USING INTERACTIVE GEOMETRY SOFTWARE BASED INSTRUCTION IN A COLLEGE TRIGONOMETRY COURSE

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ABSTRACT

Interviews of students entering college trigonometry reveal high levels of math anxiety; having to remember numerical tables and an overwhelming number of formulas unrelated to meaningful representations are to be blamed for such apprehension. Furthermore, students often perceive trigonometry as disconnected from other mathematical topics with no transition from previously acquired knowledge. Technology has proven efficient in enhancing students' learning by allowing them to rediscover mathematical properties through visual manipulations. In particular, research on using Graphing Calculators, Excel spreadsheets or The Geometer's Sketchpad in middle school and high school trigonometry courses show positive results for achieving higher conceptual understanding. A previous project focused on identifying a possible relationship between students' proportional reasoning and their achievements in trigonometry. The results being unsatisfactory, a different stance had to be chosen. In this project, we propose to make effective use of a unique lab setting in order to investigate the effect of the interactive geometry software, The Geometer's Sketchpad, in a college trigonometry course. We designed activities supporting meaningful explorations of trigonometric concepts from a constructivist perspective.

RESEARCH FOCUS

Trigonometry stands out as an overwhelmingly challenging topic for a lot of students entering college. Students' interviews show that part of the anxiety associated with learning trigonometry stems from it being seemingly unrelated to previous math classes, and requiring memorization of "meaningless" numerical tables or abstract formulas. When used appropriately, technology has been recognized as a powerful tool to enhance students' learning of mathematical concepts. In particular, it proves useful when the purpose of instruction is not to acquire fluency in numerical manipulations, but rather to bypass the numerical block for exploring other concepts in-depth. From this standpoint, using technology at the college trigonometry level seems adequate.

Psychological studies have shown that a person's prior belief about their ability to learn has a very strong effect on their academic success (Seligman, 1998). In particular, the "I can't do math" attitude has a pervasive impact on students' achievement in mathematics courses. Once again, using technology as a smoother approach to trigonometry may help focus students away from this learned defeatism.

RESEARCH QUESTIONS

Even when students are successful in their trigonometry courses, interviews reveal a "quickly learned, quickly forgotten" attitude towards the topic. It would be of particular interest to explore the effect of technology on material retention in this context. We propose to revisit the college trigonometry curriculum by designing interactive trigonometry computer labs with the Geometer's Sketchpad; questions we wish to explore are: What role does technology play in learning trigonometry concepts? Do technology-based explorations of multiple representations enable students to make better connections among mathematical topics? How does an interactive course set-up engage students in improving their problem solving schemes? How does technology affect material retention?

LITERATURE

Research in using technology to support trigonometry instruction is sparse and focused on middle school and high school (Choi-Koh, 2003; Embse & Engebretsen, 1996; Luscombe, 1998). These studies show that the graphing calculator, Excel spreadsheets, and The Geometer's Sketchpad all have a very positive impact on exploring trigonometric functions. However, traditional college trigonometry courses rely on them only sporadically. Traditional pre-calculus textbooks (Cohen, 2005) offer a sequence of instructions which delay engaging students in discovering for themselves the numerous applications of trigonometry to real world examples.

THEORETICAL PERSPECTIVE

Our theoretical perspective looks at instruction from a constructivist standpoint. In other words, we aim for the students to explore trigonometric concepts through a process of reflection-assimilation triggered by a careful design of laboratory activities. Our goal is to generate a series of investigations, each of which should stimulate the students' interests and require resolving a cognitive conflict. We hope this approach will allow students to develop higher understanding of the concepts acquired in the course by making the learning process more dynamic.

METHODOLOGY

Students will experience the computer labs in the Spring of 2007. At CSU Bakersfield, each math course includes a 2 hour, 25 minute lab component which provides instructors and students opportunities for in-depth explorations of the mathematical topics presented. For data collection purposes, students will be asked to give a written mathematical report of each lab. We will also ask the students to record in a journal any personal comments about their learning process while engaged in the lab activities. Once a week students will be required to answer open-ended questions based on the previous week's lab in order to focus the journal's reflections. In particular this should provide us with information regarding any effect the labs have on material retention. In order to gauge the progress students make in learning to learn with technology, as well as any impact this might have on the learning of trigonometry, students' interaction with the technology will be filmed during the labs. To assess the quality of material acquisition and retention, a pre-test will be administered to both project participants and a control group at the beginning of the quarter, followed by a few targeted questions on the final examination. For long term retention, we will follow the students' achievement through the Calculus sequence in the years to come.

FOCUS OF TECHNOLOGY

The trigonometry course will consist of lectures and labs, and will begin with the study of right triangle trigonometry. Labs typically consist of problems more rigorous than homework problems and are designed to engage groups of students in mathematical discussions. In redesigning the labs for incorporation of technology, we have engaged in two types of activities: first, transforming existing investigations into meaningful, technology-supported problems; and second, creating new projects aimed at sustaining our students' interest. We found that The Geometer's Sketchpad gives more meaning to questions that previously seemed artificially brought to light, and should enable the students to be more engaged in a real search process through explorations, constructions, calculations and verifications associated with a specific situation.

For example, the first lab, occurring during week one, will contain an adapted version of an activity found in *Projects for Precalculus* (Andersen, Swanson, & Keeley, 1997) entitled "Easy as Pi (pp. 109-113)." Below is a sample of changes that using technology allows while aiming at the same final product.

Original version (Andersen, Swanson, & Keeley, 1997)**:**

The number π occurs in two familiar formulas involving circles: $C = 2\pi r$ and $A = \pi r^2$. We will use these formulas, with both inscribed and circumscribed polygons, to obtain an estimate of the value of π.

1. First, the circumference formula will be used to estimate π. Since this is the formula based on the *definition of π*, it is the natural starting point.
   a) In order for the circumference of a circle to equal π, what must the radius be? Briefly justify your answer.
   b) Inscribe a regular octagon inside a circle with the radius you found in a). (See figure 1.) Our goal is to find the perimeter of this octagon. Since the octagon is inscribed inside the circle, its perimeter will be less than π.

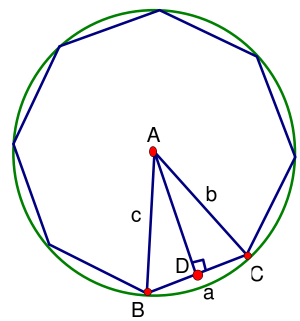

Figure 1

   i. What is the measure of $\angle BAC$?
   ii. The length of sides b and c are equal. What is that length?
   iii. $\overline{AD}$ is a perpendicular bisector of $\overline{BC}$. Show that the length of side a is $\sin 22.5^\circ$.
   iv. Find the perimeter of the octagon.

Adapted version:

The number π occurs in two familiar formulas involving circles: $C = 2\pi r$ and $A = \pi r^2$. We will use these formulas, with both inscribed and circumscribed polygons, to obtain an estimate of the value of π. **Please set your unit to inches in GSP**.

2. First, the circumference formula will be used to estimate π. Since this is the formula based on the *definition of π*, it is the natural starting point.
   Q8: In order for the circumference of a circle to equal π, what must the radius be? Briefly justify your answer.
   Using GSP, inscribe a regular octagon inside a circle with the radius found in **Q8**. Our goal is to find the perimeter of this octagon. Since the octagon is inscribed inside the circle, its perimeter will be less than π.

   Q9: The length of sides b and c are equal. What is that length?
   Q10: $\overline{AD}$ is a perpendicular bisector of $\overline{BC}$. Find the length of side a using a trigonometric expression. Check your result with GSP.
   Q11: What is the perimeter of the octagon?

The adapted "Easy as Pi" activity allows students to explore lower and upper bounds of the value of π through the use of regular polygons inscribed in and circumscribing circles.

However, for the adapted version, students will find a lower bound for π by inscribing a regular octagon inside a circle with circumference π, using trigonometry, and, unlike the original version, The Geometer's Sketchpad. The software enables students to be actively engaged in constructing a sketch of the situation to explore, which allowed us to remove some questions since they automatically become necessary steps towards a proper sketch. Furthermore, students will be able to verify their work using the measurement tools within The Geometer's Sketchpad. The students will then move their lower bound exploration from the concrete to the abstract by shifting from an octagon to the general n-gon and will continue to explore a lower bound for π through the use of trigonometry and algebra.

Part of the second lab will see the students work on a mini-project designed by the researchers (see Appendix A). The students will use The Geometer's Sketchpad to create a series of similar triangles and trigonometric functions to determine the proper roof angle and height of a window in a passive solar house. Each group of three students will be "building their house" in different cities around the northern hemisphere, thus the sun will be at different angles of elevation on the two days for each group. This will ensure each "house" is unique and will enable the researchers to determine whether students understand the relationship between similar triangles and the trigonometry involved. By allowing students to own their project we hope to bypass some of the psychological blocks and sense of uselessness that students often face when learning trigonometry.

IMPLICATIONS FOR TEACHING PRACTICE

Implications of this research are many, from hopefully shifting students' negative perception of trigonometry to redesigning the curriculum in college trigonometry courses. If preliminary results are positive, we would ultimately like for all sections of trigonometry to be supported by technology, which would lead trigonometry instructors and trigonometry students to meet everyday in the computer lab. A second project would then be to conduct similar research at the Calculus level.

APPENDIX A

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Solar House Project

It has become clear that alternative energies will become more and more in demand as we slowly deplete Earth's natural resources. This project explores some of the decisions involved with the construction of a passive solar house. In particular, it will be important for the South side of your house to include a lot of openings in order to let the light in as much as possible during the winter, but also the roof needs to provide enough shadow during the summer so that the inside does not get overwhelmingly warm.

Step 1: Choose a location for your house among the list given in class. Write your group number next to the city you choose. Each group must choose a different location.

Step 2: We are interested in designing the South face of the house. Refer to your GSP file. The ground is shown, as well as the South cardinal at the right of your screen. Decide upon a point on the ground for the South face of the house.

Step 3: Place the sun at proper elevation from the base of the house, for both the Winter and Summer solstice at noon.

The following restrictions will apply for the design of your house:

1. The South side has a glass window, the height of which you will have to determine. With that in mind your roof should not extend below the window in order to ensure a good view.
2. At 12 pm on the Summer solstice, the sun should not hit the South window.
3. At 12 pm on the Winter solstice, the South window should be bathed by sunlight.

With your group, use GSP to come up with a house design that reflects the restrictions above.

Q1: What should be the angle of inclination of the roof with the horizontal? Explain your calculation, and then check your result with GSP.

Q2: Explain how you can determine the height of the glass window. Check your result with GSP.

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