is true iff: there is a possibility π (in an initial space of possibilities) such that, *according to* π, there is an individual z such that

the following holds: (1) z is my sister and (2) z is such as to be identical to z.

The crucial difference between the two clauses involves the placement of the quantifier "there is an individual z". In the semi-disquotational version, but not in the proper semantics version, the quantifier occurs within the scope of the modal clause "according to π", which allows it to range over individuals that exist according to π, whether or not they, in fact, exist.

A semi-disquotational semantics uses an analogous strategy to characterize a semantic value for "$\lambda y\,(y = x)$" (relative to a given variable assignment). Rather than characterizing the relevant function outright, as a proper semantics would, the function is characterized within the scope of suitable semantic clauses. One uses a semantic clause of the form "according to π" to introduce a suitable value for "x" and additional clauses to ensure that the function's domain includes any merely possible individuals that are needed to secure the desired results.

The details of a semi-disquotational semantics are somewhat delicate and I won't go over them here.[37] It is worth pointing out, however, that a disquotational semantics must rely on trans-world objects, in the sense of section 13. Consider, for example:

Counterpossibles $\exists x \Diamond \exists y\,(\text{Counterpossibles}(x, y) \wedge \lambda z(z = x \vee z = y)y)$

Read: there is an individual x such that there might have been an individual y, such that it is impossible for x and y to exist together and such that y is such as to be either identical to x or identical to y.

In order to count this sentence as true, our semantics must interpret the one-place predicate "$\lambda z(z = x \vee z = y)$" with respect to values for "x" and "y" that can't both exist according to single possibility, which means that the semantic value for "$\lambda z(z = x \vee z = y)$" should be expected to be a trans-world object.

Since the Property View makes no room for trans-world objects (section 13), it is unlikely that it could be used as the basis for a semi-disquotational semantics. To my mind, this gives us an additional reason to take the Propositional Function View seriously.

[37]They are spelled out in Rayo 2020.

15 Conclusion

The contingentist faces a dilemma: she cannot embrace both the Being Constraint and β-necessity. According to a natural proposal—the Property View—she should keep the Being Constraint at the expense of β-necessity.

We have seen, however, that the Property View leads to expressive limitations. Some of these limitations are straightforward: one cannot express a notion of "external" one-one-correspondence, for example. Others are subtler but, to my mind, more important: the Property View is incompatible with transcendental n-place predicates ($n > 0$) and transcendental quantifiers.

Fortunately, there are alternatives to the Property View. We have seen that one such alternative—the Propositional Function View—enjoys independent plausibility and makes room for a transcendental arithmetic, which is able to escape Benacerraf's Dilemma. The Propositional Function View also makes room for a quasi-dispositional semantics, which can useful to a contingentist who wishes to develop a semantic theory for modal languages.[38]

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