Word Problems and Quantitative Literacy

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Introduction

In 1994 a committee of the Mathematical Association of America (MAA) prepared a report focusing on quantitative reasoning requirements for college graduates. [Reference: Sons, L. et al; "*Quantitative Reasoning for College Graduates: A Complement to the Standards*" , MAA, 1996.]

One of the recommendations of the committee report was the development of a quantitative reasoning program in colleges, where each student at each level would be required to participate in some coursework which emphasizes quantitative reasoning.

At the present time, several schools have developed their own unique quantitative literacy programs.

`http://www.stolaf.edu/people/steen/Papers/qlprogs.pdf`

According to the above mentioned MAA report, "*a quantitatively literate college graduate should be able to:*

1. "*Interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them.*

2. "*Represent mathematical information symbolically, visually, numerically, and verbally.*

3. "*Use arithmetical, algebraic, geometric and statistical methods to solve problems.*

4. "*Estimate and check answers to mathematical problems in order to determine reasonableness, identify alternatives, and select optimal results.*

5. "*Recognize that mathematical and statistical methods have limits,*"

The Course

The catalog description for the course sequence under study is as follows:

For MATH 34A: "*Introduction to differential and integral calculus with applications to modeling in the biological sciences.*

For MATH 34B: "*Continued study of differential and integral calculus with applications. Introduction to mathematical modeling with differential equations. Calculus of several variables including an introduction to partial derivatives.*"

[UCSB General Catalog 2004-2005]

The Text:

Calculus and Mathematical Reasoning for Social and Life Sciences by Daryl Cooper

1. A review of basic math skills:

   (a) **a basic review chapter on algebra** (Chapter 1),

   (b) *a chapter on graphs* (Chapter 2),

   (c) **a chapter on unit systems, conversions between units, and ideas of growth and size** (Chapter 4),

   (d) *a chapter on linearity and basic analytic geometry* (Chapter 6),

   (e) **a chapter on logarithms and exponentials** (Chapter 7).

2. Calculus:

   (a) **an introductiory chapter on preliminaries for calculus, including the concepts of error and limits** (Chapter 5),

   (b) *a chapter on derivatives* (Chapter 8),

   (c) **a chapter on integration** (Chapter 9),

   (d) *a chapter on further topics related to differential calculus, including material on trigonometric functions* (Chapter 12),

   (e) **a chapter on differential equations** (Chapter 13), and

   (f) *a short chapter on multi-variable calculus* (Chapter 15).

3. Quantitative reasoning and analytical thinking:

   (a) **a basic introduction to mathematical modelling and word problems** (Chapter 3),

   (b) *a long list of word problems (Chapter 11), and*

   (c) **a chapter on how to study mathematics and science courses and more generally how to approach quantitative subjects and problems** (Chapter 16).

   These chapters are mainly meant for self-study, although some practical instruction with respect to word problems is required for the students to benefit fully from them.

The major difference of Cooper's book from other texts serving similar purposes is its emphasis on quantitative reasoning and analytical thinking, and its concentration on word problems.

Cooper explains the relevance of this material in his introduction to the text as follows:

"*The heading of word problems is short-hand for reasoning, analytical thinking, and the ability to handle problems of a type you have not seen before by thinking. This skill is probably the most important skill you can gain from this class. Developing this skill will serve you well in many areas of life which have nothing to do with math. The ability to break down a complex problem into small manageable pieces which can be tackled one step at a time is extremely useful. Word problems bring out these issues most clearly. Often the hardest part of the problem is deciding what needs to be done, what the significance of the available information is, and how that information can be used. Then one must develop a plan to solve the problem, proceeding in small steps, without losing sight of the final goal.*" [Cooper, 2000].

Student Background and Expectations

- **The resignation to the fact that this is a required course sequence, and they are taking it because they have to:** Only 3 out of 60 students surveyed said they were taking the course "*to satisfy general interest*" as opposed to merely "*to satisfy a requirement*".

- **Lack of motivation and interest:** Among the same sixty students, twenty six have noted that even before taking the course, their interest level in the subject was low or very low. Another twenty-two rated their initial interest as average, and there were only twelve individuals who rated their initial interest as high or very high.

- **The expectation that the course should be limited to calculus:** The students were uncomfortable at first with the emphasis on word problems, which most saw as unrelated to the purposes of the course.

- **Tendency to compartmentalize mathematical experience:** Most students were comfortable with saying that they have never been good at trigonometry but are ok with algebra or vice versa. Almost all the students were annoyed when asked to use certain facts that they had happily left behind in their high school math classes, e.g. the formulas to find the circumference of a circle, or the volume of a cylinder.

The Class

- Word problems were introduced during the second week of classes in MATH 34A, and they were included in all the homework assignments starting with the second one. At least one problem was explicitly solved in lecture every other class meeting and at least five word problems were on each homework set. There were word problems on each of the two midterms and on the final.

- Word problems were still omnipresent in MATH 34B; each homework assignment (except one) still contained three to four word problems, and the midterm and the final included word problems along with more traditional calculus questions. Various types of word problems were introduced and solved regularly during the lectures.

Concluding Remarks

- When a well-planned QL program is not possible, as in the case discussed here, a modification in emphasis in a course already in existence may in fact serve the purpose to some extent.

- Students are generally averse to required courses that are not in their majors. However, the fact that the sequence was explicitly designed for UCSB helped.

- Most students were uncomfortable at first with the emphasis on word problems, but eventually they came to appreciate their use in this course.

Online Resources

`http://www.maa.org/ql/index.html` MAA website collecting information and reports concerning Quantitative Literacy that were formerly located on the website of the National Council on Education and the Disciplines (NCED) at the Woodrow Wilson National Fellowship Foundation in Princeton, New Jersey.

`http://www.stolaf.edu/people/steen/Papers/qlprogs.pdf` Selected Quantitative Literacy Programs in U.S. Colleges and Universities, by Lynn Arthur Steen, also see other links through his website `http://www.stolaf.edu/people/steen`

`http://www.statlit.org/` A page for Statistical Literacy: the study of statistics used as evidence in arguments, has various links regarding QL.