I Should Have Known Better

by Andrew Miller

This summer I had my first experience with something every homeowner dreads (and knows is coming eventually): a sudden puddle of water underneath the hot water heater. A local plumbing company quickly sent a technician out, who spared no time in pronouncing our current hot water heater dead. Then he explained that this company used flat-rate pricing: all jobs of a certain type cost the same fixed amount, which covers any expected or unexpected aspects of the job. We had two options for replacing the heater: First, he offered a standard 50-gallon electric hot water heater and installation for $1200. Second, we could install a hybrid electric hot water heater, which uses a combination of a heat pump and a heating element to heat the water, for $3200. The more efficient heating system would use, he claimed, about $30 less in electricity per month. Over the life of the heater, he claimed, the electricity savings would more than pay for the heater.

Even though I knew that I was unlikely to save as much on electricity as the plumber claimed, I was attracted to the 10-year warranty (a typical hot water heater warranty is 6 years) and the comparatively smaller environmental impact of a more efficient, longer-lasting appliance. Normally, my next step would be to take down the provider's estimate, do a little independent research, and get an additional estimate from at least one other company. In this case, though, I was under a bit of time pressure—the old hot water heater was currently leaking slowly into the garage—and this particular company was highly recommended by a major service provider ratings website. So I went ahead and had them install the hybrid electric hot water heater.

I should have known better.

A couple of days after the water heater installation, to satisfy my own curiosity I decided to do a little research on hybrid electric hot water heaters. I discovered that the exact model of water heater we had purchased could be ordered directly from the manufacturer for $1000 (MSRP $1300). Since the cost of a standard hot water heater installation was $1200, whereas the hybrid cost $3200, the marginal cost to me of the hybrid hot water heater was $3200 - $1200 = $2000. Thus, for the marginal cost I paid I could have purchased two hybrid hot water heaters outright! Things didn't add up; I was sure I was overlooking some factor to account for the cost difference. So I made a model.

The simplest model is one that assumes that the cost difference for the installation is equal to the cost difference between the hybrid and standard hot water heaters. Clearly, this model was wrong, so I tried to imagine what other factors would enter it, arriving at the following list:

- The cost of the hot water heaters, C_S for the standard heater and C_H for the hybrid.

- The labor cost to install the hot water heaters. My research indicated that the hybrid hot water heaters are designed to be as easy to install as, and to fit in the same places as, standard hot

Contents of this volume:

water heaters, so let's assume that this is the same amount L for the two hot water heaters.

- The cost to dispose of the old hot water heater. This will be the same for both types of new heater; let's call it D.

- The warranties on each type of hot water heater, denoted W_S and W_H.

- Profit. We'll model this as a constant factor $(1 + r)$ by which we will multiply the other costs.

Putting this all together, we see that the total installation cost of the hybrid electric hot water heater, I_H, is modeled by the equation

$$I_H = (1 + r)(C_H + L + D + W_H),$$

and similarly for I_S. Thus, the difference Δ in installation costs for the two heater types is modeled by

$$\Delta = (1 + r)((C_H - C_S) + (W_H - W_S)).$$

What data do we have for the values of C_H, C_S, W_H, and W_S? As noted above, the MSRP for C_H is \$1300; some online research indicated to me that a high quality standard electric hot water heater is available for around \$600 at retail. Thus, we can estimate $C_H C_S$ as about \$700.

W_H and W_S are much more difficult to model. Modeling the cost of a warranty is a standard problem for any student studying basic probability, but in this case we are missing the necessary information about the likelihood of failure for hot water heaters during the warranty period. Further complicating matters are the facts that the warranty periods are different for the two heater types (six years for the standard, ten years for the hybrid) and that, according to our plumber, the hybrid hot water heaters have a longer expected life span than the standard heaters.

Fortunately, though, we only need the difference in the cost between the two warranties. For simplicity, let's assume that the only possible loss is a total loss; that is, either the heater will last for the duration of the warranty period, or at some point the plumber will have to do a complete installation. In this case, a "fair" price for the warranty is pV, where p is the probability the hot water heater fails, and V is the value of the installation, i.e. the cost to install a new hot water heater, which we have already been modeling. In this situation though, we might assume that the company will not include a "profit" into the new installation; we'll also assume that the company

could recover the costs of the heaters from the manufacturer, as in each case we would be within the manufacturer's warranty on parts. Thus, V is simply equal to $L + D$ for each hot water heater type, and

$$W_H - W_S = (p_H - p_S)(L + D)$$

where p_H and p_S are the (potentially unequal) probabilities of failure for the two heater types. Now we can make two observations about this result: First, presumably the values for p_H and p_S will be low — manufacturers do not typically warrant products for terms that include a significant probability of failure. Second, based on the plumber's comment to me about the relative durability of the two types of hot water heater, we expect p_H to be smaller than p_S, which would contribute a decrease to the total installation costs of the hybrid compared to the standard! Thus, it appears reasonable to set $W_H W_S$ equal to zero, and doing so may, in fact, be an error in favor of the plumbing company.

According to our model, then, the difference in installation costs should be reasonably close to $\Delta = (1 + r)(C_H - C_S)$. As noted above, $\Delta = \$2000$ and $C_H C_S$ is approximately \$700; the only remaining unknown is r. Solving for r, we find that our model implies a profit margin of 186%! This is obviously an unrealistic result, if for no other reason than that it implies that on an installation of a standard hot water heater, the plumbing company makes about an \$800 profit on a \$1200 job. To pocket that much money, the company would have to get a really good deal on the hot water heater and convince the two employees who installed it to work for free. After completing my analysis, I wrote a letter to the owner of the plumbing company describing my model and its assumptions (though in less detail and without formal notation) and why, based on the model, I was very confused about the charged installation cost. In his written response, the owner pointed out that I had not taken into account taxes, which were included in the flat-rate prices I was quoted, and that the hybrid water heater, being significantly heavier than a standard water heater, is more difficult to load, deliver, and install. I'll leave it to the reader to account for these new facts in the model. He also took issue with my treatment of the warranties, and I'll grant that this is the part of the model with the most uncertainty. Finally, to his credit, the owner of the company gave me a small refund and a discount on future service. I wish, however, that he would have been willing to break down the charge into further details. As it is, we're still left to speculate as to why our model produces such unreasonable results.

Aside from containing an interesting model (and perhaps leading you to question your future SIGMAA-QL chair's consumer decision-making ability), I think this tale highlights some interesting QL lessons:

Modeling can be a useful tool for the average consumer. While I believe this is self-evident from the tale, I do want to note that the model uses only first-year algebra and a little bit of probability. That is, it's exactly the sort of basic model that exemplifies the maxim that quantitative literacy uses, for the most part, basic high school mathematics.

Assumptions matter. I had a mental pricing model that assumed in this case that the cost difference I was seeing ($3200 versus $1200) was almost entirely explainable by the extra expense of the more high tech hot water heater. In the absence of other information, I therefore made my decision on the assumption that a hybrid hot water heater costs about $2000 more than a standard hot water heater. Clearly, the plumbing company had a different pricing model, and my lack of knowledge about their model led me to make a decision I ultimately regretted.

Information asymmetry is a powerful force. Oftentimes, a supplier of a product or service knows more about that product than his or her customer: the supplier may have important information about the cost of production, potential harmful side effects, the various legal rights of the provider and customer, etc. that would, if the customer had the same knowledge, change his or her consumptive behavior. A prominent example is the modern credit card pricing model: In a recent Frontline documentary on credit cards ("The Card Game"), Shailesh Mehta, the former CEO of Providian Financial, describes how his company pioneered "stealth pricing" in credit cards: they reduced or eliminated the annual fees on their cards while increasing the late fees and other penalties that are often buried in the fine print. In the case of my plumbing adventure, the company's policy of providing flat-rate pricing hid all of the details of their pricing behind a single number that made it difficult for me to evaluate the reasonableness of the total cost or to compare it with other providers. A lit-

tle thought indicates another way in which flat-rate pricing benefits the plumbing company and has potential negative impact on the consumer: For flat-rate pricing to be a long-term sustainable business practice, the company has to overestimate the cost more often than it underestimates the cost, so the customer is more likely to overpay for a service under flat-rate pricing than he is to underpay. Failed models can teach us interesting things about how the world works. There was a hidden assumption in my pricing model: we assumed that the profit margin r was the same for each type of hot water heater. If we redo the analysis with two different margins rH and rS, we find that the difference in installation costs is now modeled by

$$\Delta = (1 + r_S)(C_H - C_S) + \delta(C_H + L + D + W_H),$$

where $\delta = r_H - r_S$. Due to the large values of the variables C_H, L, D, and W_H, even a modest value of δ can account for hundreds of dollars in Δ. I am well aware that companies often sell products at a variety of margins so as to capture more profit from those customers willing to pay more for "extras" — video game console manufacturers often sell the console at cost or even at a loss hoping to profit from the video games; gas stations sell gasoline near cost but have large markups on products in the convenience store; and movie theaters notoriously charge an arm and a leg for soda and popcorn. Nevertheless, I was surprised to see this type of differential pricing pop up in plumbing!

In the end, how do I feel about this experience? Not too bad after all. Despite being a little sour about the pricing, I love our new hot water heater. It works much better than the old one, and I can sleep a bit better at night knowing that it was a good decision for the environment. What's more, I learned something about modeling, pricing, and economics in the process and got to share some of that with you. Now that's something definitely worth paying for!

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SIGMAA-QL Cleans up on Professors of the Year

by Rob Root

Every year the Carnegie Foundation for the Advancement of Teaching and the Council for the Advancement and Support of Education select Professors of the year For the nation and in individual states. For 2012, the organization chose four national Professors of the Year and thirty state Professors of the Year. In this cadre of thirty-four distinguished educators there are three professors of mathematics, and of those three, two are part of SIGMAA-QL. "[T]he awards recognize professors for their influence on teaching and commitment to undergraduate students." [Russell (2012)] The Tennessee Professor of the Year is Mike Pinter, Professor of Mathematics and Director of the Teaching Center at Belmont University in Nashville. The Maryland Professor of the Year is John Hamman, Professor of Mathematics and Chair of the Mathematics Department at the Germantown campus of Montgomery College.
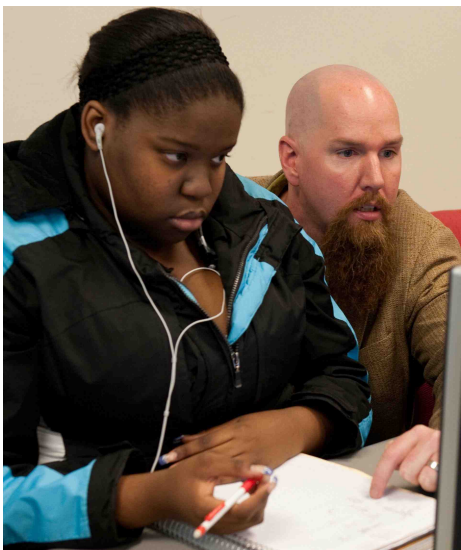

Figure 1: At right, John Hamman, Maryland Professor of the Year, with a student.

Their commitment to quantitative literacy is apparent in the announcements of their awards. In its announcement of the honor, Montgomery College quotes Prof. Hamman, [Rosano & Waugaman (2012)]

Many people and events have shaped my teaching and philosophy of education, but none as profoundly as my high school band director. It was clear that I not going to have a career involving music; however, he treated me with the same respect and attention as those students who had real musical talent... His personal, respectful, and patient approach made me realize the power of education and how it can shape the life of a student in unpredictable ways.

I try to have a similar impact on my students, most of whom are not on a career path to become mathematicians. I want to treat each of them with respect and view them as people on a journey, not just as math students. My goal... is to make sure my students have the opportunity to appreciate the beauty of what I am trying to teach and make connections to their lives outside of class, in other classes, and outside of academia.

Figure 2: Mike Pinter, Tennessee Professor of the Year.

The Belmont web site includes a perceptive quote of a colleague of Prof. Pinter, Psychology Prof. Pete Giordano, saying "Mike is not just a teacher of the technicalities and intellectual complexities of mathematics; he is somehow able to teach his students about life and living...." [Pillon (2012)] Each of these elite professors has a commitment to quantitative literacy is intimately connected to their success.

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Marcus Rosano and Chelsea Waugaman (2012) "Montgomery College's John Hamman Named Maryland Professor of the Year " Press Release: Montgomery College, November 16, 2012 `http://insidemc.montgomerycollege.edu/details.php?id=41247`

Pam Russell (2012) "Inventive Instructors Recognized as 2012 U.S. Professors of the Year" Press Release: Council for the Advancement and Support of Education, November 15, 2012 `http://www.usprofessorsoftheyear.org/Newsroom/Inventive_Instructors_Recognized_as_2012_US_Professors_of_the_Year.html#.UN4LDrbiSSB`

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In the 2012 "QL in the Media" Contest, Everyone is a Winner

by Rob Root

We received many excellent entries for the contest, and we highlight them here. From the indefatigable Larry Lesser came these gems:

- `http://kcostv.org/flowplayer/Videos/bbseason2/Video2.html` The January 24, 2012 episode of "Blast Beyond" taped live for which Larry (as the episode's featured guest) presented original mini-lessons (during the 10:30-13:20 and 16:45-21:45 intervals) on "Polls and Surveys" for local first and second graders and the band played an original parody lyric (9:35-10:20)

- `http://www.nctm.org/publications/article.aspx?id=33965` This September 2012 Mathematics Teacher "Media Clips" feature about the lottery features classroom-ready questions (and answers) generated by a published newspaper comic strip that mentions the popular "strategies" of tracking, wheeling, and pooling.

Tom Pfaff contributed this chart

- `http://mediamatters.org/blog/201112120005` Media Matters tracked down this glaring error in a line plot of US unemployment apparently broadcast by Fox News. Tom notes that even the corrected plot offered by Media Matters is suspect, since the scale on the vertical axis is still chosen to amplify relatively small changes.

Chawne Kimber tracked down this graphic

- `http://www.nytimes.com/interactive/2012/08/05/sports/olympics/the-100-meter-dash-one-race-every-medalist-ever.html` The New York Times offers this powerful visualization of the improvement in running that has been achieved in the little more than century since the first modern Olympic Games.

Finally, Aaron Montgomery offered this blog entry

- `http://squashed.tumblr.com/post/31757816989/mitt-romney-thinks-you-need-to-take-responsibility-for` Blogger Squashed offers a quantitatively rich critique of presidential candidate Mitt Romney's famous "47 percent" quote.

We hope you enjoy these entries; we find all of them helpful and worthy of investigation. Rather than choose winners, we will award prizes to all the contestants at the annual business meeting January 2013, in San Diego. (See the "Recent and Upcoming Events" for exact location and time.) Thank you and congratulations to all contestants.

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Reviews

Quantitative Literacy: Thinking Between the Lines

Cauder et al. (2012)

The textbook *Quantitative Literacy: Thinking Between the Lines* by Crauder, et al (Freeman, 2012) seems to be an attempt to redesign the traditional "finite math" general education course with the goal of teaching "quantitative literacy." The contents overall do not differ from those of a typical finite math textbook, with chapters on linear and exponential growth, finance, probability, statistics, graph theory, voting, and geometry. However, the authors do try to include some more current issues like election polling. When viewed as a finite math textbook, there are some positives to note: excellent graphs and infographics (including a section on exploring some ways data can be presented in a deceptive fashion), computational clarity and completeness (with ample examples, some formula derivations presented in the bodies of the chapters, and the usual battery of arithmetic exercises), and the chapter on personal finance is thorough, attempting to provide more computational financial literacy to students. Unfortunately, as a quantitative literacy book, the presentation falls short.

A quantitative literacy course should, at base, educate global citizens by helping them learn how to use their quantitative skills to aid in making good personal decisions and understanding macro-scale (local, national and global) community issues. A cursory glance at the "Index of Applications" might convince one that this book adheres to this principle. But it is misleading. For example, the Patriot Act appears in this index, yet it is only mentioned in an exercise; there is no explanation of what the legislation is, but rather a northern Nevada poll about the act is analyzed for potential sample size issues. There is a great deal to gain in exploring the political, demographic, and larger social context for the poll. This entire section on polling goes this way: another example is Social Security reform, which pops up in an "In the News" box. Many of our students don't know what Social Security is; they don't know why reform is being considered; nor do they know why there would need to be polling about whether to reform it. Elsewhere in the book, the chapter on personal finance goes into mind-numbing com-putational detail about savings accounts, mortgages, and stocks. But there is little mention of predatory lending and other consequential social justice issues. One might argue that teaching the context is not the goal of the book or of a mathematics course, but this rather rote and sterile approach to teaching mathematics does not make our students more literate than a traditional finite math book does.

This criticism is further evident in the lack of conceptual exercises—throughout the book they are all formulaic applications in shallow contexts. A blend is required. Students do need to practice computing, but some exercises could be replaced with short answer and essay questions that capstone a section by asking students to explain the meaning and consequences of the way things work.

All in all, this is a good textbook that does not live up to its title.

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Wolfram|Alpha Widget Gallery

Wolfram|Alpha (2012)

Wolfram Research Corporation is famous as the creator and vendor of *Mathematica*, the computer algebra system with extensive graphics and built-in access to extensive data. More recently the technically inclined have appreciated much of Mathematica's functionality freely accessible behind a friendly interface available at the Wolfram | Alpha web site and via the smartphone app of the same name.

Lately Wolfram|Alpha has been reaching out to the general public with applications of its web interface that are meant to appeal to those without pocket protectors. A recent article that described Wolfram | Alpha as able to "solve trigonometric functions" [Gaylord (2012)] shows that even people unfamiliar with high school algebra are finding this internet tool's ability to analyze Facebook data enticing.

For advocates of quantitative literacy, a graph of Facebook friends is amusing but not as obviously useful as the new widget gallery created to employ the Wolfram|Alpha engine. [Wolfram|Alpha (2012)] Using a simple API (application programmer's interface) almost anyone can create a web-based applet that harnesses to power of Wolfram|Alpha to any of a myriad of tasks, from the simplest unit conversion

to the solution of rather involved problems. While the use of such applets might hamper the development of computational facility, a survey of the thousands and thousands of applets that have been created might give students of QL a sense of the prevalence of opportunities for the use of quantitative information in understanding the world.

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Gaylord, Chris (2012) "What Facebook and Wolfram Alpha can tell you about yourself." *Christian Science Monitor*, November 22, 2012 edition. `http://www.csmonitor.com/Innovation/Tech/2012/1122/What-Facebook-and-Wolfram-Alpha-can-tell-you-about-yourself`

Wolfram|Alpha LLC (2012) Widget Gallery Web Site. `http://www.wolframalpha.com/widgets/gallery/`

Recent and Upcoming Events

MathFest 2012

Madison, WI, August 2–4, 2012
Monona Terrace Convention Center

SIGMAA-QL Panel Discussion
Friday, August 3, 4:10–5:30 PM
Ballroom B

This panel was organized by Gizem Karaali, Feryal Alayont, and Lerna Pehlivan and featured panelists Michael Starbird, Judith Garbiner, Andrew Miller, and Rachelle DeCosta. There is a description of the panel at `http://www.maa.org/mathfest/panels.html#strategies` and a summary of the presentations at the panel at `http://digitaleditions.walsworthprintgroup.com/iphone/article.php?id=1261922&id_issue=139063&src=&ref=`

National Numeracy Network 2012 Annual Meeting

QL at Work: Navigating the Worlds of Journalism, Finance, Business, and Citizenship

October 12–14, 2012, New York City

An index of presentations, with PDFs for many of them, is available at `http://serc.carleton.edu/nnn/meetings/2012annual/index.html`

AAC&U Meeting: Next Generation STEM Learning: Investigate, Innovate, Inspire

Kansas City, MO, November 8–10, 2012

This conference had 5 themes:

1. STEM course and curriculum development and assessment

2. Integrating technology into learning environments

3. Faculty development for next generation STEM learning

4. Alignment, preparation, and access

5. Institutional change for next generation STEM learning

Session materials and plenary podcasts are available at `http://www.aacu.org/meetings/stem/12/cfp.cfm`

Joint Mathematics Meetings

San Diego, CA, January 7–12, 2013
San Diego Convention Center

SIGMAA-QL Panel Discussion
Wednesday January 9, 9–10:20 AM
Room 1A, Upper Level

A survey of quantitative literacy teaching resources organized by: Caren Diefenderfer, Semra Kilic-Bahi, Maura Mast, and Eric Gaze. The panelists will be:

Jeff Bennett, Kay Somers, Eric Gaze, Andrew Miller, Bernard Madison, Judith Moran, and Maura Mast.

SIGMAA-QL Business Meeting
Wednesday January 9, 5–6 PM
Room 5B, Upper Level

Our annual business meeting is usually followed by an informal reception at a local watering hole.

MAA Session on Student Success in Quantitative Reasoning
Thursday, January 10 8–9:15 AM & 1–4 PM
Room 7A, Upper Level

Presentations by Ray Collings, A Al-Hasan, Jennifer Bruce, Aldo Maldonado, Andrew Miller, Gregory Foley, Cinnamon Hillyard & Karon Kipple, Bernard Madison, Susan Ganter & William Haver, Caren Diefenderfer, Chris Oehrlein, Theresa Laurent, and Dominic Klyve & Stuart Boersma.

MAA Minicourse #2
Thursday, January 10 1–3 PM & Saturday, January 12 1–3 PM
Room 30A, Upper Level

David Housman will present a mini course based on his book (co-authored with Rick Gilman, *Models of Conflict and Cooperation*. He describes it as "A game theory path to quantitative literacy."

MAA Poster Session Supported by NSF-DUE
Thursday, January 10 2–4 PM
Exhibit Hall B2, Ground Level

This session includes a poster about the "Quantitative Literacy and Reasoning Assessment (QLRA) Project" authored by Eric Gaze, Linda Misener, Semra Kilic-Bahi, Aaron Montgomery, Corri Taylor, and Deann Leoni.

Social Justice panel for Project NExT
Friday, January 11 9:30–10:45 AM
Room 6D, Upper Level

Organized by Abra Brisbin, Samuel Coskey, Pamela Harris, Branden Stone, with panelists Carlos Castillo-Chavez, Shandy Hauk, Lily Khadjavi, and Rob Root.

MAA Session on Mathematics Education
Friday, January 11 2:30–5:40 PM, Saturday, January 12 8:15–10:55 AM, & 1–5:10 PM
Room 33C, Upper Level

This session is organized by Stephen Davis, Gizem Karaali, and Douglas Norton and includes several talks about developing quantitative reasoning skills.

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