Roles of Revoicing in an Inquiry-Oriented Mathematics Class:

The Case of Undergraduate Differential Equations

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Introduction

In the past decades, the proposals of school mathematics reform have recommended that mathematics instruction should resemble the practice of mathematicians. In this perspective, the inquiry-oriented mathematics class was designed to provide the opportunity for students to learn mathematics through the active participation into the authentic practice of mathematics. This change in mathematics classroom requires that teachers change their teaching practice to conform to the recommendations of educational reform. In particular, reform documents emphasize the teacher's discursive role to facilitate and orchestrate students' practice of mathematics in the classroom (e.g., NCTM, 1991). In this regard, the analysis of teacher's discourse in relation to the changed teacher's role in the inquiry-oriented mathematics classroom has become more significant as a research topic.

In particular, this paper focuses on the teacher's revoicing because it is one of discursive strategies that often occurs in the teaching of mathematics, but which is not thoroughly investigated. Forman, Larreamendy-Joerns, Stein, and Brown (1998) highlighted revoicing as a critical feature of a teacher's discourse by which s/he orchestrates students' discussion. They found that a teacher recruits students' attention to point out important aspects of students' argumentation through revoicing. Also, O'Connor and Michaels (1993) characterized that revoicing affords the teacher the tools to coordinate the elements of academic task structure and social participation structure, while simultaneously bringing students into the process of intellectual socialization.

From this perspective, our research goal is to approach teacher's revoicing as a discursive move, which is defined as teacher's deliberate actions situated within the context of the mathematical communication (Krussel, Edwards, & Springer, 2004), in order to further our understanding of the complicated process of the co-construction of mathematics in an inquiry-oriented mathematics classroom. Specifically, we have investigated how teacher's revoicing facilitates the co-construction of undergraduate mathematics in an inquiry-oriented differential equations (IODE) classroom.

Theoretical Background

Since the 1970s, educational researchers have adapted the sociolinguistic perspectives to examine a teacher's discursive move in classroom settings. The early studies were interested in the sequential pattern of the interaction of teacher and students. For example, Mehan (1979) suggested an IRE pattern as a basic elicitation sequences. In the IRE pattern, the first part of this sequence has been called an "initiation", the second part a "reply", and the third part either an "evaluation" or "feedback".

Whereas Mehan's construct suggested that traditional teachers often fall into a pattern in which they funnel correct answers by evaluating students' short responses, Bowers and Nickerson (2001) observed a cyclical pattern in each phase of a concept-centered class. In the concept-centered class, when the teacher initiated a new activity, it was observed that the interaction pattern included teacher's elicitation, student's response, and teacher's elaboration. Bowers and Nickerson called this communicative routine as an ERE pattern. Their analyses of the social norms indicate that the way the teacher and the students negotiated ways of communicating served to shift conversation from an ERE pattern to another type of communicative routine in which the teacher or a student would make a proposition, and others would discuss it. They call this pattern of communication as a "proposition – discussion" pattern (PD).

While previous studies approached teacher's discourse as a communicative routine in an certain sequential order, current studies adapt the notion of discursive move to analyze teacher's discourse as an “action” that a teacher deliberately takes in the context of communication (Krussel, Edwards, & Springer, 2004). This notion of discursive move emphasizes the mutual relation between a teacher and students in classroom discourse. That is, in the sequential perspective on classroom discourse, each component is considered as isolated. On the contrary, when considering teacher’s discourse as action, it emphasizes the teacher’s intention to participate in the on-going classroom communication and to influence the flow of the communication as one of the participants. In the studies about teacher’s discursive move in the inquiry-oriented mathematics class, researchers have identified diverse verbal forms such as telling, questioning, revoicing, and their significances in teaching and learning of mathematics. Lobato, Clarke, & Ellis (2005) suggested a theoretical reformulation of telling as the set of teaching action that serves the function of stimulating students' mathematical thoughts via the introduction of new ideas into a classroom conversation. Clegg (1987) emphasized questioning as strategies to review, to check on learning, to probe thought processes, to pose problems, to seek out different or alternative solutions, and to challenge students to reflect on critical issues or values they had not previously considered. Boaler & Humphreys (2005) said that questioning develops critical concepts in student-centered learning environment.

In addition to telling and questioning, revoicing is another discursive move that teachers use to facilitate students’ learning. Revoicing involves the reuttering of another person's speech through repetition, expansion, rephrasing, and reporting (Forman et al., 1998). O'Connor and Michaels (1996) focused on the notion of revoicing to illustrate that the instructional process depends upon the skillful orchestration of classroom discussion by the teacher. They claim that revoicing by the teacher may change the way students see themselves and each other as legitimate participants in the activity of making, analyzing, and evaluating claims, hypotheses, and predictions. Forman et al. (1998) emphasize that the teacher is able to orchestrate discussion by recruiting attention and participation from students in the class, aligning learners with argumentative positions through reported speech, highlighting positions through repetition, and pointing out important aspects of their arguments through expansion. Also, Forman and Ansell (2002) found that the teachers legitimated student contributions to the discussion by revoicing their arguments. They characterized teacher's telling in reform-oriented classroom as revoicing.

Researchers have shown that revoicing is one of the significant forms of teacher’s discursive move in reform-oriented class. However, it is necessary to point out that teacher’s revoicing has not been thoroughly investigated compared to other types of teacher’s discursive move. From this perspective, we have focused on revoicing, specifically, how teacher’s revoicing supports the co-construction of mathematics in an inquiry-oriented mathematics classroom. In the analysis, we approached teacher’s revoicing situated within the context of the classroom practice of mathematics, in other words, revoicing as a teacher action to participate into the collective construction of mathematics with students. Thus, instead of singling out teacher's revoicing for the analysis, teacher's revoicing is considered as integrated with students' discourse to build the classroom practice of mathematics as a whole.

Method

Our research team has been engaged with the teaching experiment of an undergraduate inquiry-oriented differential equations (IODE) course over past seven years. The data for this analysis came from a course taught in a large state university in the southwestern area of United States in 2005. The course was taught by one of the authors of this paper. The teacher has taught the IODE over the past seven years and, as part of a research program for rethinking the teaching and learning of differential equations, and has developed a full set of course materials. The IODE materials have been inspired by the instructional design theory of Realistic Mathematics Education (RME) (Gravemeijer, 1994). The materials recruit situations (real world situations and mathematical situations) that are experientially real for learners. Instructional tasks are organized into a sequence of questions for mathematization. The materials have revised through teacher reflection and detailed analysis of student thinking over the course of several different teaching experiments (e.g, see Rasmussen & Keynes, 2003; Rasmussen, Stephan, & Allen, 2004).

In addition to mathematization, another essential feature of RME design is the proactive role by a teacher in supporting students' reinvention of mathematical ideas and methods for solving problems (Rasmussen & Marrongelle, 2006). In this regard, the IODE follows what Richards (1991) refers to as an "inquiry-oriented" instructional model, in which important mathematical ideas and methods emerged from students' problem-solving activities and discussions about their mathematical thinking. Thus, the students and the teacher are encouraged to collectively construct mathematics based on their own mathematical ideas and reasoning through the active participation into mathematical activity.

Thus, special attention was given to the classroom learning environment. In the IODE, the students worked on problems in small groups of three or four. In small group discussion, the students did not merely solve a specific problem but analyzed a question, developed reasons to support their thinking and ultimately to extend to related mathematical principles. In order to promote productive small group discussions and whole class discussions, the teacher continually fostered particular social and socio-mathematical norms regarding argumentation. Examples of social norms from this class include: students routinely give explanations of their own thinking, indications of agreement or disagreement with other students' explanations, and explanations of other students' argumentations. Likewise, examples of socio-mathematical norms are as follows: students routinely represent ideas in various ways such as graphs, tables, equations, and so on; students are asked to interpret the mathematical context qualitatively. With this view, the teacher encouraged students participate in the discussion by asking students to explain their thinking.

Also the teacher directed students to represent their thinking in various ways such as verbal form, graph, table, gesture, and so on. During most of the discussion, the teacher emphasized the critical points of the discussion or provided new problems or information through repetitions or expansions of students' utterances.

Data were collected from the video recording of every class session. In this article, we have focused on the video recordings that were captured on four consecutive class sessions. During these sessions, the class worked on a system of differential equations and how to draw solution curves using straight line solutions. In class, there were two cameras that captured whole-class discussion and small group discussion. One camera was positioned at the front of the classroom where it captured the students' participation during whole-class discussion, and then group work from the 'front group' during small group discussion. The other camera was positioned at the back of the classroom. The back camera focused on the instructor and any students presenting solutions on the chalkboard during whole class discussion. During small group discussion, the back camera captured the interaction of the 'back group'. Thus, we have collected video-recordings of the discussions in two small groups as well as whole class discussions during those four sessions.

Utterances of both the teacher and the students were transcribed and analyzed by a coding scheme developed by the research team. Our research team worked collaboratively to develop the coding scheme for teacher's discursive move. This collaborative coding procedure by multi-coder allows the opportunity for negotiation to minimize biases by each individual researcher and to eliminate interpretations not grounded on the data. When a coding scheme emerged, we applied it to whole set of data to check whether the coding scheme could cover all the cases from the classroom discourse. The coding scheme developed to a more comprehensive set of codes through this alternative process of analysis.

Through the analysis, we identified four categories about teacher's discursive move: telling (T); questioning (Q); revoicing (R); directing (D). Telling is defined narrowly as stating information or demonstrating procedures (Smith, 1996) in the more traditional sense in order to clearly distinguish this form of discursive move from revoicing, questioning, and directing. Directing is a discursive move in which a teacher states a specific behavior that s/he wants students to perform. Questioning is a discursive move in which a teacher checks for understanding, requests to explain thinking, requests to justify thinking and so on. Revoicing is broadly defined as reuttering of someone else's utterances. We applied the four verbal forms to classify teachers' discourse move in the transcripts. Thus, revoicing is treated as exclusive to questioning. For instance, when revoicing occurs in the form of questioning such as "Do you mean that it keeps the same distance?", the primary code for the discourse becomes questioning and revoicing becomes secondary code. So the code for the sentence is Q-R.

There are sub-codes depending on the forms or roles that each type of discourse move takes. In particular, revoicing has four sub-codes: repetition (R1 – a teacher repeats a student's utterance using the same words or a portion thereof); expansion (R2 – a teacher adds information to a student's utterance); rephrasing (R3 – a teacher states a student's utterance in a new or different way); reporting (R4 - a teacher attributes an idea, claim, and argument to a specific student). With these sub-codes, we extended our analysis for teacher's discursive moves to the roles of revoicing within the context of the co-construction of mathematics in the IODE.

Discussion and Analysis

How does the teacher's revoicing facilitate the co-construction of mathematics in the IODE? Our analysis indicates that the teacher utilized students' utterances through revoicing continually and, more importantly, that the teacher's revoicing carries out critical function in the process of the collective construction of mathematics in the class. What happened when the teacher participated in the mathematical communication by revoicing? The following episodes illustrate the teacher's revoicing and its roles

Mathematical Episode 1

System of differential equations	Straight line solution
$\frac{dx}{dt} = y$ $\frac{dy}{dt} = -2x - 3y$	$y = -x$ $y = -2x$

We have two different initial conditions along this straight line solution, y = -2x. How is the solution supposed to behave with those initial conditions?

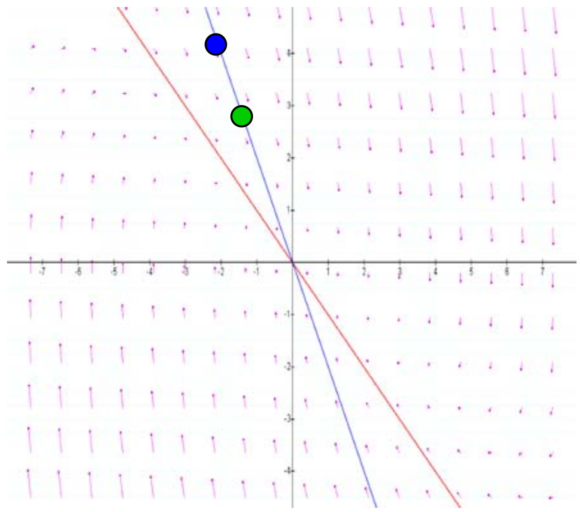

Figure 1. Task of the mathematical episode 1

The first episode is from the whole class discussion concerning how the solution curves behave. In this case, the teacher began the whole class discussion by inviting students' ideas. Harry was the first to present their groups' thinking:

Teacher: Tell us what you are trying to think about as you're moving those. (D)

Harry: ***Keeping the same distance*** and move along the straight line.

Teacher: So, you think ***the same distance***? (Q-R1)

Students: No.

Teacher: What did you mean by that then? Do it there for us because you did ***keep the same distance***, right? (Q-R1)

Harry: No.

Teacher: I mean ***the distance between the two points***. (R3)

Harry: I guess this one would go towards zero as this one moves closer to that one. Wouldn't it?

Teacher: Robert? (Q)

Robert: I don't agree. I don't think they should keep the same interval all the way towards zero. I think the top one, you got it right the first time actually go to faster.

Teacher: Do you want to come up and show us what you think? (Q)

Robert: It'll go like ***one will move faster than the other. Not necessarily meet at the same time, but meet not at the same distance*** [inaudible]

Teacher: So, you're saying ***they start here and this one starts to catch up***. (R3)

In this episode, Harry claimed that the curves move along the straight line and keep the same distance. The teacher repeated "the same distance" from Harry's claim to ask clarification and Harry elaborated his claim. Then, instead of evaluating Harry's claim, the teacher called on Robert, who challenged Harry's claim. After Robert's presentation, the teacher summarized Robert's claim by rephrasing, "they start here and this one starts to catch up."

In this episode, the teacher's major discursive move is revoicing. The teacher's revoicing fulfills several functions to facilitate and orchestrate students' communication in this episode. First, the teacher repeats or rephrases a student's claim to signal that a mathematical position has been identified and to have a speaker align with a certain position. Second, the teacher's revoicing recruits students' attention to a given claim and prompts the speaker to clarify and elaborate the mathematical meaning of the claim. With these functions, instead of directly instructing or evaluating, the teacher's revoicing ultimately leads the students to bring up diverse mathematical positions for the negotiation of mathematical meaning. So in an inquiry-oriented mathematics class, teacher's revoicing highlights diverse mathematical positions offered by students and forces students to focus for the negotiation of mathematical positions.

The following is a small group discussion that occurred after a different whole class discussion. We omit the whole class discussion due to space constraints, but note that the whole class discussion shared many of the revoicing aspects illustrated in episode 1. In the following we emphasize some of the students' discourse with bold letters because these are the students' utterances that reflect the teacher's revoicing in the earlier whole class discussion.

John: **Never touches zero**.
Diane: Okay, never touches zero because it's an **exponential**.
John: It's a **shift** on the t-axis. Same solution.
Diane: Because it's in terms of x and y.
John: It's always a **multiple** of itself, so t would give a different.
Sam: I think it's more like *uniqueness*, but oh well.
Diane: Right.
John: What would you do for that? ……
Sam: I don't know. I don't really have a strong opinion.
John: I'm interested in this thought. So, dx/dt = y. Right. And then take the partial derivative of that? NO, no, because we can say dx/dt = -2x and the partial derivative of that would be the partial of x with respect to -2?
Sam: Kay.
Aden: Can you explain to me why you wanted to take the partials?
Sam: Partials, because that's one of the things that was described by the *uniqueness theorem*. That was like one of the rules. So, I'm assuming we use that.
John: Well, I guess to build on that
Aden: How would you want to use it?
Diane: We're trying to figure out **whether or not it touches zero**

We argue that this episode illustrates how a teacher's revoicing can highlight critical concepts and ideas under discussion so that the students adapt those concepts and ideas into the follow-up inquiry in their small groups. The discussion of this small group eventually led to the uniqueness theorem of the second order differential equations followed by student's revoicing of the teacher's utterances in the whole class discussion. This tells that the teacher's revoicing ultimately works as a springboard for students' construction of mathematics.

Mathematical Episode 2

System of differential equations

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -2x - 3y$$

Straight line solution

$$y = -x$$

$$y = -2x$$

Suppose you were to start with an initial condition on the straight line solution.
What are the x(t) and y(t) equations for any solution along this straight line?

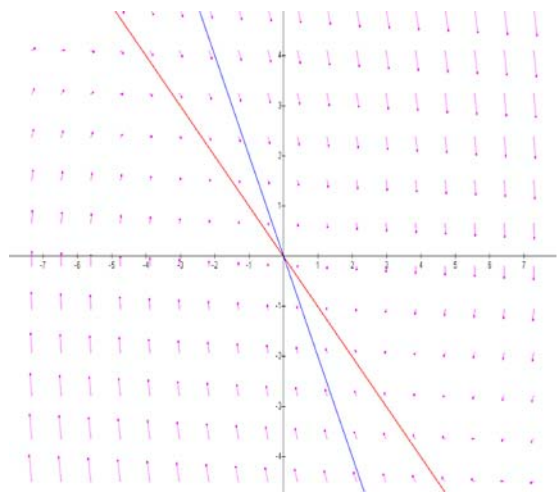

Figure 2. Task of the mathematical episode 2

This second episode is concerned with finding x(t) and y(t) equations for the straight line solution. While the students discussed their ideas in small group, the teacher walked among the small groups listening to their ideas. In the whole class discussion, the teacher revoiced the mathematical ideas generated by the students such as Young, Sarah, and Jean in their small group discussions. In this case, the teacher's revoicing functions to bring up various mathematical ideas raised by the students to reveal the mathematical connection between their ideas:

Teacher: All right. So Young and Sarah, your idea was…that ***any other solution along this straight line can be obtained by taking a multiple of your first one*** **(R2)**. So I think this is what Jean was saying when she said, ***just multiply by k*** **(R3)**. So I could have some k, I'll call it k1*e^-2t and then the original initial condition (2, -4). <u>So the point is that any other solution along this straight line can be obtained by taking a multiple of your first one.</u>

At the end of discussion, the teacher's revoicing recasts students' claims and reveals the mathematical connection between various claims given by the students. Moreover, in revoicing, the teacher refers to a specific student as an owner of a mathematical position and, as a consequence, mathematics is represented as being co-constructed by the course participants themselves –- instead of being given by the teacher -- in the class.

Mathematical Episode 3

So far, we have illustrated two cases in which students make claims without justification. Now we present an example where students are asked to provide justification to their mathematical claims. The task is to sketch the solution graph in the phase plane.

System of differential equations	Straight line solution
$\dfrac{dx}{dt} = y$ $\dfrac{dy}{dt} = -2x - 3y$	$y = -x$ $y = -2x$

Suppose you were to start with an initial condition somewhere in the second quadrant between the two straight line solutions, say at (-4, 6).

Sketch what you think the solution as viewed in the phase plane looks like.

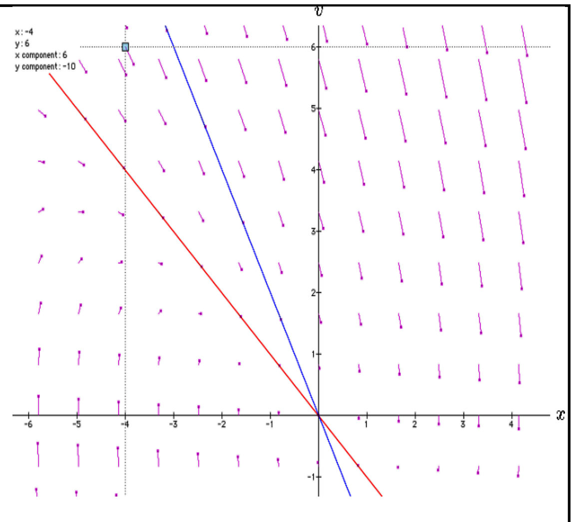

Figure 3. Task for episode 3

In the class discussion, several students claimed that the solution graph is not a straight line and the teacher asked them to provide justification. Then, Harry justified his claim about the shape of the graph. After Harry's presentation, the teacher used revoicing to expand Harry's justification by introducing useful mathematical concepts:

Teacher: Another reason. Anyone have a different reason. Harry? (Q)

Harry: Well, first of all we assumed that **there was a straight line solution and then <u>we derived it through the dy</u> or the um finding <u>x(t) and y(t) and they did not come out to have the same powers in the huh exponents</u>**. So, we had a contradiction. We concluded that there was no straight line.

Teacher: I see a couple of frowns. Like, huh? Um, let me write something on the board and tell me whether I just misconstrued. So, Harry said, **suppose it were a straight line, then if you were to calculate the dx/dt and dy/dt components, the ratio of the components, the dy/dt and dx/dt ought to be the exact same ratio as the y to the x. I mean that's how you get a straight line is that you have so, 1. <u>If it were on a straight line, then we have to have dy/dt/dx/dt = dy/dx = y/x (ratio of y/x).</u> That would have to be the case to be on a straight line. Your resultant vector, the dy/dx, would have to be exactly the same components of dy/dt, dx/dt as y to x. You have to have that proportionality going on.** Well, let's see if we do have it. All right, well, if we're at the point, um, we're at the initial condition here…**(R2)**

In this case, the teacher's revoicing provided the mathematical foundation of the validity of Harry's justification. In other words, the teacher expanded the student's mathematical arguments for elaboration by bringing up the related formal concepts. In this way, the teacher's revoicing bridges between a student's mathematical reasoning and the formal structure of mathematics. Put in another way, in this case, revoicing is a way that the teacher demonstrates how to speak in the formal language of mathematics to make a one's mathematical idea more approachable by the audience in class. Thus, the teacher's revoicing introduces another way of speaking mathematics. More specifically, the teacher as a practitioner of mathematics demonstrates the cultural way of reasoning and speaking about mathematics that is shared in the community of mathematics. Since mathematics is communal practice, there is a set of norms that confers the legitimacy to a practitioner's practice of mathematics. In addition to the system of mathematical facts and skills, the norms of how to do mathematics is an essential aspect of mathematics that students need to learn, but hardly approach through direct instruction. This episode shows that a teacher's revoicing is a way to demonstrate the cultural way of doing mathematics in order to scaffold students' mathematical practice for their socialization as practitioners of mathematics.

Conclusions and Implications

In our analysis, it has been found that teacher's revoicing constitutes a major part of teacher's utterances in class and, most importantly, it carries out very critical functions. From that perspective, we have illustrated the role of revoicing, in particular, how teacher's revoicing facilitates the co-construction of mathematics through mathematization in the an IODE classroom. Next we summarize the role of teacher's revoicing as binder, as springboard, and for ownership.

Revoicing as a binder

Teacher's revoicing works to signal that a mathematical position has been identified and that a speaker is aligned with a certain position. In addition, a teacher uses revoicing to provide an opportunity for students to bring up diverse mathematical positions for the negotiation of mathematical meaning. In this way, a teacher's revoicing enables students to attend to critical ideas in order to generate more comprehensive mathematics by connecting diverse perspective.

Revoicing as a springboard

Teacher's revoicing recruits students' attention to a specific claim and prompts the speaker to clarify and elaborate one's own claim. Thus, a teacher's revoicing scaffolds students to clarify, to elaborate, and to extend their mathematical positions through reflection. Moreover, the concepts highlighted by a teacher through revoicing come up in the small group discussion to shape students' follow-up inquiry. Also, in revoicing, a teacher can demonstrate the cultural way of doing mathematics to support students' transformation as practitioners of mathematics. In this regard, teacher's revoicing contributes to lift students' practice of mathematics and ultimately to support their socialization into the cultural organization of mathematics community.

Revoicing for ownership

Teacher's revoicing makes reference to whom the mathematical position belongs to and helps every classroom participant make sense of it. Also when the mathematical concepts or contents that the teacher wants students to discuss about do not appear fully, revoicing enables a teacher to reveal available mathematical resources rising in the voices of students. As a consequence, mathematics is represented as being collectively constructed by the course participants themselves instead of being given by the teacher. In this regard, revoicing creates a sense of classroom as a community of practice and a sense of mathematics as their own practice.

Historically, differential equations have been invented as a language to express the law of the nature. However, the conventional teaching and learning practice of differential equations heavily relies on drill and practice. It can hardly be said that students learn the historical spirit of differential equations. The development of the IODE approach has been initiated by the reflection on how to reform teaching differential equations in order for students to learn differential equation as a language for talking about their world.

It has been shown that the IODE approach positively contributes to students' conceptual understanding, problem solving, retention, justification, and attitudes toward mathematics (Cho, 2003; Ju & Kwon, 2004, 2006; Kim, 2006; Kwon, et al., 2004a; Kwon, et al., 2004b; Kwon, Rasmussen, & Allen, 2005, Rasmussen, Kwon, Marrongelle, Allen, & Burtch, 2006, Yackel & Rasmussen, 2002). However, we still have to resolve the notorious dilemma of an inquiry-oriented mathematics class for teachers, that is, "how to teach without teaching?" In this paper, we have struggled with this dilemma by deeply looking at how a teacher uses language, specifically revoicing, to guide students in the reinvention of mathematics. In this regard, this article provides an understanding of how a teacher can invite students to the classroom practice of mathematics and engage with students in the collective construction of mathematics. This study of revoicing can be extended by investigating the function of revoicing in conjunction with other verbal forms such as questioning in order to provide useful guidance for teachers how to effectively fulfill their role in an inquiry-oriented mathematics class.

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