Analyzing Effects of Status Difference in Mathematical Discourse

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Introduction. Power differences in mathematical conversations with an authority such as a teacher or tutor create unintended effects that may alter or interfere with a student's reasoning during problem solving. Using ideas from Vygotsky, Sfard and Oehrtman, we analyze transcripts of an authority interviewing second semester calculus students as they attempt to solve a covariational reasoning problem. Following Sfard, we present a framework for analyzing *object-level discourse* (i.e. student linguistic and cognitive adaptations to an authority's scientific use of language) and *meta-level discourse* (i.e. actions directed toward maintaining the social relationship, such as probes for cues of approval of linguistic adaptations, and attempts to save face if such adaptations appear unapproved).

Mathematical authorities bring to a conversation a well-defined and highly organized way of communicating. The words that mathematical authorities use are full of hidden meaning, each word hinting at a complex system of interconnections acquired through many years of self reflective study. In accordance with the research of L.S. Vygotsky (1986), such use of language will be referred to as *scientific.*

Calculus students, on the other hand, come to a conversation possessing their own, possibly idiosyncratic use of words. These uses may be formed mostly through years of everyday

experience, absent of conscious reflection. Again, in accordance with L.S. Vygotsky (1986), such use of language will be referred to as *spontaneous*.

It is worthwhile to distinguish between *scientific uses of language* and what may colloquially be thought of as *scientific language*. For example, the word "corner" is not considered to be scientific language. However, the word "corner" can be used scientifically as a way of signifying a discontinuity in the first derivative of a graph of a function. Similarly, the word "cubic" is considered to be scientific language. However, the word "cubic" can be used spontaneously, without conscious reflection, to refer to any graph that has two curves.

Based on the philosophical underpinnings of Karl Marx's dialectical materialism, Vygotsky viewed learning as being primarily spurred by a complex interplay between *scientific concepts* and *spontaneous concepts*. Similarly, Vygotsky also viewed thought and language as a complex interplay in which each spurs the development of the other. Synthesizing these two branches of Vygotsky's research together, the thesis emerges that learning may be primarily spurred by the complex interplay between scientific uses of language and *spontaneous uses of language.*

Anna Sfard (2001) puts forth a similar thesis to the one just synthesized from Vygotsky. Thinking, she argues, may be viewed primarily as a form of communication and, hence, learning may be primarily spurred by *discursive conflicts,* a term coined by Sfard that stresses "the clash of habitual uses of words" (Sfard, 2001, p. 48). The notion is illustrated through the example of clash between two different uses of the word "infinity."At the start of a transcribed mathematical conversation, an elementary student's uses the word "infinity" to refer to the biggest number-word she can think of, while her teacher uses the word of infinity to refer to limitless counting.

In order to examine such a mathematical conversation and the discursive conflict that occurs within, Sfard posits that there are two layers to every conversation, each with a corresponding form of analysis. The first layer, *object-level discourse,* has to do with the matter at hand, communication that specifically deals with the process of solving say a covariational reasoning problem in calculus. To object-level discourse Sfard applies *focal analysis*, in which the researcher looks at what is pronounced, attended to, and intended by a communicator as they work on a specific math problem. The point of this focal analysis is to ascertain whether there is any disconnect between what is said, what is done, and what is meant. If, on the other hand, there is a strong correspondence between all three, the communication is said to be *object mediated*, because it resembles the most effective communication possible: talking about an object, such as a chair, which you can "see" right in front of you.

The second layer or *meta-level discourse* has to do with maintaining social relationships between the communicators, for example communication designed to please an authority figure. To meta-level discourse, Sfard applies *pre-occupational analysis*, in which the research looks at patterns of proactive and reactive communication in the communicators. From these patterns the researcher may then attempt to determine whether a communicator is generally interested in understanding the thinking of another or say just trying to get through the conversation without showing a sign of weakness or possibly offending someone.

To address how a communicator goes about understanding the thinking of another, we turn to Michael Oehrtman (2002), who presents *metaphorical reasoning* as a cognitive tool that calculus students use to solve problems. In his conclusion, Oehrtman notes that in some instances calculus students appeared to be using metaphorical reasoning as a tool to imitate the reasoning demonstrated by a mathematical authority. Now a word on imitation: Oehrtman initially notes in

his introduction that Vygotsky viewed imitation not as the mere rote learning of animal but instead as a vital part of cognitive development, something that can only be done if a task is in a person's *zone of proximal development*. Vygotsky himself gives an example from mathematics to illustrate this point, noting that a child who can imitate a solution to an algebra problem cannot necessarily imitate the solution to a calculus problem, even if it is done by the same teacher. Synthesizing Vygotsky and Oehrtman, we emphasize that metaphorical reasoning is not just a cognitive tool for solving mathematics problems, but may also be applied specifically towards understanding the reasoning of authorities, namely by imitating their scientific use of language.

Weaving this synthesis of Oehrtman along with Sfard's framework into our original thesis synthesized from Vygotksy, we have the enriched thesis that learning may be primarily spurred by the complex interplay between scientific and spontaneous uses of language, in which object level clashes over uses of words and meta-level actions to maintain social relationships may both alter and interfere with student's metaphorical reasoning about a mathematics problem.

Research Methodology. This presentation draws upon three transcribed interviews between a Post Doctorate Graduate Student and second semester calculus students at a large southwestern public university. Subjects were given a series of problems designed to elicit their use of conceptual metaphors. Each student was given 5 minutes to work on a problem, after which they were interviewed by an authority. The interviews were audiotaped and then transcribed, and the results were coded using a grounded theory approach. We coded what Sfard called *discursive conflicts* by marking all instances of the introduction of new language in the transcript. We then coded the discourse directly following the introduction of new language for both meta-level discursive actions and metaphorical reasoning.

Results. Despite deliberate attempts to withhold any sign of approval or disapproval, power differences between the interviewer and the students appeared to affect how the students adapted to the interviewer's scientific use of language. While both interviewer and student introduced new language, students only did so in direct response to new language by the interviewer. In the student discourse immediately following the authority's introduction of new language, we identified four categories of object-level discursive adaptations as ignoring, repeating, rephrasing and translating. In addition, we identified two categories of meta-level discursive actions: probing for approval and face-saving.

Student	#Responses To New Language	#Repeating Adaptations	#Rephrasing Adaptations	#Translating Adaptations	Total # Object-Level Adaptations
Neil	88	**4**	1	0	5
Geddy	48	2	**9**	1	12
Alex	33	0	0	**7**	7

What is striking is that each student appeared to have a dominant mode of object-level adaptation, a mode that significantly impacted their subsequent metaphorical reasoning. One student Alex experienced a breakthrough in his metaphorical reasoning by simply repeating key words used in the interviewer's questions, a mode that correlated with meta-level probes of the interviewer for approval of the word's meaning and use. A second student Geddy rejected her own correct metaphorical reasoning after repeated attempts to both rephrase the interviewer's questions and save face led her to spontaneously generate incorrect metaphors based on the use of words in the interviewer's questions. In contrast, a third student Alex appeared to be utterly unaffected by the scientific use of the language in the interviewer's questions, seemingly because

he was translating these questions into a form that did not conflict with his existing metaphorical reasoning.

References

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