Teacher Decentering

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ABSTRACT

This paper will outline a year of teacher interactions within the Teachers Promoting Change Collaboratively project at Arizona State University funded by the National Science Foundation. Specifically, the focus is on the conditions and instances of teacher decentering as defined by [Steffe & Thompson (2000); Thompson (2000)] While the influence of the interactions remains individualistically subjective according to constructivist considerations, analysis highlights trends in teacher discourse when given the opportunity to construct a model of student thinking. The few observations of teachers' suspension of their own mathematical thinking in favor of a conjectured model of their students' thinking are discussed with possible explanations and implications for future interventions and research.

FRAMEWORK

The notion that individuals construct their own reality and that observers have no direct access to those constructions follows from a radical constructivist outlook. [von Glasersfeld (1996)] Using this lens, a goal of teaching would be a radical constructivist teaching model. Confrey (1990) discusses the implications of a constructivist framework in teaching; more specifically the communication processes of each participant's construction of intended meanings. She states, "a teacher must always give consideration to the possibility that a student's constructs, no matter how different they appear from the teacher's own constructs may possess a reasonable level of internal validity for that student and therefore must adapt the instruction suitably." (Confrey, 2000, p. 110) Moreover, classroom communication must not only be sensitive to the individual student's constructions, but also negotiating meanings at play. "The likelihood that a teaching communication will be successful is increased whenever the teachers' actions are guided by explicit models of the children's mathematical realities. From

this perspective, the activity of teaching involves a dialectic between modeling and practice." (Cobb & Steffe, 1983, p. 86) Thus with sensitivity to intersubjectivity teachers should work toward developing classroom interactions purposefully, allowing students to construct the intended interpretations of mathematical meanings (Thompson 2000). That is, in designing lessons, teachers must be aware of the interpretations of meanings possible by their students. These interpretations then become catalysts for classroom discourse that reveals students' misconceptions and productively transitions to more coherent mathematics. Steinberg et al. (2004) and Franke et al. (2001) demonstrated that a focus on student thinking is a mechanism for teacher change in both beliefs and practices. [Steinberg et al. (2004), Franke et al. (2001)]

> We propose that it is the engagement with student thinking that allowed teachers to develop understanding and connect ideas. As teachers engage with student thinking, they think about their daily work, about substance, content, and process, and about their own students. They come to see that they can learn through working with their own students in their own classrooms; they receive continual feedback as children discuss their thinking. Teachers can create learning communities that involve their students and their colleagues; they can learn as they engage with their students and continue that learning as they engage with their colleagues. In listening to their students and then talking about it with their colleagues, teachers are not simply sharing; they are building principled knowledge on which to base their ongoing instructional decisions. (Franke et al., p. 685)

In order to develop an understanding of their students' mathematical meanings or thinking teachers must construct models of student thinking. The idea of modeling student thinking draws from the work of Steffe & Cobb (1983), von Glasersfeld (1995), Steffe & Thompson (2000) and Thompson's (2000) work on first and second order observers and first and second order models. "First order observer addresses what someone understands, while a second order observer addresses what they understand about what the other person could understand." (Thompson, 2000, p. 423) Meaning that a first-order observer unreflectively considers others' (students or other teachers) meanings and perspectives, that is, the teacher takes for granted what

others mean. [Steffe & Thompson (2000); Thompson (2000)] However, a second-order observer consciously considers a multitude of possible perspectives that other (students or other teachers) might have to behave as they do. [Steffe & Thompson (2000); Thompson (2000)] In the analysis of the teachers' discourse it was necessary to add a third dimension to the framework of observers, namely a zero-order observer. A zero-order observer does not consider others' thinking (students or other teachers), even when the opportunity for observation is present. A first order model is a model (untested) that Person A constructs to explain Person B's behavior. Person A often imputes his/her thinking to Person B. [Steffe & Thompson (2000); Thompson (2000)] A second order model occurs when Person A imagines how Person B understands Person A. That is, Person A is actively constructing a model of Person B's thinking that includes Person B's image of Person A's meanings. [Steffe & Thompson (2000); Thompson (2000)] With a model of student thinking teachers are able to scaffold classroom interactions so that the intervention predicates a general construction of meanings based on hypothesized understandings. A teacher is said to have decentered when he or she has attempted to adopt another person's perspective. This refers to teachers' suspension of their own mathematical thinking in attempt to adopt view of students' mathematical thinking, that is, when he or she attempts to imagine one's experience from another perspective [Steffe & Thompson (2000)]. This model building, or decentering, is constructed by various information gathering tools utilized by the teacher, creating what Confrey (1990) calls a "case study" of each student. The second-order models that teachers create when teachers anticipate that the student's thinking may not correspond with the teachers' thinking. "The teacher's main goal in listening and observing is not to confirm their own mathematical thinking but to make images of and conjectures about students' mathematics." (Hackenberg, 2005, p. 49) These models are refined and evaluated

through classroom interactions. Cobb and Steffe (1983) claim that the teachers' understanding of the mathematical concepts and the students' conceptions of the mathematics implicitly guide teachers' classroom interactions. However, "teachers should continually make a *conscious* attempt to "see" both their own and the children's actions from the children's points of view." (Cobb & Steffe, 1983, p. 85) It is this conscious attempt to "see" students' mathematical thinking that is one focus of the collaboration sessions with teachers in the TPC^2 project.

REFLECTING ON PRACTICE SESSIONS

During the Fall 2005 and Spring 2006 semester the teachers in the TPC^2 project met weekly to discuss their teaching practices, to discuss student learning, and to discuss their reflection on the Functions Course that a majority of the teachers also participated in as part of the project. The tasks given to each PLC over the Fall 2005 semester and the Spring 2006 semester were slightly different in actual content; however, the inquiry tools remained primarily the same. In Fall 2005 the PLC groups were asked to interview students regarding integer operations and were asked to construct a meaningful lesson plan regarding an upcoming mathematical concept in their curriculum. During the Spring 2006 semester the groups were asked to focus on trigonometric concepts pertaining to angle measure using arc length and the corresponding trigonometric functions through that lens. During the construction of a five day lesson, the teachers were asked to interview students before and after the lesson, as well as give a pre-test and post-test comprised of the same questions. The content of the five-day lesson was introduced in the Functions Course that the teachers participated in and developed during the weekly RPS sessions. A person working for CRESMET facilitated the meetings and each meeting was video taped. The facilitator acted as a mediator, introducing topics of discussion and prompting for elaboration on teacher statements.

CODING

The coding scheme developed for the TPC^2 project through grounded research on the video data collected from each PLC group in the project. It is discussed further in other papers. This paper focuses on the category of teacher decentering as defined previously.

DECENTERING

As discussed earlier research concern decentering has shown the need for teachers to consider students' thinking. "Decentering, the ability to see a situation as perceived by another human being, is attempted with the assumption that the constructions of others, especially those held most firmly, have integrity and sensibility within another's framework." (Confrey, 2000, p.108) This project has implemented a similar definition for decentering with sensitivity to when the discourse in the RPS meetings there was an opportunity for teachers to construct a model of student thinking. These instances are then either coded 'yes' or 'no'. An instance would be coded 'yes' if the teacher considers how the student(s) are thinking and the implications that way of thinking entails, and the teachers suspend their mathematical understanding in an attempt to adopt the student's perspective. An instance would be coded 'no' if no teacher considered how the student(s) were thinking, and did not suspend their own mathematical understanding in an attempt to adopt the student's perspective. Beyond the issue of coding for the opportunity for decentering, the instances were also coded for zero-order, first-order, and second-order observers, as discussed above. Thus a teacher during a Reflecting on Practice Session (RPS) could have instances coded decentering 'no' and 'first-order observer' if they have an opportunity to suspend their mathematical thinking and attempt to construct a model of the student's understanding, however, the teacher attributes his/her own thinking to the student.

ANALYSIS OF INDIVIDUAL PLC'S

This paper will highlight the analysis of four high schools in the Tempe, AZ area.

Abraham Lincoln High School

The Fall 2005 semester presented opportunities for teachers to decenter, however, at Abraham Lincoln there were no instances of teachers suspending their model of mathematical thinking and attempting to construct possible models of what their students were thinking. During the reflection on the teachers' integer interviews with their students, the discourse focused on what the students actually answered during the interview, and the teachers were surprised with the students' inability to explain why they could perform simple integer operations. An example of the rare discussion of what students may be thinking occurred during the 5th session, and consisted of a teacher attributing her own thoughts to her students. The teachers' students were given a problem that asked the students to estimate a given angle in terms of arc length to which a majority of the students answered that the angle would be one-eighth of the circumference. In discussing the student responses this was one teachers statement:

> "I mean, they'd mark it off like, I even drew myself a sketch. Why would they do 1/8th? So I drew a 1/8th for myself. I know that's 45 degrees, I am not thinking of arc length. I know that's 1/8 of 360 and I just know that it's split up that way. I'm not using arc length."

In this statement, while the teacher is attempting to explain the student responses, she appears to be attributing her own thinking to her students. As stated above it was rare for the teachers to attempt to explain student responses, with even fewer instances of teachers decentering. The Spring 2006 semester offered more opportunities for teacher decentering with student artifacts present and greater facilitator sensitivity to probe for teachers' constructions of student thinking.

The Spring 2006 semester allowed the entire group to focus on one mathematical topic and due to the structure, more opportunities for teachers to decenter. In the dimension of not

decentering the teachers still attributed their thinking to students, but there were other distractors that were not present in the first semester. The teachers seem to put great emphasis on the correct answer to questions constructed for both the interviews and the pre/post-tests. In this area, right answers correlated to students' understanding, but the teachers rarely presented a model of student thinking beyond the correct answer. There were instances, discussed below, of teachers using student artifacts to construct a possible model of student thinking. Finally, the teachers engaged in vague generalizations about what students would think. While this is a transition to considering students' thinking, the statements appeared to be experience based with no observable evidence as to the model's construction. One example would be during a discussion during the 5th session regarding the preparation of a five-day lesson on trigonometry. One teacher stated:

> How did he get pi? The concept of pi being a ratio between circumference and diameter. And none of them, these are juniors, topics 6 students, and none of them have a clue about, and like I told you yesterday, they don't know there's a reason that pi is in a formula that has to do with circles or squares, that, there's no connection there.

The teacher makes a statement of student understanding with no evidence of how she came to make such conclusions. These vague statements seem to be based on teacher's experience, which is valuable, but not an instance of decentering. There were two conclusive instances of teachers decentering, both in the presence of student work. The first instance occurred during the 4th session when discussing the work presented by a student on the lesson's pre-test. The question asked students to create an angle with a given arc length. While one teacher initially stated that the student's need for a compass was the result of laziness, another teacher conjectured a model of what the student may be thinking.

> Teacher 1: Students were given a radian measure, and uh, we've done a lot, and they still seem to be confused about that. Like one student make an excuse about he didn't have a compass, and I said, you have a piece of string. So um...

Facilitator: So what did that tell you about that student's notion of angle measure?

Teacher 1: Well that particular student it told me that he didn't want to do his work, because usually he doesn't do his work.

Facilitator: But the fact that he wanted a compass?

Teacher 1: Oh ya, well that he had to have a compass, I guess he really didn't understand it, he just needed to strike that arc, have something to hold, hold it so that it would be the same distance away from a given point.

Teacher 2: I was thinking that he was thought he needed it for like degrees. He's still thinking of it as a rotation rather than a length, when you said it, that's what I thought he meant. He's still thinking of it in terms of degrees of rotation.

Thus Teacher 2 did in fact make a model of the student's thinking, while Teacher 1 blamed the student for not wanting to do his work. Neither teacher persisted further to test the model presented. The second instance of decentering occurred during the 14th session while the teachers were examining the post-test artifacts to determine students' understanding from the lesson. While a significant amount of the hour discussion focused on right answers revealing understanding and wrong answers revealing not understanding, the teachers used the artifacts for one student to argue about what he was thinking. The discussion was prompted by trying to explain the inconsistencies in the student's answers, which lead to a construction of a possible model of what the student was thinking to answer as he did. These two examples were the clearest evidence of teachers decentering throughout the entire semester.

Henry Kipling C

This PLC had the same opportunities presented above in the other groups. Excluding one group member there were no instances of decentering during this semester. The discourse consisted of vague statements of student understanding with no observable model of the teacher's construction of that model. The teachers also seemed to attribute understanding to

correct answers on homework and tests. For example, in the 10th session when prompted for what students should understand, one teacher focused on the correct solution.

> Facilitator: Mathematically, what is it that that student really understands?
>
> Teacher: If they could explain why the solution is the solution, is a big deal. Not just well they sold 20 cans and 4 hotdogs but why is that the solution to this problem? That's the only proper combination with the right total and the right price.

The focus on the correct answer representing understanding without a clear model of student thinking was prevalent throughout the semester. An exception during this semester was one teacher who, with the presence of student work, attempts to suspend his thinking and adopt a model of the student's thinking when looking at student interviews regarding the changing area of a window (if the height is changed by 1 foot how does that affect the area), realizing that he may not have enough information to formally state what that students truly understands or does not understand.

> Teacher: He came up with, he immediately wanted all these problems, to come up with values, like that's where he always wanted to start. He's like, well, if I have a height of 1 I have a width of 6 and here's my area. And so he calculated a bunch of values. And then he noticed that the area was increasing by two additional each time. So he said, I said, so how's the area changing? He said its double plus two. I couldn't get him to explain where the double came from, cause it wasn't, you know double, but that plus 2 part, you know was kinda interesting cause it's increasing 2 additional each time. And then when I asked to make the graph, he drew a linear graph of the slope of 2. And, you know at first I just thought, well ok so he's, you know thinking of double, or he's not understanding like how to graph this area relationship that should be increasing faster, even though he told me it was, that as you increase the foot, and as the window gets bigger, the area's going to increase more each time. But then as I was thinking back on it, a slope of 2 is actually the rate of change graph. Now, even if he was kinda stuck on that graph, was that what he was thinking I was asking him? Was to show me how the graph was increasing, or is that how he was thinking about it? In which case, he doesn't know rate of change, like that, but is that meaning he doesn't understand the problem or he was thinking about it in a different way that is still viable way to approach, think about the problem. So I sometimes feel unsure of trying to make a statement based on his response to some question, saying, oh he doesn't understand this aspect of covariational reasoning or whatever because I don't necessarily know all the questions to ask I guess to get at the bottom of all his responses.

This teacher went further in the session to develop questions that were more revealing of the student's thinking.

The Spring 2006 semester was structured similarly to the other groups, with the content differing slightly in specific lesson construction of the trigonometric concepts. While the semester presented more opportunities to examine student artifacts from the pre/post interviews and tests, the teachers rarely constructed models of student thinking. The discourse consisted of generalities regarding students' knowledge based on teacher experience and focused on how their teaching practices should be altered to allow students to understand. During the 14th session the facilitator prompted the teachers for examples of student thinking and the incorrect answers were blamed on the student's motivation without a model of student thinking. Teachers focused on students' incorrect answers, blaming students' difficulties on the student's lack of motivation. There was no evidence that teachers used or sought a model of student thinking. Another common response to opportunities for decentering was the teachers' attention to their practice, not to student thinking. For example, when looking over the results from the post-test (given after the five-day trigonometry lesson), a teacher states that students did not choose all the appropriate multiple choice answers to a particular question. When asked about the cause of the misconception she first blames the students, then her teaching.

> Facilitator: Why do you think that might have happened?
>
> Teacher: I don't know, I think that they were just hesitant to put two answers. A bunch of them, when I marked it and gave them their tests back, they were like, oh, we didn't realize it said two, and you know, the first time around [pre-test] they did. I really don't know. I think it's because I didn't stress varies directly with arc length; you know I thought I did, but I don't know.

Thus, given the opportunity to consider students' thinking that would prompt them to answer the question in such a way, she focused on her teaching. The teacher who decentered in the previous

semester's sessions had only once attempted to construct a model of student thinking in the current semester. However, the observable evidence was not conclusive enough to code that instance as "Decentering-Yes".

Martin Luther M

The Fall 2005 semester presented the same opportunities for teachers' reflections on student thinking. However, this group was not a participant in the Functions Course. This particular group's discourse consisted primarily of vague statements regarding student understanding which seemed to be based on their teaching experience, and a focus on teaching practices that would yield greater student understanding. An example of the vague statements of student understanding occurred in the 4th session with the following discourse:

> Teacher: If we give them a question, an estimate of normal systolic blood pressure depends on a person's age.
>
> Facilitator: This is out of the book, is it?
>
> Teacher: This is directly out of the book. One, they're going to go, ok, like granted systolic may be a big word, blood pressure depends on a person's age. One, they're going to go, like, depends, like just grammatically, they can't put that together. I don't think. And two, they don't have enough of a mature grasp of what the word depend means so that they can't really understand the idea well, age controls what your blood pressure is. I mean, just by definition of the word depend. I mean, that's, I don't know if that's really a misconception, its just, that kids can't read.

The teacher not only makes a vague statement, but also then blames the hypothetical students. During the same session there was one intermediate instance of possible decentering occurred during the 4th session when a teacher presented a classroom situation when the input variable of a function was the letter 'y'; her students had great difficultly with this function, and the teacher hypothesized that the students were thinking that the letter 'y' always represented an output

value, not that the students did not understand the content. The teacher then tested the model by changing the letters of the variables in the function, which enabled the students to move on.

During the Spring 2006 semester this group initially had the same structure as the other groups in the project, however, the teachers did not follow the same protocol in terms of pre/post interviews and testing, nor did their lesson reflect the content reflected in the Functions Course since they did not participate as the other groups did. Due to these differences, the presence of student artifacts was limited to what the teachers produced from teaching their lessons. The discourse consisted entirely of teachers attributing their thoughts to students or reflecting only on how their practices should be altered for better student understanding. There were no instances of teacher decentering during the entire semester, even when the opportunities (discussion regarding students' understanding) presented itself. The discussions from the student interview data consisted of explicit statements regarding what students said or answered for the questions. The following discussion took place during the 7th session.

> Teacher: No, it doesn't go through the center so it's not direct variation. It's not linear, it makes a U. Before I, you know. When I asked if it was linear, he said, no, it makes a U. I'm like, so I didn't even have to ask 1(c) in my pre-interview.
>
> Facilitator: Did he know from the table that it made a U?
>
> Teacher: Yeah, well he, what did he do? He um, actually sketched out three points, cause he was like, just a minute. Because the one thing i did gather from his is that he did know (0, 0) meant it went through the center, he couldn't just look at the table and know that it went through the center. He had to sketch it out.

The discussion of what students said/did never lead to a model of student thinking to prompt such answers.

Thomas Jefferson Q

The group of teachers at this high school was not participating in the Function Course series as the groups from Abraham Lincoln and Henry Kipling. They did however engage in activities that presented artifacts from students, but those artifacts were not from their students. During the discussion, when opportunities to decenter arose, the teachers discussed what the students said or answered exclusively on the integer interviews, showing a focus on the correct answer, without attempting to construct a model of what the students might have been thinking. One interesting discussion regarding what students should understand about slope occurred in the 1st session, during the first session the teachers discuss what students should understand about slope.

> Teacher1: You should show them your other way for showing some kids how to calculate slope, it makes them not be so confused. [The method involved seeing the differences between ordered pairs in a table versus using the standard slope formula.]
>
> Teacher 2: That doesn't give you any more profound understanding.
>
> Teacher 1: But, to have a profound understanding you first need to calculate the slope correctly, because if you can't even do that, you can't really move too much past it.

Thus in this case, the teachers focused on what they should teach, not on what students should understand. Throughout the semester the teachers primarily stated what students did explicitly, as with the previous group and had no reflection on student thinking when presented with opportunities. There were no instances of teacher decentering during this semester.

During the Spring 2006 semester this group focused on trigonometric concepts, much like the other groups, however, since they were not in the Function Course, the lessons revolved around right triangle trigonometric instruction. The discussions that were explicitly prompted by the facilitator to uncover student thinking instead involved the best problem solving strategies, or how students should "do" the problems. When presented with student interview artifacts

regarding the lesson to be taught the teachers made vague statements of what students confuse or do not understand which seem to be based on incorrect answers, with no observable evidence of model construction. The teachers also blamed students motivation with regards to remembering material covered. One teacher in particular during the 6th session could not understand why the concept of the included side between two angles of a triangle was not obvious to her students.

> Teacher: that is the craziest thing that that is a hard concept for them to grasp. It took me like three minutes with one of my students today, they had two sides and they had to say what the included angle was, and he was like pointing at other sides, and I'm like, is that an angle? No, that's a side. It's that point, is that point an angle? No, it's not. Where did the like, he couldn't even figure out where the lines met each other. <laughs> I am suppose to teach him triangle congruence. I don't understand, I don't even know how to say it any different. What do you mean you don't know what an angle or a side is? I don't…

This was a normal response to incorrect student answers, that of disbelief without reflection on a possible model to explain the students' responses. This semester revealed no instances of decentering, even with the presence of student artifacts.

SUMMARY

A radical constructivist-teaching model requires teachers to suspend their mathematics and adopt the mathematics of their students. "How the teacher uses his or her mathematical knowledge in teaching the children is crucial because, rather than embody adult mathematical practices, the teacher is obliged to be the leading mathematician with respect to the mathematics of children." (Steffe & Thompson, 2000, p. 206) Over the two semesters, there were few instances of teachers' modeling of student thinking at a second-order level. Furthermore, there were no instances of teachers decentering and then constructing second order models of their students. Artifacts of student work appear to be a necessary, but not sufficient condition for an opportunity to decenter. Modifications were made to the project's intervention to increase the

opportunities for decentering in the second semester, however the instances of teachers building models of student thinking were again extremely scarce and no second order models were constructed. Instead teachers' zero or first-order models appear to fall into four main categories of discourse: a focus on teaching not learning (zero-order model), a restatement of exactly what the student said or wrote (zero-order model), blaming students for lack of motivation to learn (first-order model), and attributing the teacher's own thinking to the student (first-order model). The majority of these zero- and first-order models occurred with the facilitator prompting the teachers to attend to student thinking. It is important to note another common instance that could not be coded as decentering due to lack of evidence of model building, and that occurred when teachers would make vague statements seemingly based on their experience regarding what students do or do not understand. We do not discount the value that stems from teachers' experience, however, without evidence that substantiate claims of understanding, it cannot be coded as decentering. Overall, while students became a greater part of the discourse during the second semester, the teachers primarily focused their attention and conversations on the five decentering distractors discussed above.

DISCUSSION

This paper reports on the first phases of the TPCC Project and the analysis outlined in this paper prompted even more changes in the intervention. We now see the necessary, but not sufficient requirement of student artifacts to propel teachers into decentering opportunities. One major addition to the Fall 2006 semester of the project was to have teachers bring in video clips of their instruction with the focus of the session being the construction of models of student thinking to reveal what students understand. Teachers found very early that they were not asking questions that revealed students' mathematics or thinking at a necessary level to construct such

models. Thus looking at their instruction during the RPSs attempted a shift away from the focus on teaching practices and toward student learning. Analysis of the Fall 2006 and Spring 2007 semesters will highlight any teacher changes regarding decentering. A second change to the intervention prompted by the analysis discussed in this paper was greater facilitator sensitivity to probe for evidence of vague teacher models. That is, when teachers make generalizations regarding what students do or do not understand, the facilitator then asks that teacher for evidence of their claim. It is too early in the analysis to attempt to assign causal factors to the rare instances of decentering, noting only the need for student artifacts and more teacher reflections. The project has also implemented a teaching experiment with a local high school to facilitate the creation of student artifacts that model conceptual teaching and act as an example of how to uncover student thinking.

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