Numerology and Mathematics

From Plato to Proclus to I. J. Good

Dan Sloughter

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  Now if the body of the universe could have been a plane without depth, one middle term would have sufficed to bind together its companions and itself. But in fact the world was to be a solid, and solids must always be conjoined not by one middle term, but by two. So God inserted water and air between fire and earth, and made them all, as far as was possible, proportional to one another, air being to water as fire to air, and water to earth as air to water.

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  - I.e., bluntness/sharpness, density/tenuousness, immobility/mobility

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  It is like this too in the case of the middle terms between two cubes like 8 *and* 27. 12 *(which is the middle term that is alongside* 8*) will have two sides from* 8 *but only one from* 27*, for* 12 *is* $2 \times 2 \times 3$*. But it is the other way around with* 18*. Since it is* $3 \times 3 \times 2$*, it has two sides from* 27 *(the two* 3*s) but one side from* 8 *— that is, the* 2*. Plato's beliefs about the physical nature of the elements of the universe are thus in concord with mathematics.*

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  After taking up the meaning of these words from a mathematical point of view, it is necessary to be directed to the consideration of their meaning with respect to physical theory. For it is not appropriate either to get bogged down in these mathematical aspects and suspend the rest of the discourse (for the dialogue is about physics) or to ignore the preceding words and examine only that which bears on the sensible realm. Rather it is necessary to bring both aspects together and constantly interweave the mathematical with the physical.

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  - Commentaries on the *Metamorphoses* of Ovid
  - Final section of a 14th-century Old Norse text on the Indo-Arabic numerals and arithmetical algorithms (*algorismus*)

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- The movie π (the search for a number with $216 = 6 \times 6 \times 6$ digits)

Bode's law

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  The immediate practical implication for any one who accepts this conclusion is an increase in the "evolving" probability that a galactic government already exists, and that the earth is a galactic zoo.

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  - Let ζ be the zeta function. For any complex number s, with $\mathrm{Re}(s) > 1$,

$$\frac{1}{\zeta(s)} = \prod_p \left(1 - \frac{1}{p^s}\right) = 1 - \frac{1}{2^s} - \frac{1}{3^s} - \frac{1}{5^s} + \frac{1}{6^s} - \cdots = \sum_{n=1}^{\infty} \frac{\mu(n)}{n^s}.$$

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  - Probability: If for square free integers n, $\mu(n)$ is equally likely to be $+1$ or -1, then $M(x)$ is $O\left(x^{\frac{1}{2}+\epsilon}\right)$ for every $\epsilon > 0$ a. s.

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  Heuristic reasoning is reasoning not regarded as final and strict but as provisional and plausible only, whose purpose is to discover the solution of the present problem.

  We need heuristic reasoning when we construct a proof as we need scaffolding when we erect a building.

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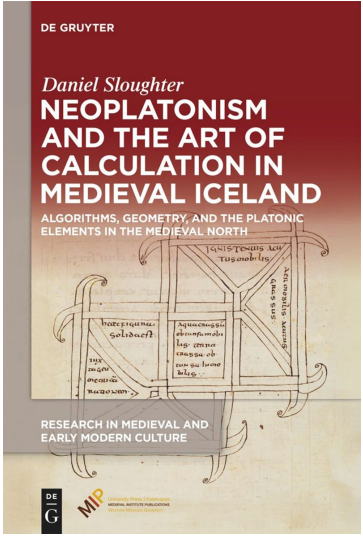
  - E.g., Ising models explain phase transitions in matter with probabilistic models
  - States of matter: solid – liquid – gas – plasma

Reference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