**Title: The use of pragmatic reasoning schemas to improve undergraduate students' logical reasoning skills**

Mika Munakata
Montclair State University

## Introduction

When teaching an undergraduate mathematics course, one of our main goals is to help students develop their logical reasoning abilities. In particular, in our courses for mathematics majors, we would like students to improve their abstraction and formal mathematical reasoning abilities to aid them in writing proofs. In our liberal arts mathematics courses, we encourage students to be able to recognize mathematics and mathematical thinking in their everyday lives. The question of how we teach logical reasoning to our students is one that many undergraduate educators face. For example, studies have found that teaching formal logic in tandem with concrete examples improves students' reasoning abilities [1]. Furthermore, skills gained when considering problems related to permission and obligation have been shown to transfer to abstract contexts [2]. These problems, representing pragmatic reasoning schemas, provided the conceptual framework for this study.

Pragmatic reasoning schemas consist of "context-sensitive rules which…are defined in terms of classes of goals (such as taking desirable actions or making predictions about possible future events) and relationships to these goals (such as cause and effect or precondition and allowable action)" [3, pp. 395]. In contrast to the specific-experience view and the natural-logic view, pragmatic reasoning schemas do not depend on specific memory or on context-free syntactic rules.

For this teaching experiment, data were collected to assess the impact of an instructional unit based on pragmatic reasoning schema. As part of the instructional unit, students interpreted, analyzed, and translated street signs into logical statements. Using the instructional unit and a pre- and post-test design, this exploratory study addressed the following research questions: 1.) How successful are students in translating rules for permission and obligation into logical statements? 2.) What are the effects of a context-based instructional unit using rules for permission and obligation on students' logical reasoning skills?

Data were collected from students enrolled in two sections of a course called Contemporary Mathematics. The course is one of three courses students can take to fulfill the general education mathematics requirements at Montclair State University. Of the 61 students from whom data were collected, only 44 ($n$=44) of the data points were usable due to absences, incomplete surveys, and situations where students forgot to write their names. None of the students in the course were declared mathematics or science majors; elementary education, psychology, and undeclared majors were the most prevalent.

The participants completed pre- and a post-tests, each consisting of 25 multiple-choice items, on which they were asked to indicate the truth or falsity of logical statements. Of the 25 items, 13 were based in geometry, and 12 were logic statements based in everyday contexts. The following sample problems were included on the surveys. The first pair of examples (Example 1a and 1b) illustrates the converse in an everyday situation, where the latter (Examples 2a and 2b) represents the inverse set within a geometry problem.

**Examples**

**1a) Pretest:**

If it rains Monday night, then I will mow the lawn on Tuesday. I mowed the lawn on Tuesday. Did it rain Monday night?

(a) yes (b) no (c) cannot tell.

**1b) Posttest:**

If it is cold on Monday, we will go to the park on Tuesday. I went to the park on Tuesday. Was it cold on Monday?

(a) yes (b) no (c) cannot tell.

**2a) Pretest:**

If two rectangles are congruent, then they have the same perimeter. Rectangles A and B are not congruent. This means rectangles A, B __________ have the same perimeter.

(a) must (b) could (c) cannot

**2b) Posttest:**

If two angles are vertical angles, then they are congruent. Angles A and B are not vertical angles. This means angles A, B ________ be congruent.

(a) must (b) could (c) cannot

**The instructional unit**

The instructional unit was implemented over two class meetings—each 75 minutes in duration. The unit began with a brief overview of symbolic logic, followed by discussions about interpretation of street signs using symbolic logic. For example, the first street sign presented to students was one found in the park of the Museum of Natural History in New York City. It stated “No dogs off leash at any time.” First, students were asked to describe any peculiarities they noticed about the sign. Students stated that the “at any time” seemed odd because the park commissioner probably wouldn’t care whether dogs are on or off leash when they are away from the park. Also, students responded that in “No dogs off leash”, the “No” and the “off” seemed to cancel each other out. This was used as a springboard to use symbolic logic to represent the double negative. Second, the sign was used to contextualize the direct implication, $p \rightarrow q$ , where $p$ represents "I have a dog at the park" and $q$ represents "my dog is on a leash". Students were asked to use the sign to verify that each row of the truth table made sense within the context of the situation: namely that the implication is false (the sign is being violated) only when $p$ is true and $q$ is false. Also, various relationships such as $p \to q \equiv \sim p \vee q$ (I am following the sign if I don't have a dog at the park or my dog is on a leash, or both) and $\sim(p \to q) \equiv p \wedge \sim q$ (I am violating the sign if I have a dog at the park and he is not on leash) were discussed.

Other street signs were used to discuss the contrapositive, inverse, converse, disjunctive, modus tollens, modus ponens, and DeMorgan's laws. For example, a sign with two placards, one stating "No parking, 7am-4pm (on) school days" and the other "No parking, 11am-12:30 pm Monday(s) and Thursday(s)" was used to illustrate DeMorgan's laws: Namely, that $\sim(p \vee q) \equiv \sim p \wedge \sim q$ where $p$ represents 7am-4pm on school days and $q$ represents 11am-12:30 pm on Mondays and Thursdays. For each sign, students were asked to translate the signs using a truth table and to interpret the sign under various conditions.

**Results**

A paired-samples t test was conducted to see if there was a difference in scores between the pretest and the posttest. The results (Figure 1) indicated that the mean posttest scores (M=16.69, SD=5.11) was significantly higher than the mean pretest scores (M=12.49, SD=4.13), $t(40)=5.99$, $p<0.001$). The mean difference was 4.20 on the 25-item survey, indicating that the instructional unit had a positive effect on students' logical reasoning skills as they were measured by the instruments.

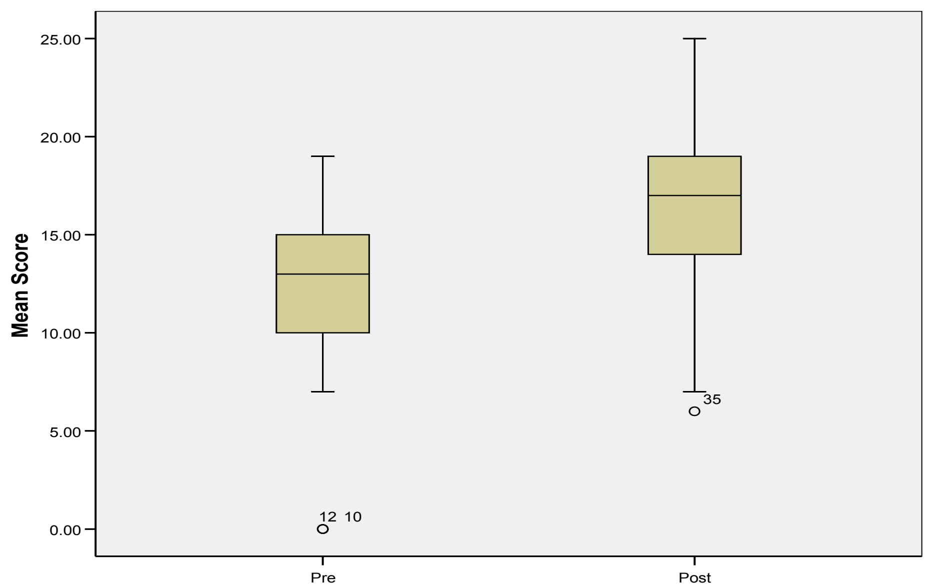


Figure 1

These results suggest that even a two-day instructional unit using pragmatic reasoning schemas to discuss logic statements has a positive effect on students' logical reasoning abilities. As stated earlier, however, this is merely a preliminary report, so further studies need to be conducted to verify and replicate these results. Possible avenues for future research include refining the instruments, designing a study with control-treatment groups in order to determine the difference in gain between a traditional unit on symbolic logic and one using the pragmatic reasoning schema, and conducting a study involving a more comprehensive unit. Though the gain in scores was significant, the mean score on the post-test (16.69 out of 25 items) leaves much to be desired. It is the hope that a more extensive instructional unit implemented over a longer duration of time would allow for deeper analysis and discussions, resulting in higher post-test scores.

The results of this and future studies have implications for various undergraduate courses. It is the hope that students enrolled in mathematics courses for non-mathematics majors will be encouraged to examine the utility and frequency of logical reasoning in making decisions in their everyday lives. Through the instructional unit using street signs, they practice formalizing their reasoning processes and become exposed to the deductive reasoning and abstraction skills necessary in mathematics. In geometry and other courses for mathematics majors, this type of instruction has the potential to influence how students approach proofs.

References:

[1] S.S. Epp, The role of logic in teaching proof. The American Mathematical Monthly 110 (2003) 886-899.

[2] P.W. Cheng, K.J. Holyoak, R.E. Nisbett, and L.M. Oliver, Pragmatic vs. syntactic approaches to training deductive reasoning. Cognitive Psychology 18 (1986) 293-328.

[3] P.W. Cheng, K.J. Holyoak, Pragmatic reasoning schemas. Cognitive Psychology 17 (1985) 391-416.