Carroll's Regress, Revisited

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Carroll's Papers in *Mind*

- Late in his career, Lewis Carroll published two papers on logic in the academic journal *Mind*. The one we are interested in was published in 1895.
- Interestingly, he published these papers under the Lewis Carroll name. Normally, his mathematical work was published under his real name, Charles Dodgson.
- Both papers presented logical puzzles and were written as humorous dialogues.

Russell on Carroll

Bertrand Russell remarked:

> *I think he was very good at inventing puzzles in pure logic. When he was quite an old man, he invented two puzzles he published in a learned periodical,* Mind, *to which he didn't provide answers. And the providing of answers was a job, at least so I found it.*

Russell on Euclid

It has been customary when Euclid, considered as a text-book, is attacked for his verbosity or his obscurity or his pedantry, to defend him on the ground that his logical excellence is transcendent, and affords an invaluable training to the youthful powers of reasoning. This claim, however, vanishes on a close inspection. His definitions do not always define, his axioms are not always indemonstrable, his demonstrations require many axioms of which he is quite unconscious. . . . [T]he value of his work as a masterpiece of logic has been very grossly exaggerated.

Russell on Aristotle

I conclude that the Aristotelian doctrines with which we have been concerned in this chapter are wholly false, with the exception of the formal theory of the syllogism, which is unimportant. Any person in the present day who wishes to learn logic will be wasting his time if he reads Aristotle or any of his disciples.

The Tortoise and Achilles

We will discuss his paper, "What the Tortoise Said to Achilles."

What *Did* the Tortoise Say to Achilles?

The dialogue centers around the following logically valid argument.

(A) Things that are equal to the same are equal to each other.
(B) The two sides of this triangle are things that are equal to the same.
(Z) The two sides of this triangle are equal to each other.

The Tortoise says, "[Z] follows logically from A and B, so that any one who accepts A and B as true, must accept Z as true?"

Achilles agrees to this.

What *Did* the Tortoise Say to Achilles?

The Tortoise continues, "And if some reader had not yet accepted A and B as true, he might still accept the sequence as a valid one, I suppose?" Achilles agrees.

The Tortoise: "And might there not also be some reader who would say 'I accept A and B as true, but I don't accept the hypothetical?' " [The hypothetical is, "If A and B are true, then Z is true."]

Again, Achilles agrees.

What *Did* the Tortoise Say to Achilles?

"And neither of these readers," the Tortoise continued, is as yet under any logical necessity to accept Z as true?" Once more, Achilles agrees.

"Well now, I want you to consider me as a reader of the second kind, and to force me, logically, to accept Z as true."

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"Well now, I want you to consider me as a reader of the second kind, and to force me, logically, to accept Z as true."

"I'm to force you to accept Z, am I?" Achilles said, musingly. "And your present position is that you accept A and B, but you don't accept the Hypothetical:

(C) If A and B are true, then Z must be true.

"That is my present position," said the Tortoise.

"Then I must ask you to accept C," said Achilles.

Adding Premises

The Tortoise says that he will accept C if it is presented explicitly as a premise in the argument, leading to this:

(A) Things that are equal to the same are equal to each other.
(B) The two sides of this triangle are things that are equal to the same.
(C) If A and B are true, then Z must be true.
(Z) The two sides of this triangle are equal to each other.

Achilles now insists that, "If A, B, C are all true, then Z must be true."

The Regress Appears!

But now the Tortoise complains that this is just one more hypothetical and therefore must be given as another premise.

So we now add

(D) If A, B, and C are all true, then Z must be true

as yet another premise in the argument.

The story ends with Achilles adding an infinite sequence of ever more complicated implications without ever forcing the Tortoise to accept the conclusion.

What Do We Learn From Carroll's Regress?

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Pascal Engel (2016) identifies four branches of philosophical logic to which philosophers have applied insights drawn from Carroll's regress:

- The nature of logical inference.
- Knowledge and understanding of logical rules.
- The justification of logical laws and the epistemology of logic.
- The normativity of logic.

Thomson's View

J. F. Thomson (1960) wrote:

> *The extreme eccentricity of the behavior of both of the characters may well make us wonder whether Lewis Carroll knew what he was up to in writing the story. Certainly it cannot be merely taken for granted that he intended to advance some moderately clear thesis or theses about inference but chose to do so in a veiled and cryptic way. It is just as likely that the story is the expression of a perplexity by someone who was not able to make clear to himself just why he was perplexed.*

Thomson's View

The proposition that such-and-such an argument is valid can itself be a premise of an argument. But it cannot be a premise in the argument to which it refers. If you want to say of some argument that it is valid you must be able to say what argument it is that you want to make this claim for. The argument must be identifiable. And the identification must be such as to allow the claim that it is logically valid to be assessed. To assess that claim we need to know what the premises are and what the conclusion is. So the premises must be identifiable independently of the claim that there are enough of them.

Was Carroll Even Talking About Logical Inference?

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But is it really clear this was Carroll's point? The Tortoise seems to accept the validity of the argument. What he denies is the notion that this validity forces him to accept that Z is true. In other words, the issue has to do with the Tortoise's mental state, as opposed to the mechanics of what follows from what.

Knowing Logical Rules vs. Accepting Propositions

Suppose that Joe believes Q on the basis of P. Then there must be something in his mind that links P and Q. We might think that the thing that links them is some other proposition R, which can be represented as $P \rightarrow Q$.

At this point we might naively think that settles things. He believes Q because he believes P and R.

But if R is just another proposition in his mind, then we would need to find an S that links P, Q, and R. In this way we are once again plunged into regress.

Stroud's View

Philosopher Barry Stroud (1979) ran with this. In his view, the regress does not manifest itself in the infinitely long lists of premises we allegedly must add to an argument before we are compelled to accept its conclusion. Instead, it appears when we ask: What does it mean to believe a proposition "on the basis of" some other propositions?

Stroud's View

The moral is that for every proposition or set of propositions the belief or acceptance of which is involved in someone's believing one proposition on the basis of another, there must be something else, not simply a further proposition accepted, that is responsible for the one belief's being based on the other. . . . A list of everything a person believes, accepts, or acknowledges must leave it indeterminate whether any of those beliefs are based on others.

Blackburn's View

In 1995, Simon Blackburn expressed a similar view to Stroud's.

> *The problem Carroll raised can succinctly be put like this: can logic make the mind move? . . . I want to ask whether the will is under the control of fact and reason combined. I shall try to show that there is always something else, something that is not under the control of fact and reason, which has to be given as a brute extra, if deliberation is ever to end by determining the will.*

The Normativity of Logic

Maybe Carroll's regress is teaching us about the normativity of logic. Jan Wieland (2013) recasts the problem in deontic terms. He asks us to consider arguments of the following form:

(A') S intends to ϕ.

(B') S believes that ϕ-ing requires S to ψ.

(Z') Therefore, S ought to intend to ψ.

This argument seems reasonable, but it is not classically valid. So, what else is needed to warrant the inference to Z'? Skipping the details, certain ways of expressing this warrant can lead you into Carroll-like regresses.

Wieland's View

Why should I adopt certain attitudes given certain other attitudes that I have? By many eyes, Carroll's Tortoise has something important to say about this problem. I agree. Yet, her importance does not lie where commentators usually think it lies. [T]he Tortoise does not demonstrate that no extra premises (rather than rules) should be introduced in our reasoning, nor that whatever is to govern our attitudes (premises or rules) should remain external to our attitudes. . . .

Rather, she shows that no solution to this problem should entail that our obligations to adopt certain attitudes depend on additional obligations to adopt further attitudes.

Has the Tortoise Really Been Answered?

Everyone would agree that the modern discussions almost certainly go well beyond anything Lewis Carroll had in mind when writing his story.

However, the fact remains that this all started when the Tortoise leveled a specific challenge. I think the Tortoise could read every word of the literature he inspired and still not feel his challenge had been answered.

I would like to suggest a different interpretation of the story. Rather than lecture the Tortoise about rationality or normativity, we should instead lecture him about logical instrumentalism!

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- If a reason is sought for why the Tortoise ought to align his reasoning with classical logic, then Achilles should say, "Attempts to solve geometric problems with methods other than those sanctioned by classical logic tend to produce undesirable results."

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Presumably, we mean classical logic. And for the sake of argument we can suppose that the Tortoise's intent is to derive correct geometrical theorems.

Clarifying the Challenge

But once we clarify these points, the Tortoise's challenge becomes, "Force me to use classical logic to solve problems in Euclidean geometry."

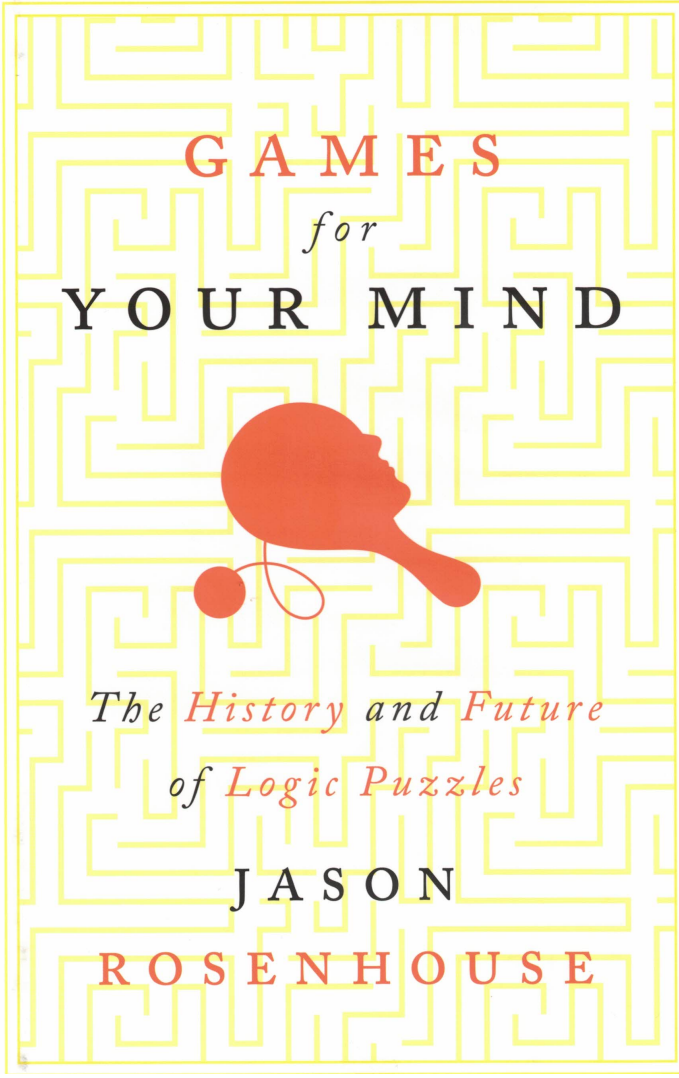
Confronted with this challenge, it is doubtful that anyone would hold forth on rationality or normativity. If he had challenged us to force him to use a hammer to pound a nail, we would not hold forth on the philosophy of hammers. We would just say, "Good luck pounding a nail with any other tool."

And so it is here. We should take an instrumentalist view of logic. The Tortoise is only guilty of using the wrong tool for the job.

For More Information . . .

"Logic is one of the most important subjects in mathematics and Jason Rosenhouse's welcoming style makes it easy to understand. A fantastic achievement!"

James Grime,
Numberphile



Thanks

Thank You!