LOOKING AT TEACHER COLLABORATION THROUGH THE LENS OF INTERSUBJECTIVITY

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Abstract

This paper is part of a collection of documents that report on the ongoing research project "*Teachers Promoting Change Collaboratively (TPCC)*", which is intended to improve mathematics teachers' instruction by improving their mathematical knowledge and attention to students' thinking. Specifically, we present the case study of two middle school teachers, Quinton and Marcy, who participated in the above mentioned project. We examine the ways in which Quinton's and Marcy's conceptions of trigonometric functions were refined as they collaboratively attempted to build a unit on trigonometry. We also examine how holding different ideas about trigonometry affected their intersubjectivity over time. As their planning of the unit progressed, differences in their conceptions of both trigonometry and how it needed to be taught became evident as they struggled, unawarely, between trigonometry based on special right triangles and trigonometry based on the unit circle.

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Introduction

This paper is part of a collection of documents that report on the ongoing research project *"Teachers Promoting Change Collaboratively (TPCC)"*, which is intended to improve Mathematics teachers' instruction by improving their mathematical knowledge and attention to students' thinking. Specifically, we present the case study of two middle school teachers, Quinton and Marcy, who participated in the above mentioned project.

As participants in the project, Quinton and Marcy were part of a group of teachers that took a mathematics course intended to support their development both of covariational reasoning and of the process conception of functions, as well as to re-conceptualize the middle school mathematics curriculum. One of the modules of the course was about developing a coherent understanding of angle measure and trigonometric functions.

Quinton and Marcy met each week with other teachers in their school in "*Reflecting on Practice Sessions (RPSs)"*, which are guided by a facilitator. One of the activities that were assigned to the teachers during the RPSs in the spring of 2006 was to develop and implement a unit on trigonometry. Each session was videotaped and analyzed.

Based on the framework of intersubjectivity (Steffe &Thompson 2000, Thompson 2000, Thompson and Miller 2006), that has as its aim to understand mathematics teachers' professional interactions in order to be able to better support them, we examine the ways in which Quinton's and Marcy's conceptions of trigonometric functions were refined as they attempted to build their unit in a collaboratively manner. We also examine how holding different ideas about trigonometry affected their intersubjectivity over time. As their planning of the unit progressed, differences in their conceptions of both trigonometry and how it needed to be taught became evident as they struggled, unawarely, between trigonometry based on special right triangles and trigonometry based on the unit circle.

Theoretical Framework

Intersubjectivity

From a radical constructivist perspective, people do not communicate meanings per se. Rather, the communication of a meaning is accomplished by the listeners, who attribute meaning to what they hear, making it compatible to their understandings (Thompson 2000).

A model of communication as described by Von Glasersfeld (1995) is as follows: to say two people communicate successfully means that they have arrived at a point where their mutual interpretations of each other's meanings are compatible.

Intersubjectivity is the state of dynamic equilibrium where each participant in a conversation feels assured that the other participants think pretty much as he or she anticipates they do. There is no need for consensus or agreement for this dynamic equilibrium to take place. Rather, it is a claim that no one sees a reason to believe that others think differently than he or she presumes they do (Thompson 2000).

A state of *intersubjectivity* can be sent into mild ("*ruffled*") or severe ("*punctured*") disequilibrium in several ways, two of which are the following: 1) one or more participants detect that someone does not mean or believe what they had presumed she means or believes; and, 2) participants detect that the interactions of intended and construed meanings are incompatible, but they cannot locate the source of this incompatibility.

Equilibrium can be re-established in several ways, including: 1) individual participants rethinking their own meanings so that they become compatible with their new understandings of others' meanings; 2) individual participants rethinking their understandings of others' meanings so that they are more compatible with their own; 3) both the two above reasons simultaneously; and, 4) individual participants accept the disequilibrium as a persistent state (Thompson and Miller 2006).

Angle Measure as Arc Length

In order to have meanings of angle measure and trigonometric functions that are coherent not only while working with right triangles but also while working with functions, Thompson, Carlson and Silverman (in press) propose to define “degree”, “angle measure in degrees”, “sin” and “cos” in the following way so that they are compatible in both contexts:

> Suppose we have an angle. Assume an arbitrary circle centered at the angle’s vertex. By “degree” we mean an arc on the circle whose length is 1/360 of the circle’s circumference; by “angle measure in degrees” we mean the length of the arc subtended by the angle, measured in arcs of length 1/360 of the circle’s circumference. Imagine an embedded right triangle made such that its hypotenuse is the circle’s radius and one of its angles is formed by the angle in question. By “sine of an angle” we mean the percent of the radius’ length made by the length of the side “opposite” to the origin in the embedded right triangle. By “cosine of angle” we mean the percent of the radius’ length made by the length of the side “adjacent” to the origin.

An important aspect of defining things in such a way is that we are able to answer questions such as the following: How much is sin(cos 35°) in degrees? And, we can do so in a meaningful way, since both the input and output of each of the functions involved are lengths. Otherwise, if by “cos” we mean adjacent over hypotenuse, then how can a ratio of two lengths be measured in terms of rotation (degrees)?

Covariational Reasoning

Covariational reasoning refers to coordinating an image of two varying quantities and attending to how they change in relation to each other. (Carlson, M., Jacobs, S., Coe, E., Larsen, S., & Hsu, E. 2002). In this context a function is defined as an invariant relationship between two covarying quantities. In particular, covariational reasoning may be considered to play an important role when studying trigonometric functions. For example, if the arc length *x* varies from 0° to 90°, then the values of *sin x* vary from 0 to 1.

Method

The two teachers in this case study, Quinton and Marcy, were middle school mathematics teachers that taught in a school located in a southwestern metropolitan area in the United States. They were part of a group of teachers that volunteered to participate in the TPCC project.

As part of the project, the teachers took a mathematics course intended to support their development of both covariational reasoning and of the process conception of a function, as well as to re-conceptualize the middle school mathematics curriculum. One of the modules of the course was intended to develop a coherent understanding of the concepts of angle measure and trigonometric functions. From the data obtained in the course (videotapes of the classroom sessions and written artifacts) there is evidence that Quinton and Marcy might have developed different ideas about the trigonometry module.

Quinton and Marcy were members of the same group of teachers that met each week in RPSs guided by a facilitator. During the Spring semester of 2006, the teachers were asked to develop and implement a unit on Trigonometry. Each of the 15 RPSs was videotaped and analyzed by members of the research project team.

The framework of intersubjectivity was used to analyze RPSs as follows: first, to analyze each teacher's basis in meaning and, second, to examine interactions among meanings and their repercussions. In order to accomplish this, the discourse of the sessions was used to create a map of teachers' meanings and their interplay with other teachers' meanings. The statements made by the teachers and their reactions to others' statements were used to construct the hypotheses about their meanings. For the second phase of the analysis, our focus was on the interactions and the discourse surrounding perturbations of the intersubjectivity. The reason to do this is because these perturbations can reveal incompatibilities in teachers' understandings of each others' meanings, creating opportunities for the teachers to re-evaluate their held meanings.

Discussion

Purpose of the unit on Trigonometry

At the beginning of the semester, the teachers decided that they were not going to include the specific material of the course in the development of the unit, since they were going to implement it with Algebra I students and they considered that the material was too sophisticated to use it with these students. Although they were not going to make use of content of the course, the teachers emphasized the importance of building connections to prior knowledge. They argued that at this level (Algebra I) students have already seen the ratios and formulas in trigonometry, but that they only make use of them as a tool to solve simple problems related to triangles. They agreed that the focus of the unit would then be to introduce angle measure as arc length as well as to stress the covariational nature of sine and cosine.

In order to accomplish the goals of the unit, the teachers began to develop different activities for their students based on the unit circle. These activities included measuring angles by focusing on the arc length of the circle instead of focusing in "the amount of rotation" and determining the values of sine and cosine of an angle by comparing the vertical distance and the horizontal distance with respect to radius of the circle.

Differences in what Quinton and Marcy intended to be the purpose of the unit became evident during RPS 7, when the facilitator suggested a shift to focus on the coordinates of the point on the unit circle instead of the triangle that was drawn on the unit circle.

In the case of Quinton, he reconsidered the weight that he had given to right triangles in the lesson and became aware that if the main purpose of the lesson was to connect students' knowledge about right triangles with the idea of sine and cosine as functions, then the purpose of drawing the triangle on the unit circle while working with the horizontal distance and the vertical distance, was to avoid the idea that sine and cosine were now used in a completely different context, but eventually triangles should not be the focus of attention anymore. Instead, he would stress the importance of the coordinates of the point that was drawn on the unit circle and how the values of sine and cosine would vary as the arc length varies as well. Marcy's reaction, on the other hand, was the following:

> **Marcy:** *We are expecting them to unlearn... [Students will think] I've already been taught SOHCAHTOA, but now they want me to learn this [the unit circle] …*

For Marcy, introducing the unit circle in the lesson was only as a new tool to help the students work with right triangles. The use of the unit circle was convenient because the hypotenuse of the triangle was 1 and the calculations were made easier. She did not agree with the facilitator in that the focus should be the coordinates of the point rather than the triangle itself because what the students needed to know was right triangle trigonometry.

Angle measure-arc length

Another situation in which Quinton and Marcy held different meanings was in relation to angle measure; even after the teachers stated at the beginning of the semester that they wanted to include in the unit *"angle measure as arc length"*. In Marcy's case, she would still think of measurement of an angle as an amount of rotation. This was made evident during RPS 3 when she was trying to explain what a degree is:

> **Marcy:** *The 360, is a way of measuring, a degree is the measurement. A degree is the amount of rotation from the initial side.*
>
> **Facilitator:** *is there always rotation?*
>
> **Marcy:** *if you are talking about measurement yes.*

In the case of Quinton, his idea of angle measure was not related to amount of rotation. His focus was on the arc length and not on the amount of rotation. The next excerpt is from RPS 4 in which Quinton was reading the main ideas that he wanted to include in the unit:

> **Quinton:** *A degree is a portion 1/360 of the circumference of the circle…A degree is a unit of measurement that is relative to the size of the circle…Different circles have different arc lengths that have the size of 1 degree…An angle is the measurement of the portion of the circumference of the circle it intercepts.*

From this perspective, to eventually shift the focus from the triangle to the coordinates of the points on the circumference would be more coherent than if there is no reference to a circumference when measuring angles.

Covariation

One of the main ideas developed in the course in which the teachers participated was about the importance of covariational reasoning as a means to develop a better understanding of the concept of a function. The facilitator encouraged the teachers to include the idea of covariation in the unit as a way of thinking about trigonometric functions.

The idea of covariation that Quinton wanted to include in the unit was the following: "*as the angle increases from 0° to 90°, the sine increases from 0 to 1(radius). As the angle increases from 0° to 90°, the cosine decreases from 1 to 0(radius)*". The quantities included in this idea of covariation are the arc length and the values of sine or cosine.

Every time that the members of the group would talk about covariation, they would agree with Quinton, except for Marcy. Marcy's idea of covariation was: *"as the angle changes, sine and cosine change simultaneously"*. The idea that she wanted to reinforce was not that of sine and cosine as functions. Instead, her focus was on complementary angles. Once again, this was completely connected to trigonometry based on right triangles.

During RPS 6, when the teachers were working on the assessments that they were going to use as part of the unit, Marcy expressed her idea of covariation:

> Marcy: In my point of view, we haven't asked anything about covariation, as far as sine and cosine varying together. Are we going to ask about sine and cosine changing simultaneously?
>
> Quinton: As the angle changes what happens to sine and cosine…it would be three variables changing at the same time.

Quinton's reaction was that they shouldn't focus on three quantities changing at the same time. She apparently agreed with Quinton, but as the semester went by, Marcy would keep coming back to the same idea of sine and cosine changing together and complementary angles as the only valid idea of covariation.

Implications for the Research Project

As a result of teachers holding such different meanings in the development of the lesson and never being prompted to negotiate those different meanings, the implementation of the unit did not progress as expected.

Some of these unexpected results were that the number of days assigned for the implementation of the unit, were not enough to develop all the activities that were planned, because of the time that was devoted to explain things that were not clear, even for Marcy. In her case, although the lesson was supposed to be a connecting link between right triangles' trigonometry with functions, she was surprised that the students did not use *SOHCAHTOA* anymore while working with trigonometry. So, after the implementation of the unit she went back to teach special right triangles.

In terms of the research project, the modifications that were made due to the analysis of cases such as the one of Quinton and Marcy was that the project goals and theory were made more explicit to the facilitators. It was paid special attention to orient the facilitators to help teachers speak meaningfully. This means that the teachers were continually asked to explain what they meant when they talked about mathematics.

Finally it was placed greater emphasis on teachers imagining the effect of their teaching on students' thinking. To continually ask themselves: what sense are the students making of what I say?

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