MEDIA clips

(Im)Perfect Storm

> [El Paso, Tex., city council representative] Eddie Holguin stated, "It's taken at face value that we have to pay for a 500-year flood construction project right away, but we still have 490 more years to deal with this and not tax people out of their home in order to get things done."

Source: Gustavo Reveles Acosta, "Mayor Says City Will Review Stormwater Fee," *El Paso Times*, April 25, 2008, pp. A1-2

"Media Clips" aims to offer readers contemporary, authentic applications of quantitative reasoning that are based on print or electronic media. The source material may contain mathematical errors or inaccuracies. We encourage submissions that include questions as well as clips. Submissions must include a complete citation of the source, along with the full text of the cited material. If possible, submissions should also include a copy of the actual clip. Please send submissions to "Media Clips" editors.

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In August 2006, El Paso experienced a series of severe, damaging floods, described as a "500-year-flood." Adapting the National Weather Service's definition (www.weather.gov/glossary) of a 100-year flood, we will define the term *500-year flood* as "the magnitude of flood which can be expected to occur on average with a frequency of once every [500] years." For the following questions, assume that the numbers of 500-year and 100-year floods are binomially distributed.

Consider 500-year floods.

1. What is the probability that a 500-year flood will occur in any given year?
2. Is a 500-year flood guaranteed to occur exactly once during any given 500-year period? Why or why not?
3. What is the probability that a 500-year flood will occur at least once in a given 500-year period?
4. Describe the shape of the graph of the probability distribution for the number of 500-year floods in a given 500-year period.
5. How would you assess the El Paso city council representative's comment quoted in the news story?

Consider 100-year floods.

6. What is the probability that a 100-year flood will occur in any given year?
7. What is the probability that at least five 100-year floods will occur in a given 500-year period?
8. Is the probability that at least one 500-year flood will occur in a given 500-year period the same as the probability that at least five 100-year floods will occur in the same 500-year period?

MEDIA CLIPS solutions

"(Im) Perfect Storm" answers

1. The probability that a 500-year flood will occur in any given year is 1/500 = .002.

2. No. Within a given 500-year period, there may be no 500-year flood, exactly one 500-year flood, or multiple 500-year floods.

   Consider the analogy of six rolls of a fair six-sided die. Although the probability of rolling a 3 is 1/6, within any six rolls the number of 3s may be 0, 1, 2, 3, 4, 5, or 6.

3. The probability that a 500-year flood will occur at least once in a given 500-year period is approximately .63.

   Consider the probability that no floods will occur in a given 500-year period. We have assumed that the number of 500-year floods is binomially distributed, so it follows that the trials are independent, and we can then use the multiplication rule for independent events. Therefore, the probability that no floods will occur is given by $(499/500)^{500} \approx .3675$.

   Because the event of at least one flood is the complement of the event of no floods, the probability that at least one 500-year flood will occur is given by 1 – .3675 = .6325.

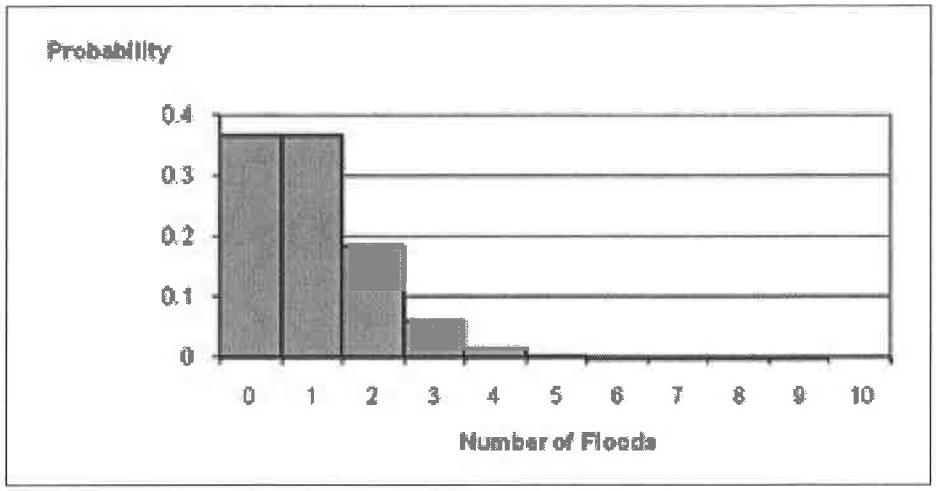

Fig. 1 ["(Im)Perfect Storm"] Binomial distribution (n = 500, p = .002)

4. The graph of the probability distribution for the number of 500-year floods in a given 500-year period is highly skewed right, with the result that mean = median = mode = 1.

   See **table 1** and **figure 1 ["(Im)Perfect Storm")]**. Use a TI-83/84+ to sum the probabilities for more than 10 floods.

   $$1 - binomcdf(500, .002, 10) \approx 9.2 \times 10^{-9} \approx 0.$$

5. Although the El Paso city council representative is understandably concerned about the financial impact of making huge repairs in a short period of time, he is naïve to suggest that the weather will remember its history and wait 500 years before the next 500-year flood.

6. The probability that a 100-year flood will occur in any given year is 1/100 = .01.

Table 1 "(Im)Perfect Storm"
Binomial distribution (n = 500, p = .002)

Number of Floods	Probability
0	.3675
1	.3682
2	.1841
3	.0613
4	.0153
5	.0030
6	.0005
7	7.1E-05
8	8.8E-06
9	9.6E-07
10	9.4E-08

7. The probability that at least five 100-year floods will occur in a given 500-year period is about .56.

   Consider a binomial distribution (n = 500 years, p = .01) for the probability that a 100-year flood will occur in any given year.

   The probability that four or fewer 100-year floods will occur is given by

   $$\sum_{i=0}^{4} {}_{500}C_i\,(.01)^i\,(.99)^{500-i} = binomcdf(500, .01, 4) \approx .4396.$$

   Because the event of at least five 100-year floods is the complement of the event of four or fewer 100-year floods, the probability that at least five 100-year floods will occur is given by 1 – .4396 ≈ .5604.

8. No. Comparing the answers to questions 3 and 7, we see that in a given 500-year period, the probability that at least one 500-year flood will occur (.63) is greater than the probability that at least five 100-year floods will occur (.56).

   Note that to model the number of floods we could also use a Poisson distribution, which approximates the binomial distribution when p is very small and n is very large. We obtain very similar answers when using these TI-83/84+ expressions:

   1 – *poissonpdf*(1, 0) = .6321 and
   1 – *poissoncdf*(5, 4) = .5595

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