Mathematics teaching assistants learning to teach:
Recasting early teaching experiences as rich learning opportunities

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Abstract

Graduate students typically have significant training in their field of research but little preparation to teach. This lack of preparation prompts many departments to provide instruction in the mechanics of teaching. In contrast, research at the K-12 level indicates that professional development (PD) programs focusing on teachers' knowledge of student thinking contribute to improved student performance. The success of these programs calls into question the focus on basic teaching skills for graduate students and prompts several important questions. What knowledge of student thinking would help mathematics teaching assistants (MTAs) provide their students with rich opportunities to learn? How might MTAs use that knowledge? How might MTAs gain that knowledge?

In this paper we take a theoretical look at these questions, dividing our analysis into two parts. First, we examine the work of MTAs, identifying the activities they engage in which could be better informed by having detailed knowledge of student thinking. Second, we look at the ways in which MTAs could build this knowledge on-the-job, in the context of their teaching. We close the paper by examining implications for the professional development of MTAs. This analysis provides a new, reverse-engineering perspective on the PD of MTAs, starting with the knowledge that would help them in teaching, moving to the ways in which they might gain that knowledge, and finishing with the PD that might help them gain such knowledge. Thus we are recasting TA PD from instruction in *teaching* to support for *learning from teaching experiences*.

A Scene From The Life of a Calculus Teaching Assistant

We open this paper with this short fictional vignette to set the context for our examination of how mathematics teaching assistants (MTAs) use and acquire knowledge of student thinking.

> Beth sits in her office a few moments before she has to go teach her calculus class. They are in the midst of the section on sequences and she remembers that at this point last year, her students somehow got it stuck in their heads that only monotone sequences

(ones that are either increasing or decreasing) converge. This year she wants to help them avoid this difficulty so she carefully chooses her examples to include a non-monotone, convergent sequence. Beth is confident that this example will make the relevant characteristics of convergence clear to her students.

In class, after handling a variety of simple examples and talking through both informal and formal definitions of convergence, she puts her new example on the board:

$$1,0,\frac{1}{2},0,\frac{1}{3},0,\frac{1}{4},0,\ldots$$

Beth says, "OK, now I want you to take a minute to discuss whether this sequence converges or diverges."

Beth then wanders by a pair of students who are deep in conversation. George thinks it diverges: "The terms aren't *going* to zero." Maria thinks it converges: "No, you see they *both* go to zero."

On the spot, Beth is faced with a host of questions. George is exhibiting the ideas she saw last year, but Maria, who has the correct answer, appears to not be reasoning correctly either. What is she thinking?

What question would prompt the two students come to a better understanding of convergence? Should she have one or both of them explain their reasoning more fully – and if she asks both of them to do so, who should she ask to go first? What sequence should she give them to clarify their thinking? How can she use their knowledge of limits (in the context of derivatives and integrals) to elucidate the ideas when it comes to sequences?

While teaching assignments for MTAs vary greatly by institution and course, the preceding scenario, focusing on a graduate student teaching a calculus course, is illustrative of the many MTA experiences we will analyze in the balance of this paper. Decades of research have illuminated factors that shape students' learning opportunities. Among these factors, teachers' knowledge of various kinds has been shown to be especially influential on teachers' practices and thus on students' learning opportunities. This work has been concentrated at K-12 levels, with little attention to the knowledge used and needed by college teachers. What types of knowledge will help TAs provide opportunities for students to learn? Where might TAs gain that knowledge?

Before we address these questions, let us look back to the vignette. What mathematical knowledge did Beth use in planning this class? What knowledge of student thinking did she use? In addition, what knowledge did Beth draw on to even *pose* the questions she is considering when the vignette ends? What knowledge would she be drawing upon to answer her questions?

Let us take another step back. This situation could be a "learnable moment" for Beth. What knowledge might she *gain* through these interactions, both about mathematical content and about student thinking? How should she approach the situation if one of her goals is to acquire more knowledge of student thinking?

Finally, let us take one last step back. What professional development (PD) could have been provided beforehand to help Beth use this situation as an opportunity to develop her knowledge of student thinking? What activities would make her more likely to *want* to do so? What support could her graduate program give to help her productively reflect on this incident?

These last questions introduce a potentially quite different perspective on the preparation of TAs. Instead of viewing PD as preparing TAs to *teach* mathematics, we suggest that PD experiences be viewed as preparing them to *learn* how to teach mathematics. Just as doing mathematics creates opportunities to learn mathematics, "doing teaching" creates opportunities to learn to teach. It is often presumed that what is needed to teach mathematics is knowledge of mathematics and presentation skills. Research in mathematics education over the past several decades has demonstrated that this view is incomplete and that there are many mathematics-specific aspects of knowledge essential to the effective teaching of mathematics (see references in the next section).

In this paper we explore this new perspective. In doing so, we are "reverse-engineering" the development process. We begin by examining types of knowledge an experienced teacher would draw upon while teaching and work backward to see where TAs might have opportunities to acquire such knowledge. Thus we start this paper where we hope TAs end up, by looking at what it takes to be a knowledgeable, experienced, effective college teacher. We then return to what we, as PD planners, start with – namely, new graduate students with solid content knowledge and little teaching experience – examining the types of experiences that might be generative of such knowledge. Finally, we discuss PD activities that might lead TAs to be more likely to turn their own teaching experiences into *learning* experiences during which they might build the knowledge of the sort held by experienced teachers. An overview of this structure is provided in Figure 1.

Practice ↓	Using knowledge	Gaining knowledge	PD to support learning
Planning			
Instructing			
Reflecting			

Figure 1

Framework of use, acquisition, and development of knowledge

In the next section, we provide a tour through research literature that addresses the role of teachers' knowledge (particularly, knowledge of student thinking) relationships between teachers' knowledge and their students' achievement, and how PD can play a role in helping teachers develop knowledge of student thinking. Following our discussion of this literature and how it as applicable at the college level, we turn our attention to the activities of TAs. We first examine the ways in which knowledge of student thinking can help inform TAs' teaching decisions. We then turn to how TAs might build this knowledge of student thinking. This section is organized by the types of activities TAs engage in, with analysis of the ways in which each activity is potentially generative of new knowledge of student thinking. In the final section, we bring all of this work back to bear on the PD we provide to TAs. Here we weave together three different strands: the knowledge of student thinking we want TAs to gain, the activities through which they might gain that knowledge, and the ways in which PD can support their learning. Throughout, we refer to the vignette above to bring the focus back to the context of TAs in college classes.

Research on Mathematics Teachers' Knowledge

We begin this section with an overview of research literature that addresses issues of teacher knowledge acquisition and use. Because of the relative scarcity of research on the effectiveness of PD at the college level, we extrapolate from work done in K-12 settings. At that level, especially for the early elementary years, several programs of research have successfully tied increases in teacher knowledge, and especially knowledge of student thinking, with improvements in student performance. We give a brief overview of those programs and their findings in this first section, extrapolating from them what teacher knowledge might lead to improved student understanding at the college level. We also discuss two relatively new branches of research, one that looks at PD programs that focus heavily on student thinking, and another that examines the mechanisms by which teachers learn while teaching. Taken together, these areas of research inform our focus on student thinking and our discussion of the PD needed to help TAs build that knowledge found in subsequent sections of this paper.

Identifying knowledge that matters in teaching

Recent years have seen an explosion in research on the knowledge teachers need to know in order to teach effectively. Early work in this area had proven challenging (Ball, Lubienski, & Mewborn, 2001). For example, studies in the 1970s showed the counter-intuitive finding of no correlation between the number of mathematics courses teachers had taken and their students' performance (Begle, 1979). Since then, educational researchers have searched for and found other aspects of knowledge that teachers draw on that do correlate with their students' achievement.

While it is undoubtedly the case that teachers make use of many kinds of information when designing and carrying out instruction, research from the past two decades has identified certain types of knowledge that appear to be especially influential in shaping teachers' practices. The mid-1980's saw the introduction of the term Pedagogical Content Knowledge (PCK) to describe knowledge needed for teaching a subject that was neither purely subject knowledge nor purely pedagogical knowledge (Shulman, 1986). Two important components of PCK are knowledge of student thinking and knowledge of the curriculum. In the vignette above, Beth exhibits such knowledge when she chooses an example based on the misconceptions she knows her students are likely to have (knowledge of student thinking). She might also be drawing on knowledge of the curriculum, for instance, that sequences with every other term equal to zero appear in the Taylor expansion of sine and cosine. What Beth knows is a product of interaction between her knowledge of mathematics and her knowledge of students' typical experiences while learning particular mathematical ideas. This portion of the vignette corresponds to where the "planning" row intersects with the "using knowledge" column in the framework in Figure 1.

Researchers have recently also endeavored to identify the knowledge needed to teach mathematics effectively, including knowledge of (or ways of thinking about) mathematics that develops while doing the work associated with teaching mathematics (Ball & Bass, 2000; Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Hill, Rowan, & Ball, 2004, 2005; Hill, Schilling, & Ball, 2004; Ma, 1999). This work focuses on the particular mathematical ideas that teachers develop as they analyze students' mathematical thinking or think deeply about connections among ideas present in school mathematics. These activities are not necessarily part of "doing mathematics," the activities engaged in by people using or studying mathematics itself. This is an example of teachers gaining knowledge as they interact with students during instruction (represented in the Figure 1 framework by the box for "gaining knowledge" in the "instructing" row.)

The role of knowledge of student thinking

In the early 1980's, a group of researchers at the University of Wisconsin at Madison began to design and study PD programs for in-service teachers structured around research on how early elementary students approach addition and subtraction problems. This work eventually evolved into the program known as Cognitively Guided Instruction (CGI). Improving teachers' knowledge of the relationships between problem types and students' solution strategies improved teachers' abilities to assess children's knowledge and adapt instruction based on that knowledge (Fennema, Franke, & Carpenter, 1993). These gains in knowledge and changes in instructional practices were linked to increases in students' achievement (Fennema et al., 1996).

The CGI researchers developed an assessment tool to measure teachers' use of student thinking in their classrooms. Level 1 teachers characterize student thinking only in terms of procedures students had been taught. The remaining levels described teachers with increasing attentiveness to student thinking, culminating with Level IV teachers who let children's thinking drive instructional decisions and create opportunities to build greater knowledge of their particular students' thinking (Franke, Fennema, & Carpenter, 1997). These most advanced CGI teachers had created a feedback loop, using knowledge of student thinking to plan student activities and manage classroom discussion in ways which helped them learn more about student thinking– which further informed their instructional decisions. This program of research examined all three columns in the framework from Figure 1 (using knowledge, gaining knowledge, as well as the PD that can support teachers' learning) and focused in particular on how they were manifested in teachers' in-class practices (as represented by the "instructing" row of the framework) and planning practices. Beth's teaching choices in the vignette could be evidence of this type of teaching, as she used knowledge of student thinking to guide her construction of an example. The vignette closes before we can determine if she will use George and Maria's responses to build further knowledge of student thinking, but that potential exists. In the case of the CGI teachers, researchers were able use their framework to connect teacher understanding of student thinking with student success:

> This study provides strong evidence that knowledge of children's thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. These findings, when viewed in conjunction with those of other studies, provide a convincing argument that one major way to improve mathematics instruction and learning is to help teachers understand the mathematical thought processes of their students. It also appears that this knowledge is not static and acquired outside of classrooms in workshops, but dynamic and ever growing, and can probably only be acquired in the context of teaching mathematics. (Fennema et al., 1996)

Other programs have come to similar conclusions: that knowledge of student thinking can be a fruitful focus for PD programs (see, e.g., The Purdue Problem-Centered Mathematics Project (Cobb, Wood, & Yackel, 1990) and SummerMath for Teachers (Schifter, 1993)).

Although the above work focuses on the K-12 level, the guiding principle would seem to apply at the college level as well – that Beth and other college instructors will provide a more productive learning environment if they have greater understanding of and attend more to student thinking. While studies have yet to verify this claim, some research suggests that many MTAs do not possess this rich knowledge of student thinking (Speer, Strickland, & Johnson, 2005; Speer, Strickland, Johnson, & Gucler, 2006). In that study, MTAs, including those with several years of teaching experience, were largely unable to identify solution strategies other than their own and were generally unaware of many of the conceptual difficulties detailed in the literature.

There is, however, hope. Research on TAs who have had significant experience facilitating group work in calculus courses suggests that TAs can gain rich knowledge of student thinking during their graduate school teaching experiences (Kung, in press). These TAs reported that observing students working challenging problems afforded them the opportunity to develop knowledge of student thinking– which they used in teaching later courses.

The above research, when taken together, suggests two things. First, improving TAs' knowledge of student thinking is likely to improve their teaching and their students' learning. Second, this goal is necessary (novice MTAs largely do not possess this knowledge) and reachable (MTAs are able to gain such knowledge during graduate school).

The question then becomes: How might TAs come to learn about student thinking? For possible answers, we turn to the literature on PD in mathematics education, noting, however, that the research community has only focused specifically on teachers' knowledge of student thinking for the last twenty years, and so it is not surprising that there is much less research on how that research can be used in PD activities.

Programs to develop knowledge of student thinking

One very successful effort to actively support teachers' acquisition of knowledge of student thinking is the CGI program mentioned earlier. The core of the program is to provide in-service teachers a set of frameworks that categorize key problems in the area of addition and subtraction problems, along with the strategies children use to solve them (Carpenter, Fennema, Peterson, & Carey, 1988; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Program activities typically include discussions of the research basis for the frameworks, structured interviews with children to assess how their strategies fit within the framework, discussions of those interviews, and practice writing word problems that fit various parts of the framework. Follow up PD is also typically provided and usually includes meetings among a team of CGI teachers all at the same school.

The results of the CGI PD are impressive. "Teachers realized that they needed to listen to their students' mathematical explanations, create strategies and questions to elicit those explanations, and understand enough about children's thinking and the content to know what to do with what they heard" (Franke & Kazemi, 2001). Furthermore, as is the hope for Beth above, being attentive to students' thinking created opportunities for the teachers to continue to develop their knowledge while teaching. In other words, their teaching practices became generative of new knowledge relevant to teaching. "It transformed teachers into learners. They learned in the context of their practice about the teaching and learning of mathematics…" (p. 104). The effects of the PD were strong enough that in a follow-up study, four years after the PD ended, some teachers were still engaged in generative growth of the sort targeted in the PD.

Learning from teaching experiences

Two other lines of inquiry shed light on the mechanisms by which PD might help TAs such as Beth be more open to learning about student thinking while teaching. Sherin (2002) examined the implementation of high school algebra curriculum materials and found that the teachers used three different types of content knowledge: subject matter knowledge, knowledge of the new curriculum materials, and knowledge of student learning. These types of knowledge were not completely distinct nor static for these teachers: they drew upon different types of knowledge at different times. Analysis of the data indicates that observing students using novel approaches can catalyze changes in teachers' thinking. She concludes that, as was the case for Beth, "novel student ideas have emerged as a key trigger for teacher learning during instruction" (p. 145). This program of research illustrates how PD can help teachers gain knowledge in the context of instructing and reflecting on that instruction (captured in the Figure 1 framework by the intersection of the "instructing" and "reflecting" rows with the "gaining knowledge" and "PD support" columns.)

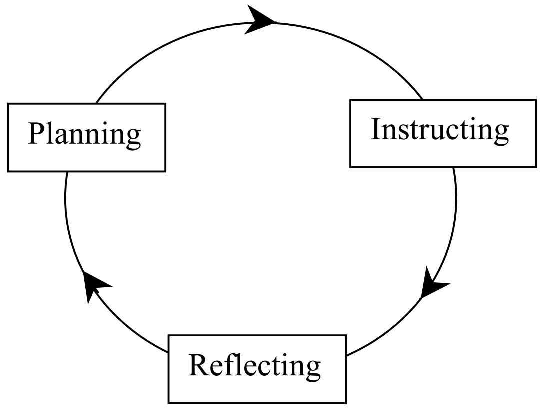
Finally, Little and Horn (in press) have examined discourse among teachers as a way of investigating what types of interactions prove powerful enough to generate new understanding. This is yet another example of how teachers can learn through reflection on their instructional practices. While many conversations involved teachers "normalizing" colleague's experiences (i.e. reassuring them that their difficulties were not uncommon), only some of these conversations went further to "open up opportunities for learning in, from, and for practice" (p. 4). Productive groups followed the normalizing comments by explicitly eliciting greater detail and initiating an analysis of the situation. These questions served as a crucial transition between non-productive and productive interactions. This work sheds light on the types of conversations Beth might have with other TAs to help them all interpret and learn from the interactions they have with students.

Taken together, these studies provide a basis from which to extrapolate to the context of novice TAs teaching in college classrooms. Although differences certainly exist between the K-12 setting of the studies above and the post-secondary context of TAs, the commonalities of mathematics and teaching provide a fertile ground for examining the work of MTAs and the types of PD that might be provided to help them learn more from their own teaching. As such, they shed light on the opening vignette and how PD might help Beth pursue her own teaching questions in ways that are generative of knowledge that can improve her students' opportunities to learn.

How Activities of TAs Use Knowledge of Student Thinking

In this section, we take a closer, more detailed look at the "using knowledge" column of the framework and examine the activities of TAs and focus on the many ways in which their teaching could be informed by knowledge of student thinking.

Graduate students who have responsibility for instructing students plan for that instruction, enact that instruction in classrooms with students, and plan for their next class by taking into consideration what happened during the class. As we saw in the case of Beth, teachers engage in a cyclic process of planning, instructing, reflecting, and planning again and such learning might take place at any point in this teaching cycle:



Knowledge of student thinking is potentially useful in all three phases of this cycle. Here we detail how TAs might make use of such knowledge. Throughout, we will refer to Beth's situation in the vignette, without restricting ourselves to that particular situation.

Reflecting. In the vignette we stepped into the teaching cycle at the reflecting stage and so it is there that we begin our examination of teaching practices. Beth, while reflecting on her previous teaching experiences, used her general knowledge of student thinking to interpret students' actions as misunderstanding the definition of convergence of a sequence. Beth's knowledge of student thinking provided a lens that enabled her reflection on past classes to shed light on how students approach sequences.

Beth returned to the act of reflection at the end of the vignette, processing how her class transpired in real time. She reflected on students' reactions, informing her teaching decisions and possibly providing background information to inform the planning of future classes. Additional consideration of how the class session met particular goals is apt to occur after class, as planning for the next class begins. This processing of the class events might include thinking about how the class matched the plans and the extent to which students learned what was intended. Then, using knowledge of how students think about the ideas in question as an interpretive lens, plans can be designed to get students closer to the learning objectives in the next class or to build on what was learned in service of tackling the next topic.

Planning. Now we move to the top of the "using knowledge" column and look at Beth's use of knowledge as she planned for class. In the vignette, Beth considered experiences from her previous time teaching sequences as she planned the examples to use with her class. Recalling the specific difficulties that students had with situations involving particular kinds of sequences, she decided to construct an example to use in class. The example that Beth created was intended to help students make the relevant distinctions and strengthen their understanding of convergence. When she made the decision to include such an example, Beth considered a variety of factors, including where students were in the process of learning the ideas, what kinds of sequences they had already worked with, and what ideas the created example was likely to cause the students to think about. Thus she was *using* knowledge of student thinking; however, it was unlikely that she was *generating* new knowledge.

The planning portion of the teaching cycle could be made more generative in several ways. Having groups of teachers do their planning together, learning from each others' insights into student thinking, might provide an opportunity to build new knowledge of student thinking. A teacher's planning might also include reading research on student thinking and how it might be incorporated into their classroom. Finally, some Just In Time Teaching (JITT) techniques involve using on-line student responses to plan class, which could be generative of new knowledge of student thinking.

Instructing. Once in the classroom, plans may play out just as the teacher had envisioned or modifications may occur in the midst of class. As each part of the plan is implemented, the outcome is compared with the goals for the class and then the decision is made about whether or how to alter the next part of the plan. For example, in the middle of class students may ask questions that indicate they have misunderstood some idea from a previous class (or course). In such situations, teachers must decide whether to change their lesson plan to address the students' difficulties. Such choices can be informed by various factors, including the teacher's sense of the importance of the concepts in question, ideas about how students are making sense of the new concepts, and how to best create opportunities for students to learn the material. Such decisions can be shaped in significant ways by what a teacher knows about how typical students think about the ideas, how the ideas relate to students' prior knowledge, and how the current ideas will be built upon in future lessons. Teachers who possess richer and more detailed knowledge of student thinking are positioned to make decisions that are better informed and more closely tied to how students are likely to be experiencing the lesson.

In the vignette, the initial part of class seemed to go as Beth had envisioned. As students discussed her special example, however, she faced some unanticipated decisions. When the discussion between the two students generated opposite answers, Beth needed to decide, on the fly, how to respond in ways that would work toward her goal of resolving the confusion that she believed (from her knowledge of student thinking) students were having about the nature of monotonic sequences and convergence. Deciding what to do next could be guided by interpreting the students' comments correctly, using knowledge of typical student misconceptions. For example, when Maria says, "They *both* go to zero," she is probably interpreting the sequence in question as a combination of its two disjoint subsequences, a misconception detailed in the literature on student thinking (Tall & Vinner, 1981). Knowing this, Beth's response might be, "Interesting, is this one sequence or two?" then leaving the students to investigate the source of their conflict.

TA Activities Generating Knowledge of Student Thinking

As illustrated above, a TA can have ample opportunities to *use* knowledge of student thinking in the course of going through the teaching cycle. Where might a TA *gain* that knowledge? Given the pressures of graduate schools, the most realistic answer is "on-the-job." This section is a tour of the "gaining knowledge" column of the framework, providing an examination of how the activities TAs engage in might provide opportunities for them to learn.

Here, we analyze the teaching activities of college mathematics teachers, with a particular focus on the aspects of those activities that could be generative of knowledge of student thinking. Beth's product rule and the chain rule and it occurs to the MTA that students have not yet reached the chain rule section of the course, and thus determines that the problem is appropriate for this quiz. In constructing the example for her class, Beth knew that her students understood some things about sequences and that they were familiar with certain classifications for sequences such as monotonic. Those elements of knowledge enabled Beth to write a problem that was accessible to her students and, by knowing what content was yet to be discussed, she also avoided creating an example that depended on understanding concepts her students were not yet familiar with.

In these circumstances, MTAs also have opportunities to consider the relative importance of topics within the curriculum. When constructing her problem for class, for example, Beth had to decide that this particular aspect of understanding sequences was of sufficient importance to warrant time in class. In a similar fashion, when an instructor writes a quiz, decisions need to be made about which of the many aspects of the topic merit being assessed in the finite number of available questions. No assessment can include all aspects of a topic and no class can address all ideas related to a concept, so aspects must be prioritized. To make such decisions, MTAs need to decide how important the different aspects of the topic are in the field of mathematics in general as well as how necessary they are in equipping the students to be able to learn the topics that are still to come in the course. Making such decisions makes use of and builds knowledge of how the discipline and the particular course are structured and how the different concepts/topics connect to and support one another.

Observing Students Working on Problems

After planning for class, MTAs' opportunities to acquire knowledge continue during their in-class interactions with students (represented by the "Instructing" cell in the "gaining knowledge" column of the Framework.) As suggested in the vignette, when MTAs observe students who are working on problems, they have opportunities to access many kinds of knowledge of student thinking. Such access to how students think, in real time, about content creates rich opportunities for MTAs to build a mental catalog of ways that students think about and make sense of particular ideas. In the research and PD programs discussed earlier (e.g., CGI), these are the very sorts of experiences that enabled teachers to acquire detailed knowledge of how children think about particular mathematical ideas. In Beth's case, she knew from a prior teaching experience that students did not fully grasp the concept of convergence, but she did not have a clear idea of what specific thinking was behind students' incorrect reasoning. Observing George and Maria's argument could potentially shed light on that question. While some of the knowledge gained at such times may be fairly generic, much of it might connect particular ways of thinking to particular topics and content (in the vignette, concepts of monotonicity, convergence, and sequences are all evoked).

In addition to how students think about particular content and the errors and strategies they use while working with that content, observing students as they work on problems creates opportunities to see how students think about *prior* content as they connect it with new ideas. For example, while watching students practice techniques for differentiation, MTAs may observe the varied ways that students approach simplification of algebraic expressions or they may notice students' implicit use of the idea of limit. In these situations, MTAs have access to knowledge of how concepts from earlier in the course or prior courses interact with the learning of the content at hand. Armed with such knowledge, MTAs can anticipate difficulties students will encounter and design instruction that helps students overcome those lingering difficulties while also making progress on understanding new ideas.

Finally, observing students working on problems potentially allows TAs to build knowledge of the many coping strategies students use to "get through" problems, with or without understanding the underlying concepts. For instance, watching students work on limit problems might lead a TA to learn that some students will simply plug in the limiting value without considering the underlying concept of the limit. Also, some questions are more difficult for students than others. As MTAs observe students working on problems, they have opportunities to acquire knowledge of the relative difficulty of different tasks.

Discussions with Students during Class

Discussing mathematics and mathematical problems with students during the course of a teacher-directed lecture also presents especially rich opportunities for MTAs to learn about student thinking. Asking open-ended questions, having students discuss a question in pairs and then share thoughts with the class, and other teaching techniques provide teachers access to student thinking. Even while answering homework questions, TAs might elicit (and learn about) some student thinking with many of the same questions suggested by the CGI program.

Grading Student Work

One of the major teaching activities TAs engage in is grading. Graduate students often have responsibility for grading homework, quizzes, and/or exams. Although "grading" probably conjures up images of examining students' written work, grading actually begins when MTAs create or select problems to use to assess students' learning. This is particular kind of planning that is part of MTAs' teaching practice where decisions are made about how best to gain access to what students know and can do. Next, students' written work is examined. One way to describe this part of the process is that of comparing students' answers to the known correct answer and assigning all or none of the points. While there may be some circumstances where that is how grading proceeds, in many circumstances graders instead assess *the extent to which* the student's work is correct and/or represents an understanding of the concepts being tested in order to determine how many of the allotted points the answers merits. This kind of diagnostic work requires that MTAs try to imagine what the student's thought process might have been to lead them to create the written work on the paper. Thus reflection on the grading process creates opportunities to build knowledge of both how students typically think about the ideas as well as the typical difficulties/mistakes that students make while learning these topics. Although the details of students thinking may not be as clear as it can be during observations of students working on problems, grading can provide an opportunity to examine the results of their thinking.

Office Hours and One-on-One Tutoring

Graduate students frequently interact with students outside the traditional classroom in office hours and in drop-in tutoring programs. This is another example of the intersection of the "gaining knowledge" column and the "instructing" row from the Framework in Figure 1. While the approaches taken during these encounters may vary, the general goal is to figure out what students are struggling with and help in ways that are more tailored to their specific issues than is possible in a classroom setting. The kind of diagnostic work that occurs during a class or during grading could possibly occur in a more intense way in these one-on-one settings since graduate students could ask a series of questions of the student until the nature of the difficulty is uncovered. These interactions may make use of the graduate student's knowledge of how students think about the topic while also helping to build on that graduate student's knowledge of student thinking. Potentially, these situations are rich sources of such knowledge.

Implications for TA Preparation, Professional Development, and Graduate Programs

In this section we describe what our analysis of the research literature and the teaching practices of TAs could imply for the design of experiences that support novice college mathematics instructor development. This is the discussion of the final column in the framework from Figure 1. We provide both some general thoughts as well as descriptions of several PD activities that could be used with TAs. In addition, we consider how to make good use of graduate students' non-classroom teaching-related assistantships as sites for learning about teaching.

Guiding Principles and Sample PD Activities

Teaching takes more than knowledge of mathematics and presentation skills, and it is impossible to completely prepare anyone for the enormously complex task of teaching, However, rather than just sending graduate students into the classroom with the expectation that they will eventually acquire the necessary knowledge, we can equip them with the skills and dispositions to seek out and acquire that knowledge in a more efficient way. Here we set out the principles for PD suggested by the literature review and our analysis of TAs activities. Given the successes of K-12 PD programs that focus on student thinking and the evidence that TAs do not begin their graduate careers with such knowledge, we suggest that PD at the collegiate-level needs to adopt a similar focus.

In an ideal world, interactive seminars taking several years would give novice MTAs ample opportunities to conduct numerous clinical interviews of students, plan and carry out classroom activities with groups of students, and reflect on these activities in diverse groups including all levels of graduate students, professors, and mathematics education experts. The financial reality of research institutions is that graduate student TAs provide teaching labor and their services are needed in the classroom from their first or second years. Thus, instead of providing a lengthy pre-service experience based on research on student thinking, circumstances dictate a more scaled-back approach focused on helping MTAs to learn *while* teaching.

Luckily, the research described above provides guidance as to how such PD might be structured. Much of what is done in pre-semester or on-going PD for TAs could be reshaped to include a focus on acquisition of knowledge of student thinking. By giving TAs brief opportunities to engage with student thinking and providing targeted, informed support, TAs will be more likely to transform their early teaching experiences into learning experiences. In this section we describe several ways in which this might be done.

Predicting student thinking.

One important function of knowledge of student thinking is helping teachers anticipate the mathematical concepts that particular tasks will prompt students to think about. Being able to make accurate predictions about typical solution strategies and difficulties for a particular task enables teachers to tailor questions to tap into the particular concepts/skills they wish to assess and enables teachers to anticipate different ideas that students might need to understand in order to tackle the task successfully. These kinds of predictions come more easily for experienced teachers and this activity can help develop beginning teachers' knowledge in this area and also sensitize them to the volume of such knowledge.

This activity encompasses all three rows (planning, instructing, reflecting) in the "PD to support learning" column of the Framework from Figure 1. In this activity, an MTA would analyze a problem that they are going to use with their students – an aspect of planning. It could be a problem from a homework assignment, for a quiz, or any other venue where the graduate students will have access to the students' written solutions. It could be a problem given to the graduate student or one they have selected or written on their own. They examine the problem and write out predictions about what solution strategies students will use—both correct and incorrect ones, what particular techniques/procedures they will use (appropriately or inappropriately), and what specific difficulties or mistakes will come up. Then the graduate students use the problem with their students, examine the students' spoken and/or written work, and compare/contrast what students actually did with what they predicted would occur, reflecting on the extent to which their predictions during the planning portion held up. Then graduate students can revise their initial list so it represents what the students actually did. This activity could also be done as a pre-semester activity using samples of existing student work. One possible variation on this activity is to have graduate students make their predictions and talk with an experienced TA or professor to gather their predictions, then conduct the rest of the activity. This variation could help graduate students recognize that this type of knowledge is potentially something acquired by instructors as they gain teaching experience. Graduate students could also share and discuss their lists so they can see which student strategies and difficulties are common across different topics.

Using research articles to examine students' thinking.

While a central message of this article is the point that graduate students need to learn about student thinking from their teaching experiences, there is a substantial collection of research articles that represent what is known in the field of mathematics education about how students think about particular mathematical concepts. This activity provides graduate students with opportunities to become aware of this literature base and also to see how the findings of this kind of research may provide insight into how their students are learning/understanding particular mathematical ideas.

There are articles that report on research about student thinking in many areas, including function, limit, and derivative (see, e.g., *Making the connection: Research and practice in undergraduate mathematics education* (Carlson & Rasmussen, in press)). Such articles often present analyses that include categorizing student ideas or difficulties in various ways. After reading such an article, graduate students can examine examples of students' work on problems in the area of choice and reflect on the extent to which the categorization scheme used by the researchers fits the students' work. This activity (representing the "reflecting" row in the Framework) could be done with student work from the graduate students' classes or with existing or created work that represents students' thinking about the ideas.

Interviewing students about their thinking.

Findings from the CGI-based research on teacher practice indicate that as teachers learn more about how students think about particular ideas, they are more inclined to both use that information in teaching decisions and to interact with students in ways that provide more access to that thinking. One of the major ways that teachers can both base their teaching on students' thinking and learn more about that thinking is by asking many questions of their students. Having students explain their ideas, both correct and incorrect, provides teachers with insight into different ways of thinking. Incorporating questions of this sort into one's practice is something that can be learned. This next activity is designed to give graduate students opportunities to work on asking these kinds of questions, to reflect on what they learn about student thinking, and to develop an appreciation for the fact that there is much to know about how students think.

MTAs select or are given a problem from a topic that is coming up in their course. Graduate students without classroom-based assignments can also participate in this activity— for them, it might be most useful to use a problem that connects to some major concepts in a course they are likely to teach in the future. Students are recruited to work on the problem and to discuss their ideas with the MTA. During this time, the graduate student is NOT coach students towards a particular solution, but is only to ask questions that prompt the student to explain what they were thinking and why. In educational research, this kind of interaction is called a *clinical interview*. Graduate students could write up what happened during the interview, summarizing the students' thinking. In a variation of this activity, all graduate students could use the same task in the interviews and then meet and describe the ways the students thought about the ideas, comparing and contrasting students' ways of thinking. By concentrating on asking questions that elicit student thinking in an interview setting, graduate students may be more inclined and prepared to ask such questions in the context of their teaching, either for the particular topic from the interview or more generally.

Assignment Decisions

Decisions about which courses are taught by novice versus experienced TAs are made in different ways across institutions. In some departments, TAs are assigned to teach the mathematically least advanced courses first and then move up through the curriculum over time. In other departments, the particular challenges of teaching lower-level courses are seen as more appropriate for TAs with substantial teaching experience and thus beginning TAs are assigned to teach more mathematically advanced courses. In addition to (or perhaps in place of) these priorities, it could be constructive to consider which teaching assignments provide graduate students with the most access to student thinking and then assign novice TAs to those assignments. These teaching assignments might be ones where TAs have increased one-on-one time with students and have opportunities to reflect in groups on their interactions with other TAs and with facilitators who are trained to help move discussions from normalizing to transformative.

As noted above in the discussion of possible PD activities, there are a variety of ways that graduate students with non-classroom assignments can learn from their experiences. In fact, absent the time (and other) pressures of teaching a class, graduate students may be especially well-positioned to inquire into and learn about student thinking. As a result, there is much that can be done to structure learning opportunities for graduate students who are grading or tutoring. Graduate students doing this kind of work have a great deal of access to individual students' thinking and PD programs should take advantage of this and help graduate students learn as much as possible during these times.

Concluding Thoughts

Being a professor entails many different things and graduate school, no matter how intense, cannot possibly prepare people completely for the varied demands of academic life. Given the multifaceted goals of graduate education and the finite time spent in graduate school, it would be constructive to focus attention on helping graduate students learn how to learn from their experiences. This approach of enhancing on-the-job learning has shown promise as a means for professional development and instructional improvement in K-12 settings. With attention to the particular features and constraints of the college setting, such approaches are apt to be effective in equipping graduate students to learn in the context of their experiences as they begin their teaching careers and in subsequent years.

Much remains to be examined in this area. From findings at the K-12 level, there is reason to expect that this approach to PD will lead to increased student achievement, but such an outcome needs to be established empirically. To inform the design of PD activities as well as potential research programs, a richer understanding is needed of the developmental trajectory of graduate students' knowledge of student thinking. For example, what do TAs enter graduate school knowing? How does a shift of focus from PD for teaching to PD for learning shape TAs' practices? What do MTAs leave graduate school knowing and how might that be enhanced so that their students (now and in the future) can have the best possible learning opportunities?

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