Intermediate mechanics students' coordinate system choice

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Abstract

We investigate the interplay between mathematics and physics resources in intermediate mechanics students. In the mechanics course, the selection and application of coordinate systems is a consistent thread. Students start the course with a strong preference to use Cartesian coordinates. In small group interviews and in homework help sessions, we ask students to define a coordinate system and set up the equations of motion for a simple pendulum where polar coordinates are more appropriate. We analyze the video data from these encounters using a combination of Process/Object theory(Sfard, 1991) and Resource Theory(Hammer, 2000). We find that students sometimes persist in using an inappropriate Cartesian system. Furthermore, students often derive (rather than recall) the details of the polar coordinate system, indicating that their knowledge is far from solid.

As part of ongoing research into cognitive processes and student thought, we investigate the interplay between mathematics and physics resources in intermediate mechanics students. We collect data from student interactions to build models of student cognition, which are then used to inform curriculum development.

Theoretical Development

We use theoretical perspectives from both mathematics education research (Process/Object(Sfard, 1991)) and physics education research (Resource Theory(Hammer, 2000)). In this section, we present an overview of Resource Theory and its connections to Process/Object. We introduce the idea of plasticity, a continuum which extends Resource Theory to describe the development of resources. We then present heuristics for identifying resources and their plasticity *in situ*.

Resource Theory

Resource Theory is a constructivist schema theory which bridges neuro-cognitive models of the brain and results from education research to describe the phenomenology of problem solving(diSessa, 1993). Resources are small, reusable pieces of thought that make up concepts and arguments. In contrast to Process/Object's focus on conceptions of mathematical entities, Resource Theory focuses on the connections between different ideas in physics.

Examples of resources in the literature primarily focus on primitives(diSessa, 1993). Some examples include "effect dies away," which describes the motion of a box sliding on a floor, the ringing of a struck bell, a person's motivation, and other phenomena. A mathematical equivalent exists in symbolic forms(Sherin, 2001). Though most described resources are primitive and thought of as having no internal structure, we describe a larger resource, *coordinate systems*, with much internal structure in this paper.

As originally published(Hammer, 2000), resources were intentionally vaguely defined. Later papers elaborate on the theory and make more explicit connections between resources and other theories(Hammer & Elby, 2003; Hammer, Redish, Elby, & Scherr, 2004; Lising & Elby, 2005; Redish, 2004; Sabella & Redish, 2005; Sayre, Wittmann, & Thompson, 2003; Tuminaro, 2004; Wittmann & Morgan, 2004; Wittmann, 2006). As Resource Theory has developed, different aspects of student cognition have been found important, including epistemology(Hammer & Elby, 2003), metacognition(Lising & Elby, 2005), physics and mathematics content knowledge(Tuminaro, 2004), and problem solving skills(Sayre et al., 2003). Representations of linked resources have been described(Wittmann, 2006) and made consistent with the model of coordination classes(diSessa & Sherin, 1998). Many aspects have not been explored, including their self-efficacy[1] and literacy.

Resources serve an adaptive function in thinking by uniting a variety of specific experiences (e.g., experiences in which effects die away) into a general statement. By doing so, they organize the experiences into chunks of information for use in working memory. Working memory is fast but limited; it can hold only a few items at a time, and those only for a few seconds. However, the items in working memory may encode considerable structure. We model this structure in terms of resources. Resources may contain resources as elements, in the same fashion. Resources, then, are blocks of data with which reasoning takes place. Most problem solutions require the coordination of many resources.

Based on the literature and our own work, we summarize that individual resources and groups of resources have the following properties:

Resources are small, reusable pieces of thought that make up concepts and arguments(Hammer, 2000). To be considered a resource, an idea must have sufficient duration and stability to be reused. Resources are individually nameable. Individual students hold resources; they are

[1](except inasmuch as it might affect their epistemological resource activations)

not socially negotiated[2] (unlike social norms).

Resources have two states: active, and inactive. The physical context and cognitive state of the user determine which resources are available to be activated. The activation of resources occurs when their invocation, express or implicit, is used to support or form an argument. Once activated, they form a network with graph-like structure(Sayre, 2005; Wittmann, 2006), which is consistent with a model of coordination classes(diSessa & Sherin, 1998). Just as neuronal links may be excitatory or inhibitory, links between resources may promote or demote activation(Sabella & Redish, 2005). If the network tends to have the same structure repeatedly, then it is stable. If not, then it has been built "on the fly"(Hammer, 2000).

Resources are nestable(Sayre, 2005): they may have internal linked structure made up of other resources. If their internal structure is explorable (but currently not explored) by the user, they may be called concepts. If their internal structure is no longer explorable, they may be called primitives. A large body of literature has identified both concepts and some kinds of primitives(diSessa, 1993).

Resources can be epistemological(Hammer & Elby, 2000, 2002, 2003; Lising & Elby, 2005), metacognitive, or content-oriented. Multiple kinds of resources are active in any given situation(Hammer & Elby, 2003). Unlike the concepts in Conceptions, resources are not necessarily inherently correct or incorrect(diSessa, 1993; Hammer, 2000).

Resource Theory has several strengths. It describes reasoning done by both experts and novices. It can describe learning in many different curricula(Wittmann, 2006). It is well linked to

[2]Though individual resources are held by individuals, their expression and use in a specific context may be socially negotiated. However, we wish to draw a distinction between resources (held individually) and their appropriate expression (which may be socially determined)

may other theories of learning and social interaction(Redish, 2004). It builds on a long tradition of small-scale models of learning(diSessa, 1988; Minstrell, 1992; diSessa, 1993; diSessa & Sherin, 1998) while generalizing and expanding those theories to more accurately reflect learning in physics.

Plasticity

The plasticity continuum(Sayre, Wittmann, & Donovan, 2007) is an extension to Resources which describes the generation and development of resources. Simply put, the two directions in the continuum are more solid and more plastic. (We think of "more plastic" things as if they are a soft gel, and not yet hardened.) A solid resource can be considered a durable concept: its connections to other resources are plentiful, and its internal structure is unlikely to change under typical use(Scherr, 2007). Plastic resources, in contrast, are less durable in time or less stable in structure; they are not the reified objects that more solid resources are. The more plastic a resource is, the less likely the user is able to apply it to new situations, and more explanation is needed to justify and explain its use. The more solid a resource is, the more likely the user is to refer to the resource in diverse contexts without explaining its internal structure. As with all resources, the plasticity of a resource is independent of its veracity.

We use the Recognize/Build-With/Construct (RBC) model(Tsamir & Dreyfus, 2002) to inform and improve the plasticity continuum, noting that the RBC model was originally intended to describe the reification and abstraction of concepts and not all resources need be thought of as concepts. The RBC model proposes three epistemic actions through which abstraction occurs and which may be inferred from behavior. These three actions - recognizing, building-with, and constructing - are dynamically nested.

Recognizing, the simplest action of the three, occurs when a student realizes that a "familiar mathematical notion, process, or idea ... is inherent in a given mathematical situation"(Sfard, 1991). These recognized cognitive objects may be the resources in Resource Theory. Recognition is thus synonymous with activation. Content resources such as these need not be restricted to mathematics; physics is another appropriate subject area. The specifics of which resources are recognized gives insight into students' thought structure. Ease of recognition is therefore a marker of solidity.

Once a familiar idea has been recognized, a student may build-with that idea to solve a local goal, such as solving a problem or justifying a statement. Several resources may need to be recognized and built-with at once. Under Resource Theory, activated resources form a web or graph that may be built on the fly. Thus, building-with is akin to building graphs on the fly. Because building-with and recognizing are two separate actions, the RBC model can describe behavior when students mention an idea, but don't appear to know what to do with it.

In contrast to building-with, constructing has purpose and duration beyond solving a local goal. Constructing creates a less-local, more abstract entity. As a construction becomes more durable, it becomes more consolidated and is no longer necessarily built on the fly. It becomes a resource in its own right, and therefore can be recognized or built-with in later local goals. The new-formed resource may be quite plastic, but as further constructions are added to it and as it compiles further, it can become more solid. Thus constructing is a mechanism for increasing the solidity of specific resources. Extremely solid resources – rigid resources – have been so tightly compiled that their internal structure is not readily accessible to the user.

The process of abstracting and consolidating resources can be of long duration.

Heuristics

We have heuristics for defining the plasticity of a resource, as described below. Examples for many are given elsewhere(Sayre et al., 2007) and in this paper:

P1. Ease of use. The more solid a resource is, the more easily it can be recognized or built-with. This ease of use is directly related to the number and strength of connections that a resource has. Well-connected resources are more likely to activate in a variety of situations.

P2. Recency of construction. Often, but not always, the more recently a resource was constructed, the more plastic it will be. Counter-examples include infrequently used resources, which may be old but plastic (like a physics professor's criterion for critically damped harmonic motion), or recently constructed "flashbulb" resources, which are so vivid that, despite their newness, they are etched solidly upon the mind.

P3. Need for elaboration to evaluate. Users need to explicitly test plastic resources against other (often more solid) resources to determine if the plastic ones should be used in a given context. These tests often take the form of elaborative sense-making. In contrast, solid resources can be apprehended whole and are often quickly recognized without elaboration.

P4. Justification. Because plastic resources often are tested against solid ones, solid resources can justify the use of plastic ones. The degree to which a resource justifies another can be used to see how nearby the two resources are and their relative solidity.

P5. Need rejustification or rederivation for extended use.

Research Setting

Intermediate mechanics is a particularly rich place to study the interplay between physics and mathematics ideas, as students often enter with a solid intuitive grasp of the physics (which may be incorrect), but have not yet applied sophisticated mathematics. At the University of Maine, intermediate mechanics is a one-semester physics course which meets for three one-hour periods each week. Generally, one of those periods is devoted to small-group work on research-based guidedinquiry tutorials. The other two are lecture-based. Though there is not a formal discussion section, an optional Homework Help Session runs once weekly for students to ask a graduate student for help on their homework. In Spring 2006, the course followed a typical intermediate mechanics schedule, starting with air resistance and continuing to damped and driven harmonic motion, energy considerations, Lagrangians, and rotational motion. Typically, about half of the students are concurrently enrolled in Differential Equations; the other half have already taken it.

One thread that runs through the entire course is the selection and application of coordinate systems. Upon starting the course, students have a strong preference to use a Cartesian coordinate system where the positive directions are up and to the right. However, many problems in physics can be made simpler through using other coordinate systems. For example, finding the position of a pendulum a function of time is simplest using polar coordinates. As these students develop as physicists, choosing appropriate coordinates for a problem becomes more important.

A problem involving coordinate systems

To investigate students' developing understanding of coordinate systems, as well as other questions, we collect video data from the Homework Help Sessions, from weekly small group short interviews, and from class discussion. We also collect written data in the form of pretest, homework, and exams. In this paper, we focus on video data from one pair of students during short group interviews in weeks 4 and 10 during the Spring 2006 semester.

The two students, "Derek" and "Wes", volunteered to be interviewed together. Derek was a conscientious student who submitted thorough solutions to assigned problems. He started the semester averse to small-group tutorial work, and finished a loyal supporter. In contrast, Wes rarely submitted complete solutions and had a poor work ethic.

In both interviews, students are presented with the same problem: given a (drawing of) a simple pendulum (Figure 1, with polar coordinates shown), find the position of the pendulum bob as a function of time. So that students do not spend time figuring out the forces on the bob, and to predispose the students into thinking of force-based solutions, the forces on the bob (a weight force and a tension force) are given diagrammatically. So as not to predispose students into choosing a particular coordinate system, the forces are not described as being "vertically downwards" (weight force) or "radially inwards" (tension force).

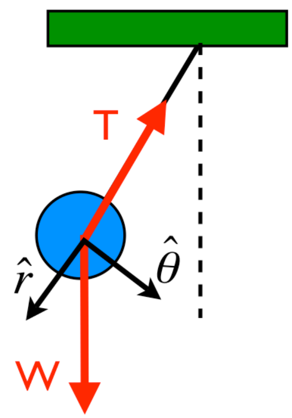

Figure 1. The forces on a simple pendulum, with a physicist's polar coordinate system shown.

To solve for the position of the bob as a function of time, a physicist would first write Newton's Second Law for the system, a vector second-order differential equation for an unspecified coordinate $\boldsymbol{x}$:

$$\sum \mathbf{F} = \mathbf{W} + \mathbf{T} = m\frac{d^2\mathbf{x}}{d^2t} \tag{1}$$

The physicist would then choose a polar coordinate system where the radial coordinate, r, points outwards from the attachment point of the pendulum and the angular coordinate, θ, is measured counter-clockwise from the equilibrium position of the bob (straight down). This coordinate system takes advantage of the natural geometry and symmetry of the situation, and it is a calculationally easy choice. With the coordinate system in place, the vector equation of motion can be split into two scalar differential equations, and then solved. As the focus of the interviews was on the coordinate system choice, the students were not expected to solve the differential equations.

We note that these students have encountered the pendulum problem in some detail in a calculus-based introductory physics course. The pendulum is but one (relatively simple) example of harmonic motion, one of the most important models taught in an undergraduate physics major. We expected students to be familiar with the problem but to have forgotten the specifics of the modeling they previously did.

The resource structure of coordinate systems

One property of resources is that they are nestable: one resource may contain graphs of other resources. As researchers, therefore, we choose a level of specificity to examine, noting that other levels are possible and may yield interesting results. We designed the research task to match our investigation of the structure and development of resources.

We break the *coordinate systems* resource into three subgraphs of resources nested within it:

- *Properties* resources, which describe general properties that coordinate systems bear;
- *Use* resources, which describe when to use coordinate systems and which coordinate systems to use; and
- *Case* resources, which hold the specifics of given coordinate systems.

An example of resources in each subgraph is available in Table 1. Some common *case* resources are also listed; the list is meant to be suggestive, not exhaustive. The exact breakdown of which resources belong in which sub-graphs, as well as the inter- and intra-subgraph connection details, are user specific and time specific. However, naming the one possible set of components and their interplay gives a baseline against which users' ideas can be tested.

Table 1: Subgraphs in *Coordinate Systems*

Properties	directionality	positive and negative, forward and backward
	orthogonality	each coordinate cannot be obtained through linear combination of other coordinates in the same system.
	span	a set of all coordinates expresses all possible dimensions of the space.
	equivalency	different coordinate systems are interchangeable
Use	choice	coordinate systems must be chosen
	explicitness	use may be implicit or explicit
	natural	"preferred" coordinate system based on geometry
	ease	"preferred" coordinate system based on calculational ease
	arbitrariness	choice of coordinate system is not predetermined
	consistency	Within a problem, coordinate systems should not change
Case	Cartesian	A rectilinear coordinate system (x, y, z)
	polar	A circular coordinate system in two dimensions (r, θ)
	numberline	A one-dimensional coordinate system
	phase space	A coordinate system where coordinates do not refer to locations
	...	Presented are examples only; others abound.

Using our breakdown of subgraphs within *coordinate systems*, it is possible to examine which resources activate in given situations and show intra-*coordinate systems* linkages. It is unreasonable to expect that all of these resources would activate in every episode; typically, only a few need be active, depending on the context.

In this paper, we focus on only a few examples of resources which are closely tied to the pendulum problem. The *case* resources commonly activated in this problem are *Cartesian* and *polar*. These are often activated because they are *natural* to the geometry or *easy* to use mathematically. Finally, the issue of *span* arises in determining whether the chosen coordinate system can actually describe the entire system (and the space in which movement occurs) appropriately.

Data

Though *coordinate systems* contains a large number of other resources, not all of them are activated in the interviews discussed here. In addition, a great many resources unrelated to coordinate systems are activated. To help focus the discussion, we restrict the analysis to looking at the activation of and connections between five resources within *coordinate systems* (*polar, Cartesian, natural, ease,* and *span*) and the plasticity of only two resources (*polar* and *Cartesian*) for each student. In the process, we draw resource graphs of these five resources for each student.

We do not give direct quotes from the interviews for several reasons. First, space restricts us from the full transcript. Second, Wes and Derek were good friends whose interactions included much banter; their language was often not appropriate for the general public. We describe their actions as best possible, as a result.

Preliminary resource graphs and plasticity, Week 4

In the first interview (Week 4), after presenting the problem described in Figure 1, the interviewer asks the students about their coordinate system. Wes replies that he's using "the standard Cartesian [system]", and draws coordinate axes next to the diagram, indicating that the positive directions are up and to the right. Derek immediately interjects with, "Why not use polar?", supporting his assertion with the claim that the angles are changing. Wes objects, but he redefines his coordinate system, relabeling the upward-pointing y-axis as the r-axis, and defining the angle between the x-axis and r-axis as θ.

After some discussion, the interviewer asks Wes and Derek to apply their coordinate system, which is drawn next to the sketch of the pendulum, to the sketch itself. Wes labels the polar angle as measuring counterclockwise from horizontal to the position of the bob, and both students readily volunteer that $\mathbf{r}$ should be measured outwards from the attachment point of the pendulum.

Because Wes's definition of θ indicates a curved path, but unit vectors are always drawn as straight lines, the interviewer presses Wes to show the direction of θ at the instant shown, hoping that he will choose a direction tangent to the path. Wes demurs, asserting again that θ is "counterclockwise".

We interpret this interchange as showing Derek and Wes using a different *natural* coordinate system, *polar* for Derek and *Cartesian* for Wes. Furthermore, *polar* coordinates are connected to physical examples for Derek. They are less connected for Wes, for whom Cartesian coordinates are "standard" for any given problem and polar coordinates are not well defined. This indicates differing levels of plasticity: Cartesian coordinates are more solid for Wes, polar coordinates more plastic, and polar coordinates are more solid for Derek than they are for Wes.

Their coordinate system defined, the students move on to discussing how to solve for the position as a function of time. Wes asks for clarification about the task: does the interviewer want to know the up-down position of the bob, or the side-to-side position of the bob? He explains that he "[doesn't] know any functions that would give you two parameters" and thus he can't solve the two-dimensional problem using the traditional x-y coordinate system.

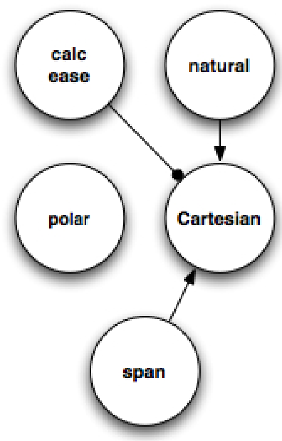

Figure 2. A resource graph describing Wes's use of *coordinate systems* in the Week 4 pendulum interview. Note the explicit lack of connection between *ease* and *Cartesian* resources.

Wes then wonders if it would be smarter to measure position based on displacement from starting angle. When the interviewer asks if knowing the angle as a function of time is sufficient, Wes replies that he doesn't know and that he'd "have to figure that out". When the same question is put to Derek, he replies quickly that "yes, of course" knowing θ(t) is sufficient.

From the continuing interchange, we have more evidence that Wes's *polar* resource is not well connected to other resources. He seems to have activated *span* as a relevant measure of modeling, but is unable to find a *natural* or *easy* connection based on his chosen coordinates and the mathematical models available to him. He explicitly speaks against the calculational ease of using Cartesian coordinates. Derek, on the other hand, sees the *span* of *polar* and *Cartesian* coordinates as sufficient for this problem and sees *polar* coordinates as *natural* for this problem. We represent this description of Wes's and Derek's resource use in Figures 2 and 3. Note that one arrow is drawn to indicate the explicit lack of connection between Wes's *ease* and *Cartesian* resources.

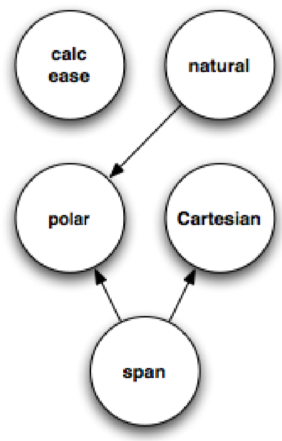

Figure 3. A resource graph describing Derek's use of *coordinate systems* in the Week 4 pendulum interview

Revisiting polar coordinates, Week 10

In Week 10, the task in Figure 1 is posed to the students again. In the intervening weeks, students have studied damped and driven harmonic motion in class, and have been assigned a homework problem on this question. Their responses typify their approaches to the class: Wes says that he "tried to use radians, but got stuck and gave up"; Derek says that to solve this problem, he would just "assume a solution".

When the interviewer asks them to define a coordinate system, Derek chooses a polar coordinate system for the same reasons he did in week 4. Wes once again chooses a Cartesian system. However, instead of choosing the standard system where positive is up and to the right as he did in week 4, he tailors his system to the problem at hand, defining positive down and to the right. The downward direction is consistent with the weight vector, showing a better match of coordinates to physical situation.

At the interviewer's prompting, Wes continues to write Newton's Second Law for the system and starts to break the forces into components, defining θ as the angle between the horizontal and the position of the bob (θ_1 in Figure 4) . His choice of Cartesian coordinate system complicates the problem, and he gets stuck.

Derek uses Wes' confusion as evidence that a Cartesian system is inappropriate. As illustration, he writes Newton's Second Law (Equation 1) and breaks it into **r** and θ components. Derek writes,

$$\sum F_x = m\frac{d^2\theta_x}{dt^2} \tag{2}$$

$$T\cos\theta = m\frac{d^2\theta_x}{dt^2} \tag{3}$$

After writing the Equations 2 and 3, Derek reads them aloud. In reading them, he corrects himself: because θ is a coordinate in its own right, he does not need to use the subscript x. His equation, amended, reads

$$T\cos\theta = m\frac{d^2\theta}{dt^2}. \tag{4}$$

This equation differs from the standard physics equation because the angle they have defined as θ is the complement of the typically chosen angle. Furthermore, it is dimensionally inconsistent: the left-hand side has units of force (Newtons) and the right-hand side has units of mass per time squared (or Newtons/meter). These differences aside, Derek's equation has the right form of the differential equation.

An equation in place, the interviewer again asks the students to label their coordinates on their diagram. Derek first copies over Wes' definition of θ, (θ_1 on Figure 4), then argues that by alternate interior angles, it is equal to θ_2. The interviewer asks where θ is equal to zero, and Derek redefines θ to be θ_3, the common and calculationally easy physicist response. When the interviewer asks the direction of θ at the instant shown, Derek argues that the bob is moving in a circular arc and that, at any point along the arc, θ is tangent to the arc. He draws θ_4. With all four θ definitions arrayed before him, Derek expresses doubt that he has a sensical answer.

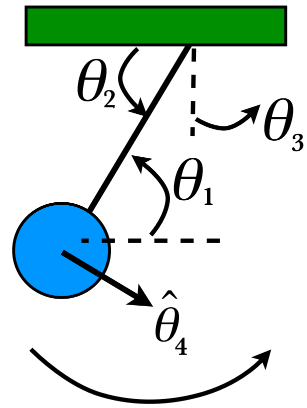

Figure 4. Derek and Wes use four definitions of θ.

We interpret the evidence from the week 10 group interview to indicate even more strongly that Wes's *polar* resource is very plastic. It is not well connected to other resources, in particular with calculational *ease*. Derek, whose *polar* resource seemed solid, shows evidence of problems with the *ease* of applying the *natural* coordinates for this problem. We represent this description of Wes's and Derek's resource use in Figures 5 and 6. Because *span* was not a topic of the Week 10 interviews, we have left that resource out of our graphs.

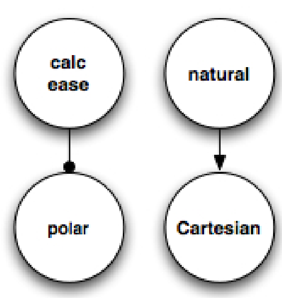

Figure 5. A resource graph describing Wes's use of *coordinate systems* in the Week 10 pendulum interview. Note the explicit lack of connection between *ease* and *polar* resources.

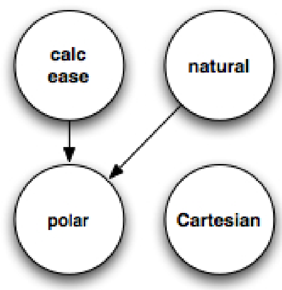

Figure 6. A resource graph describing Derek's use of *coordinate systems* in the Week 10 pendulum interview

Discussion

In this paper, we hope to have fulfilled two separate tasks. The first is to introduce a theoretical structure by which we can understand the use of mathematics when reasoning about intermediate and advanced physics topics. We build off Resource Theory, but add ideas from Process/Object and RBC theory to help us develop observable tools for understanding the plasticity of resources as they develop over time. One of the elements of Resource Theory is that resources can be nested, containing other resources. We represent some of this structure through resource graphs.

To show that our first task can be fulfilled, we have applied our theoretical structure to help explain student reasoning about coordinate systems in a canonical physics problem that nevertheless presents difficulties to students. We have shown resource graphs of two students applying resources that are parts of the *coordinate systems* resource, and used these graphs to indicate the level of plasticity of the different sub-resources with *coordinate systems*.

We find that students sometimes persist in using an inappropriate Cartesian system despite professed knowledge of polar coordinates, indicating that Cartesian coordinates are quite solid to these students. Furthermore, students must rederive (rather than recall) the details of the polar coordinate system, indicating that polar coordinates are quite plastic. Detailing the interactions between resources gives us better insight into the working of student minds and lets us build better models of our students and their learning. These models may lead to better curricular design.

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