Abstract

This paper will use analyses of classroom discourse to characterize the effect that deliberate efforts within the projects’ courses to have teachers speak meaningfully had on their conceptions of mathematics and mathematical explanations. Data from videotaped sessions of the courses indicate a synergy between their attempts to speak meaningfully, as opposed to speaking in jargon or shorthand, in regard to specific problems and ideas and their creation of more powerful and more coherent meanings regarding those problems and ideas.

Teachers Promoting Collaborative Change (TPCC)

Teachers Promoting Collaborative Change (TPCC) is a five-year research project aimed to produce a model to help mathematics teachers develop professional learning communities (PLC). The model is designed to help teachers to acquire content knowledge and adopt methods that will improve student learning. (Thompson & Carlson, 2006) To support teachers in developing a PLC, the TPCC designed a two-sequence course to affect their mathematical knowledge in ways that would help promote the transition of a collaborative group of teachers to a PLC.
Likewise, the courses were designed to run in tandem with *Reflecting on Practice* sessions (RPSs) that were designed with the intent that teachers learn to imagine students’ performances as being grounded in underlying concepts rather than procedures and algorithms. Each RPS had a facilitator to help teachers discuss classroom practices and student thinking.

The assumption is by affecting teacher’s mathematical knowledge and focusing teachers on student thinking this will increase the probability to teach mathematics for understanding. It is hoped that this new focus and knowledge will serve as a model for students to strive for coherence in their own learning.

*TPCC, Observations of PLCs and RPSs of the first year*

In the implementation of the RPSs and the functions course, it was anticipated that by affecting teacher’s mathematical knowledge there would be a transfer into their classroom practices and a focus on student thinking. However, it was observed that what the teachers learned in the classroom remained in isolation from classroom practices. That is, the activities the teachers engaged in to promote conceptual learning were viewed belonging in the functions course but not as models of activities for teachers to use in their own classroom. Moreover, the teachers were engaged in understanding their own learning process but did not use this engagement as a means to understand student thinking.

In the RPSs, it was observed the teachers were exchanging ideas and meanings. However, the images of the meanings they discussed were exchanged in such a way that the teachers assumed that they all shared the same meanings. There were failures in communication that occurred among teachers because of lack of negotiation of meanings.
The Intervention of Speaking Meaningfully placed for the second year

To address the failures of communication in the RPSs, the format of the course was redesigned. The purpose of the course became essentially two fold. First, the course designs were predicated on the assumption that teachers could not teach for understanding without themselves understanding the mathematics they teach as comprising a coherent body of meanings—a richly and coherently connected body of ideas, images, and ways of thinking (Thompson, 1984). The coursework and classroom activities are purposely designed to enhance the content knowledge of the teachers. Secondly, the classroom norm created by the course instructor was to be a model for the participating teachers to speaking meaningfully.

Speaking Meaningfully

The term speaking meaningfully can be open to many interpretations and the teachers’ constructed their own understanding of this idea. To help them to begin a formulation of a definition, the teachers were given at the beginning of the course a short article that defined conceptual conversations. A conceptual conversation is “…one that has a diminished emphasis on technique and procedure, and an increased emphasis on images, ideas, reasons, goals and relationships” (Thompson, 2006). The implications behind the ability to maintain a conceptual conversation is to focus student understanding on the underlying mathematical ideas and to emphasize on the process of thinking rather than focusing on the final answer. Therefore, a teacher who can speak meaningfully is to be able to engage in a conceptual conversation about mathematical concepts.

As part of their professional development, teachers who will speak meaningfully are teachers who have a coherent understanding of the mathematics. With the
establishment of this classroom norm to speak meaningfully, we wanted to see how this would impact the teachers enrolled in the course.

The methodology to observe this impact is based on video observations of cohort of teachers enrolled in the functions course where speaking meaningfully made explicit by instructor and teachers, analyzed the structure of the teachers lesson designs and conducted one on one interview with two particular teachers.

The paper will focus on of two teachers, Janine and Clark, who had developed consequentially different understandings resulting from this norm. We will then use a comparison and contrast of Janine and Clark’s efforts to examine how their mathematical knowledge developed over time and how this affects their design of instruction. Both teachers were enrolled in the functions course and in different PLCs.

**Four factors in the Teachers’ construction of Speaking Meaningfully**

The instructor made a continuous effort throughout the course for the teachers to speak meaningfully. Since the course served as a model of speaking meaningfully, Janine and Clark developed very different interpretations of speaking meaningfully over the first sequence of the course.

Many factors played a role in their formulation of and effort to speak meaningfully. However, there seemed to be four predominant factors that indeed play a strong role in their concept of speaking meaningfully. These influences are:

1. The teachers’ personal views and beliefs about mathematics. The impact of the course played a different role on their views and beliefs of mathematics. Over the time period, the two teachers in this case study have views and beliefs that took different trajectories.
2. The amount of time both teachers spent on reflecting to understand mathematics as interconnected ideas rather than compartmentalized topics. This includes the time spent on understanding a ‘functions’ approach to their secondary school
The amount of time spent on reflecting how speaking meaningfully will help in student thinking.
3. The development of their content knowledge over the period of the course.
4. Their understanding of what it means to be a concept and what it means to be a topic.¹

The construction of the definition of speaking meaningfully for Janine and for Clark

As the course progressed over the semester, each teacher developed their own distinct understanding of what it means to speak meaningfully. Their construction of their definition is closely related to their beliefs of mathematics, their development of their content knowledge, their definition of a concept, and their time spent reflecting on the emphasis of a ‘functions’ approach to the mathematics in their curriculum. These four aspects are recurrent themes that appear in the interviews and their discourse in the classroom.

In the case of Janine, her views and belief of mathematics did not change much over the period of the course. Moreover, her definition of what constitutes a concept deeply affects how she views speaking meaningfully.

With Clark, his views and beliefs of mathematics did change. In fact, he values the importance of ways of thinking and his image of mathematics is to view concepts as interconnected. However, his definition of concept is closely tied to topics.

Case 1: Janine

At the beginning of the course, Janine had been teaching four years at the high school level. It was her first year teaching precalculus and also taught an algebra course. Her PLC consisted of other mathematics teachers who were also in the functions class.

¹ To clarify, we use the term concept as a way of thinking and the term topic as mathematical subject matter.
Her mathematical knowledge she admits has improved significantly over the course of her four years teaching. Her knowledge is highly dependent on the textbook. That is to say, she can calculate and solve problems in the book and her understanding of topics is well aligned with the format of the textbook.

It is clear from her participation in the course that she enjoys mathematics. She is genuinely engaged in the tasks given in the course. She enjoys solving math problems. Her enjoyment of mathematics stems from the fact that the arguments to solve a problem are clear and concise. To her, mathematics lies in the end result or the final presentation. During classroom discourse, her style of speaking focuses on clear presentations and well thought ideas.

She views mathematics as topics rather than concepts. Janine has topics compartmentalized much like a textbook. Janine’s content knowledge was procedural at the beginning of the course. Over the period of the course, she has equated concept with topic. Her view is to understand a concept is to understand the topic at hand. By equating concepts with topics, Janine has expanded her mathematical knowledge. For example, by incorporating covariational reasoning in her problem solving abilities, she views this way of thinking isolated from her mathematical knowledge. Since she views concepts and topic as the same, Janine uses concepts as another topic that she has learned in the course.

Janine's views of mathematics are consistent with her definition of speaking meaningfully. That is, her view is to give what she sees as coherent. In an interview after the first part of the course ended, when asked what speaking meaningfully meant,

*If I have something to say I try to make it concise and clear and make sure students understand. What I focus on more in this [functions] course is being more conceptual and focusing less on the process and more on the concept behind it.*
Her focus is on clarity and presentation centered on her own coherence. It seems that if she focuses on being clear and concise that students will understand her. Student thinking never became an issue because the concepts were coherent to her. Moreover, in her definition she assumes this is sufficient for student learning.

In another portion of the interview, she mentions that with the experience she has improved her mathematical knowledge therefore her ability to speak meaningfully has improved. With the following quote, she focuses on what she is saying rather than how her students may understand what she is saying. There is no indication whether she is incorporating a model of student thinking in her speaking meaningfully.

*I’ve been speaking more in meaning every single year that I teach. By taking this class it makes me think more about what I am going to say. It has probably helped me do that a little bit better. Every year that I teach I understand the content better and better... and as far as I’m concerned it’s my own increased understanding of the content that has helped me to explain it more clearly and be more conceptual with my students.*

**Case 2: Clark**

At the time, Clark had been teaching mathematics at different age levels. It was his first year teaching high school mathematics in his district. He was previously a middle school teacher for four years. He is an enthusiastic teacher that genuinely wants to help his students and is willing to find alternative ways to teach. He is the only member of his PLC currently enrolled in the functions class.

Clark admits his mathematics and style of teaching is purely procedural before he began the course. He was an active participant in the class. His enthusiasm for mathematics was clear throughout the course. Clark asked questions to understand how to solve a problem and tried to understand his own thought process as he was solving
problems. More often than not he used metaphors or analogies to describe concepts to himself. On more than one occasion, Clark has reflected on the materials from the previous class. He is constantly asking for feedback and suggestions on how to understand something. His desire to learn is planted in how this all fits in the big picture. His views of mathematics and content knowledge were challenged. By his reflecting on his own struggles, he realized the impact speaking meaningfully may have on student learning. Hence, he made explicit attempts in decentering. He was beginning to see the power of teaching mathematics with a focus on a thinking process rather than focusing on answers.

Clark has equated a concept with a mode of thinking that may be applied across topics. However, Clark is still struggling with the definition of concept. There is a difficulty in his distinguishing between using a topic to illustrate a concept and seeing the topic as a concept.

At an interview conducted at the end of the course, Clark discusses his definition of speaking meaningfully. His definition is a reflection of his understanding of how speaking meaningfully can be a window to his own mathematical meaning and how it can influence student understanding. Student thinking is an issue for him and he wants to express himself in a way that is optimal for students to construct meaning.

*My definition of speaking in meaning would be to make sure your words make sense and somebody else can process those words and the meaning behind the words. Sometimes though in the class I feel like I am not saying it the right way. Like there is this cookie cutter phrase I’m supposed to say and I can’t seem to remember those, I’m not good with script…. If you are going to say something don’t just say the answer, explain to me how I can understand what you are saying and seeing what is in your mind…So that’s what I mean by speaking in meaning. [Later on in conversation] if you start talking about stuff like a calc book would talk to an 8th grader then BOY you just lost them. Then each word, if I say thingy ot an 8th grader that’s still confusing to them too so I have to be*
more specific but yet not over their head... This variable is moving at some constant rate, now does that make sense? Then you have to ask yourself does that make sense to them? Does it make sense to me? Is it what is actually going on in the math?

In summary, Janine’s definition of speaking meaningfully, was to present clear, concise, and well thought out conversations. However, the coherence was centered around her own understanding rather than focusing on student coherence. In her understanding of speaking meaningfully, she was in fact doing this in the functions class as well as in her classroom practices.

Clark’s definition of speaking meaningfully was shifting the focus of his conversations to student thinking. As a result of his own reflection, he realized the need to speak in a coherent manner is a way to orient students’ way of thinking towards concepts.
References


