Preservice Teachers' Number Sensible Mental Computation Strategies

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This report briefly describes results of a classroom teaching experiment around number sensible mental math in a semester-long content course for preservice elementary teachers. The instructor sought to encourage students to develop their own number sensible mental computation strategies. Of the 50 study participants, 13 were interviewed pre and post. Analysis of interview data suggests that students did develop significantly greater number sense. Results can inform mathematics teaching at various levels.

BACKGROUND

The development of number sense in students is a widely accepted goal of mathematics instruction (NCTM, 2000). Reys & Yang (1998) state that:

> Number sense refers to a person's general understanding of number and operations. It also includes the ability and inclination to use this understanding in flexible ways to make mathematics judgments and to develop useful strategies for handling numbers and operations. It reflects an inclination and ability to use numbers and quantitative methods as a means of communication, processing, and interpreting information. It results in an expectation that numbers are useful and that mathematics has a certain regularity. (p. 226)

Good number sense is especially essential for elementary teachers. Without it, they are ill-equipped to make sense and take advantage of children's often unorthodox but very number sensible solution strategies. Mental math ability is considered a hallmark of number sense (Sowder, 1992). Much work has been done with the aim of identifying the characteristics exhibited and strategies used by individuals who are skilled at mental math (cf. Reys, Bestgen, Rybolt, & Wyatt, 1980, 1982; Hope & Sherrill, 1987; Markovits & Sowder, 1994). Of particular relevance to this study is flexibility in thinking about numbers and operations (Sowder, 1992).

In this paper, I describe the results of a classroom teaching experiment (CTE) (Cobb, 2000) with preservice elementary teachers enrolled in a mathematics course focused on Number & Operations. The instructor attempted to foster students' development of number sense with regard to mental math. In previous semesters, the instructor had seen evidence that such development had not occurred. The most poignant evidence for this came in final exam results. Many students' responses to mental math problems betrayed a conspicuous lack of number sense. An answer of 1800 to a problem that asked students to "Use benchmarks to estimate 0.32147 x (67.557% of 89.515)" is one such example. Also of note is that, while most students had at least attempted to answer mental math problems in a seemingly appropriate manner (i.e. employing some strategy to make the calculation simpler, rather than applying a standard algorithm), such approaches were not often used for other problems where they could have come in very handy. Without the cue to solve a problem "mentally," students had rarely used nonstandard or novel approaches. Thus, their inclinations appeared unchanged.

How could one design a class to support students' development of number sense? Drawing on the related literature, I developed a local instruction theory (Gravemeijer, 1999, 2004) aimed at students' development of number sense in the area of mental math. Researchers have observed that number sense develops gradually (Howden, 1989). Sowder (1992) notes that "[t]here is consensus on the fact that number sense should permeate the curriculum… rather than being relegated to 'special lessons' designed to 'teach number sense' " (p. 386). First and foremost, my idea was not to treat mental math as an isolated unit in the curriculum but to integrate authentic mental math activity[1] throughout the course.

Teaching with the goal of students developing number sense involves a catch-22. Attempting to teach the symptoms of good number sense may not be an appropriate route to students' development of number sense. Explicit, direct instruction in the use of particular strategies may actually be counterproductive (Greeno, 1991; McIntosh, 1998; Schoenfeld, 1992). This is due to the flexibility inherent in skilled mental computation and the necessity that choices be made. Appropriate instruction for the development of number sense would encourage the development of flexibility and the habit of making choices. Yet, direct instruction would have the opposite effect. As Schoenfeld (1992**)** points out, when strategies are taught directly, "they are no longer heuristics in Pólya's sense; they are mere algorithms" (p. 354). McIntosh (1998) suggests instead that students be given the opportunity to invent their own strategies, that those strategies should be shared and discussed, and that the instructor should take advantage of students' spontaneous interest in each others' strategies.

The above recommendations from the literature greaty influenced my instrucional approach. As instructor and mental math practitioner, I sought to engender a learning environment in which the doing of mental math appeared not as a separate exercise, but as a natural and practical aspect of our activity. The orientation toward mathematical

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[1] The term *authentic activity* is used in the sense of Brown, Collins, and Duguid's (1989) discussion of situated cognition, in which "Authentic activities… are most simply defined as the ordinary practices of the culture" (p. 34). In this case, the culture is that of persons who regularly use number sensible mental math in their daily lives.

activity that I intended to model and engender was that of a sense-making approach, accompanied by a disposition toward justification and critical analysis of problem-solving strategies. I endeavored to foster a classroom culture in which mental math was an ordinary practice. Fundamental to this practice would be a conceptual orientation and a preference for number sensible, context-appropriate strategy use. I hoped that students' participation in this kind of mathematical activity would result in their developing the sensibilities that are characteristic of good mental calculators and estimators.

Analysis of the data suggests that students did develop greater number sense with regard to mental math. The notion of flexibility particularly informed the analysis of interview data. This report describes that analysis. This work was part of a larger study, which constituted the author's Master's thesis.

THEORETICAL PERSPECTIVE

The theoretical orientation for this study can be characterized as emergent. Students' individual mathematical activity is recognized as taking place in a social context, while the social environment of the classroom is constituted by collective mathematical activity. As such, the teacher's role was to support collective learning, while also concerning himself with the negotiation of norms and practices (Cobb, 2000).

The perspective on number sense that I take is rooted in Greeno's (1991) metaphor of situated knowing in a conceptual domain. As Greeno (1991) puts it,

> The metaphor is quite suggestive regarding the role of a teacher. As learning is analogous to acquiring abilities for finding one's way around in an environment, teaching is analogous to the help that a resident of the environment can give to newcomers. (p. 197)

Thus, as instructor, I endeavored to provide students with experiences that would enrich their ability to navigate the domain of numbers and operations. In keeping with this metaphor, one does not become adept at navigating a domain by merely acquiring specific sets of directions. Rather, a key aspect of knowing is a person's ability to construct and reason with mental models (Greeno, 1991).

SETTING

This study was conducted with undergraduates at a large, urban university in the United States. The participants were preservice elementary teachers enrolled in two sections of a first-semester mathematics course, belonging to a four-course sequence. Of the 50 students who agreed to participate in the study, 42 were female. The author was also the instructor of the course. He had taught it for two prior semesters. Basic course topics included quantitative reasoning, place value, meanings for operations, mental computation and estimation, meanings for fractions, and operations involving fractions.

DATA & METHODOLOGY

The data sources drew from classroom events, written artifacts, and individual interviews. Specifically, the data corpus consisted of the following:

- Transcripts of early- and late-semester clinical interviews with 13 students;
- The instructor/researcher's journal, which included accounts and interpretations of classroom events, as well as rationales for instructional design decisions;
- Students' relevant written work, both in-class and take-home, including responses to exam questions;
- An adapted version of the Number Sense Rating Scale (Hsu, Yang, & Li, 2001), which was administered to students at both the beginning and end of the semester as a quantitative measure of number sense.

I report here on the analysis of interview data.

Thirteen subjects participated in early- and late-semester structured, task-based interviews (Goldin, 2000). The interviews were designed to reveal the variety of mental computation strategies available to students for a given operation. Interviews consisted of one-step story problems in context, which students were asked to solve mentally. Students were then asked exploratory, metacognitive questions (Goldin, 2000) such as how they arrived at their answer and what decisions they had made in the process. For a given operation, students given the same story problem several more times with different numbers each time, the succession of numbers used depending on the students' previous responses. Another common question was to ask if a student had an alternative method for performing the calculation. While repetition of such questions is bound to affect subjects' responses to subsequent questions, Goldin (2000) states that "[t]here is nothing in this to discredit the methodology" (p. 521).

Interview data was coded via constant comparative analysis (Creswell, 1998). The researcher sought to appropriately and meaningfully categorize each of the strategies subjects had used. This effort resulted in short lists of subjects' strategies for addition, subtraction, and multiplication of whole numbers. Each strategy was defined in detail in consideration of each instance of its use. In summarizing the strategy use data, it was a student's primary strategy for solving a problem that was recorded. This convention parallels that used by Hope & Sherrill (1987), in which the use of ancillary strategies was noted, but ultimately the student's solution method was classified according to the strategy that played the "primary calculative role" (p. 103).

Framework

Markovits & Sowder (1994) formulated an overarching framework for mental calculative strategies. In a seminal paper entitled *Developing Number Sense*, the authors used the following taxonomy for describing the mental computation strategies that students used:

> *Standard:* The student used a mental analogue of a standard paper-and-pencil algorithm.
> *Transition:* The student continued to be somewhat bound to the standard algorithm. However, more attention was given to the numbers being computed and less to algorithmic procedures.
> *Nonstandard with no reformulation:* A left-to-right process was used.
> N*onstandard with reformulation:* The numbers were reformulated to make the computation easier. (p. 14)

Markovits & Sowder's (1994) taxonomy is useful in that it is general enough to apply to each of the basic operations. It is also useful for analysis since it suggests a progression from one end of a spectrum to the other. Students who rely heavily on Standard methods evince poor number sense. Their understanding of an operation seems to be tied to symbol manipulation, so that they lack flexibility. At the other end of the spectrum, students who readily employ Nonstandard methods, especially Nonstandard with reformulation, exhibit good number sense. Their understanding of the operations is independent from any particular algorithm, so that they have good flexibility. Those primarily using Transition strategies can, indeed, be seen as in transition from Standard to Nonstandard, from tied to the algorithms to independent, from inflexible to flexible.

Categorization of Strategies by Operation	Standard	Transition	Nonstandard	Nonstandard w/Reformulation
Addition	*MASA*	*Right to Left, Left to Right*	*Empty Number Line*	*Giving, Round-Compute-Compensate*
Subtraction	*MASA*	*Right to Left, Left to Right*	*Empty Number Line, Adding On*	*Round-Compute-Compensate, Shifting the Difference, Difference Below*
Multiplication	*MASA*	*Partial Products*	*Nonstandard Additive Distribution*	*Subtractive Distribution, Aliquot Parts, General Factoring, Derived*

Table 1. *Subjects' Strategies in Terms of the Standard-to-Nonstandard Spectrum.*

RESULTS

Subjects' responses to story problems were recorded and coded according to the solution strategy the student employed. Many of the strategies seen were anticipated, but the coding scheme was not predetermined. It emerged in analysis of the data. Markovits & Sowder's (1994) taxonomy was then used as an organizing framework for ordering subjects' strategies along the Standard-to-Nonstandard spectrum (See Table 1). In this way, the strategies observed were not merely catalogued but were ranked according to the degree to which they departed from the standard algorithm. The further from Standard a strategy is, the more indicative it is of good number sense (Markovits & Sowder, 1994).

Interview subjects used six distinct strategies for addition, eight for subtraction, and seven for multiplication. In the following sections, subjects' strategies are described, as are the changes seen in strategy use from first to second interviews.

Addition Strategies Observed

This section describes each of the six distinct addition strategies that the 13 interview subjects used in their first and second interviews.

Mental Analogue of the Standard Algorithm (MASA)

The student used the mental analogue of the standard (US) addition algorithm. Language such as "carry the one" often accompanied the use of this strategy.

Right to Left (RtoL)

The student added place-value-wise from right to left but without actually visualizing the numbers aligned as in the MASA. Students generally used place-value language in describing their steps.

Left to Right (LtoR)

The student added place-value-wise from left to right, rather than right to left.

Empty Number Line (ENL)

The student began with one of the two addends (usually the larger one) and added the other one on in convenient chunks, generally working from big to small and keeping a running subtotal. This strategy was called Empty Number Line since it resembled a typical usage of the empty number line.

Giving

The student altered the problem such that part of one addend (usually a small number of ones) was given to the other prior to performing the addition. Since the sum was maintained, no compensation was necessary.

Round-Compute-Compensate (RCC)

The student altered one or both addends (usually rounding them up or down to the nearest multiple of ten) prior to performing the addition. The student added the rounded numbers and then compensated for rounding.

The above list encompasses all of the strategies for addition that were observed amongst the 13 interview subjects in both first and second interviews, modulo occasional combinations of strategies and slight variations.

Shift in Addition Strategy Use

Figure 1 shows the percentage of total strategy uses belonging to each strategy observed for addition. These appear in order along the Standard-to-Nonstandard

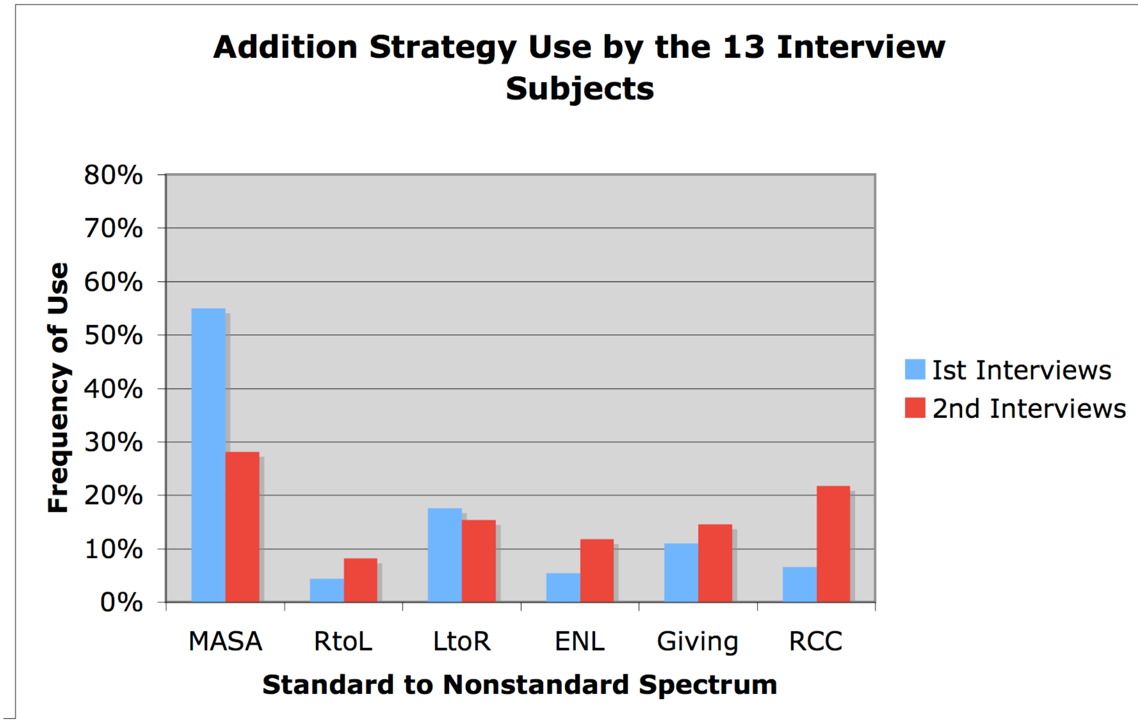

Figure 1. *Addition Strategy Use by the 13 Interview Subjects.*

spectrum. In first interviews, the MASA for addition was used by interview subjects for the majority of addition problems they were given (approximately 55%), RtoL accounted for 4.4% of strategy uses, LtoR 17.6%, ENL 5.5%, Giving 11%, and RCC 6.6%. In second interviews, the frequency of use of the MASA dropped to 28.2%, about half of what was seen in first interviews. The frequency of use of four of the five alternative strategies increased, RtoL to 8.2%, ENL to 11.8%, Giving to 14.6%, and RCC to 21.8%.

Subtraction Strategies Observed

This section describes the eight distinct subtraction strategies used by the 13 interview subjects in first and second interviews.

Mental Analogue of the Standard Algorithm (MASA)

The student used the mental analogue of the standard (US) subtraction algorithm. Language such as "borrowing" often accompanied the use of this strategy.

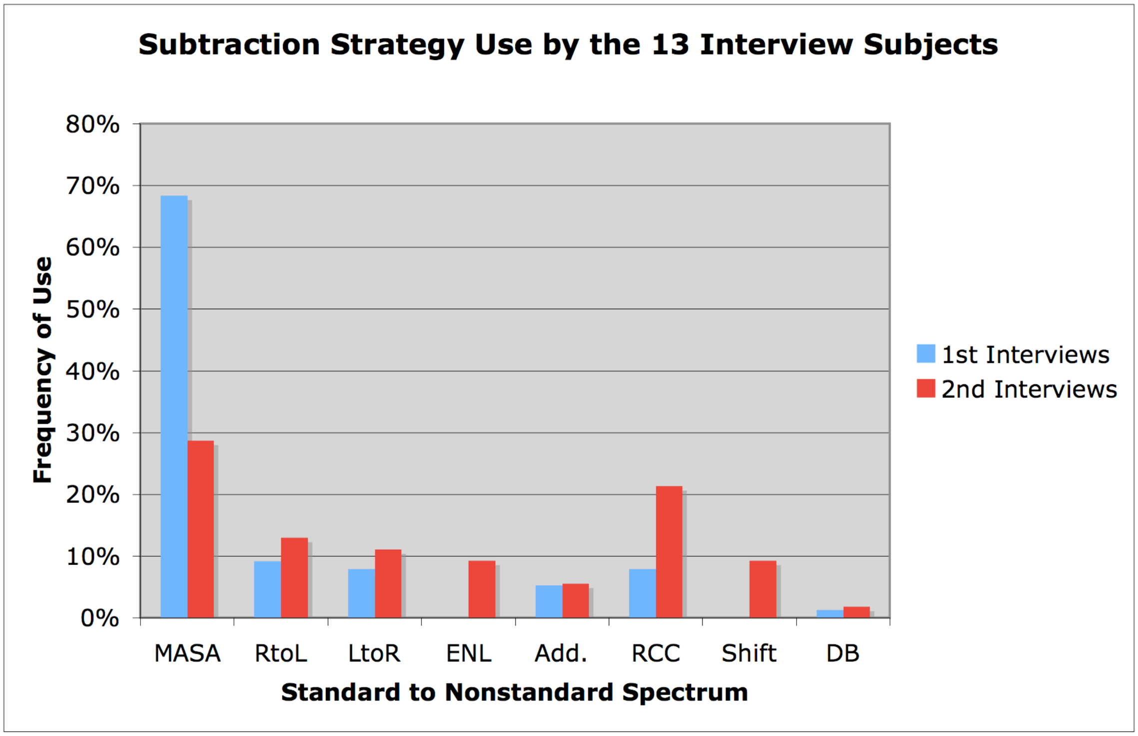

Figure 2. *Subtraction Strategy Use by the 13 Interview Subjects.*

The above list encompasses all of the strategies for subtraction that were observed amongst the 13 interview subjects in both first and second interviews, modulo occasional combinations of strategies and slight variations.

Shift in Subtraction Strategy Use

Figure 2 shows the percentage of total strategy uses belonging to each strategy observed for subtraction. These appear in order along the Standard-to-Nonstandard spectrum. In first interviews, the MASA for subtraction was used more than two-thirds (68.4%) of the time, while RtoL accounted for 9.2% of strategy uses, LtoR for 7.9%, ENL was not used, Add. was used 5.3% of the time, Shift. was not used, and DB was used only once. In second interviews, the frequency of use of the MASA dropped to less than half of that seen in first interviews (28.7%). The frequency of use of all alternative strategies increased, RtoL to 13%, LtoR to 11.1%, ENL to 9.3%, Add. to 5.6%, RCC to 21.3%, Shift. to 9.3%, and DB was used twice.

Multiplication Strategies Observed

This section describes the seven multiplication strategies used by the 13 interview subjects in first and second interviews.

Mental Analogue of the Standard Algorithm (MASA)

The student used the mental analogue of the standard (US) multiplication algorithm. Students spoke in terms of digits, rather than using place-value language, and often made reference to carrying, to using a "place holder" on the second line, or to adding two "lines" of digits together.

Partial Products (PP)

The student decomposed one or both factors place-value-wise and then applied the distributive property of multiplication over addition. Most students who employed PP used place-value language.

Nonstandard Additive Distribution (NAD)

The student decomposed a factor non-place-value-wise, then applied the distributive property of multiplication over addition. Thus, the partial products added were not those that one would add in the standard algorithm.

Subtractive Distribution (SD)

The student applied the distributive property of multiplication over subtraction. This strategy was often used when there was a benchmark number slightly greater than either of the factors.

Aliquot Parts (AP)

Aliquot Parts refers to a special case of factoring. Interview subjects only used AP for products involving 25s. When one of the factors was 25 and the other a multiple of 4, some students divided the latter factor by 4 to find out the number of hundreds in the product.

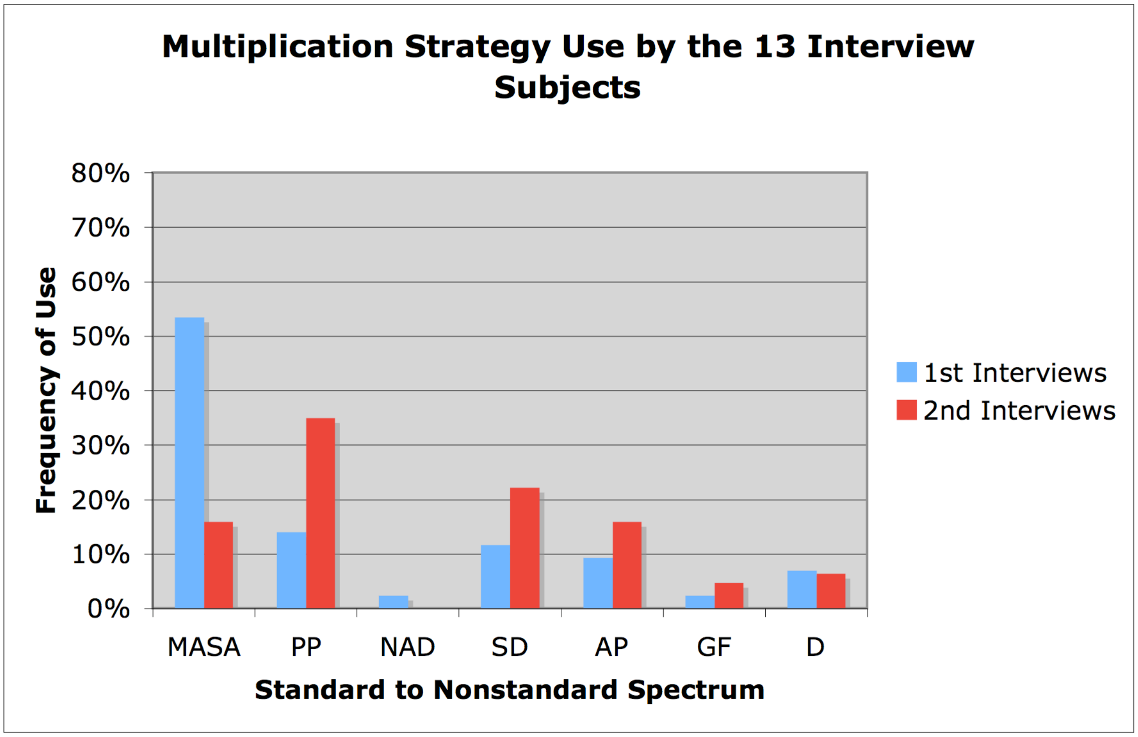

Figure 3. *Multiplication Strategy Use by the 13 Interview Subjects.*

General Factoring (GF)

The student treated one of the given factors in terms of two of its factors. The student then either converted the product into an equivalent one, or found it in two steps, multiplying by one factor at a time.

Derived

The student made two or more distinct compensation steps to derive the desired product from a known (or readily calculated) one.

The above list encompasses all of the strategies for multiplication that were observed amongst the thirteen interview subjects in both first and second interviews, modulo combinations of strategies and slight variations.

Shift in Multiplication Strategy Use

Figure 3 shows the percentage of total strategy uses belonging to each strategy observed for multiplication. These appear in order along the Standard-to-Nonstandard spectrum. In first interviews, the MASA was used the majority (53.5%) of the time, while PP accounted for 14% of strategy uses, NAD was used once, SD accounted for 11.6% of uses, AP 9.3%, GF was used once, and Derived was used three times. In second interviews, the frequency of use of the MASA dropped to less than one-third of that seen in first interviews (15.9%), while the frequency of use of six of the seven alternative strategies increased. PP became the most common strategy for multiplication, with students using it 34.9% of the time. SD accounted for 22.2% of strategy uses, AP for 15.9%, GF 4.8%, and Derived 6.4%.

Overall Strategy Use Summary

For each operation, there was a large decrease in the frequency that the MASA was used, accompanied by an increase in the frequency of use of alternative strategies. Thus, students exhibited greater flexibility by making use of a wider variety of strategies in second interviews than they had in first interviews. Furthermore, strategies used in second interviews were more number sensible. This is apparent in the movement we see along the spectrum from Standard to Nonstandard. This data is summarized in Figure 4, according to Markovits & Sowder's (1994) framework. Overall, for addition, subtraction, and multiplication problems, students in first interviews used Standard (S) methods the majority (59.5%) of the time, while Transition (T) methods accounted for 18.6% of uses, Nonstandard with No Reformulation (N) 4.8%, and Nonstandard with Reformulation (N w/R) 17.1%. In second interviews, the frequency of use of Standard methods dropped to less than half that seen in first interviews (25.6%), while Transition and Nonstandard methods were used more often. The frequency of use of Transition strategies increased to

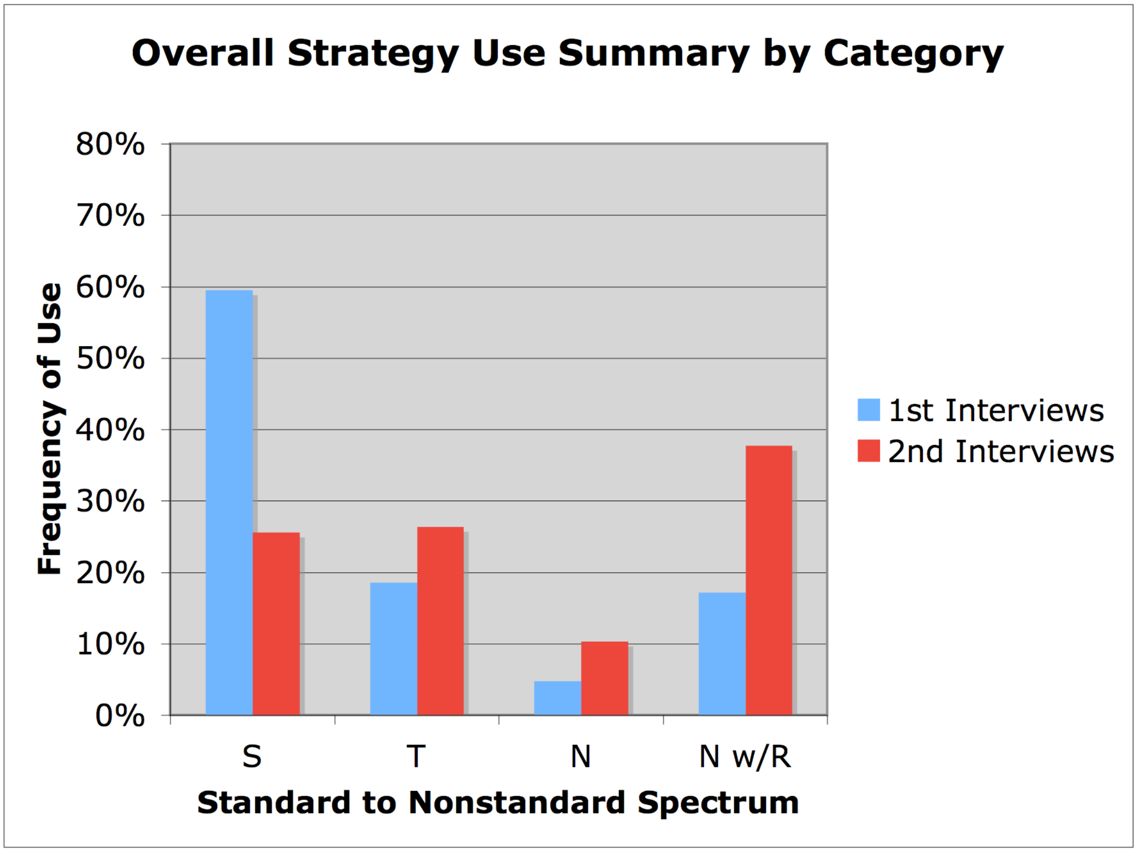

Figure 4. *Overall Strategy Use Summary by Category*.

26.3%, Nonstandard with No Reformulation to 10.3%, and Nonstandard with Reformulation became most common, at 37.7%.

Discussion

It is compelling evidence for change in the direction of number sense that Standard methods were most common in first interviews, while Nonstandard with Reformulation became most common in second interviews. At the same time, however, a complete shift from a left-heavy to a right-heavy distribution is not necessarily desirable. Flexibility manifests itself in this picture in the more even distribution of strategy uses across the four categories. There are times when it seems perfectly appropriate to use Transition, and even Standard, strategies in practice. Figure 4 reflects the phenomenon that subjects in second interviews tended to make a choice of strategy based on the particular numbers given and that the chosen strategies tended to be more number sensible. While an algorithm can be applied without understanding, the use of Nonstandard strategies generally requires understanding (McIntosh, 1998). These students were never tested on the strategies seen in class nor given any inducement to memorize them. They appear to have been used as true heuristics, grounded in understanding.

Note that I depart from Markovits & Sowder (1994) regarding the classification of Transition versus Nonstandard strategies. When subjects used left-to-right processes, their thinking appeared to be rather closely tied to the MASA. When it came to addition and subtraction, for example, several subjects worked from left to right when no regrouping was required, and right to left otherwise. Thus, their choice seemed to be dictated by an aspect of the standard algorithm. I concluded, therefore, that the use of this strategy reflected thinking that was "somewhat bound to the standard algorithm" (p. 14). The classification of subjects' strategies was made in the spirit of Markovits & Sowder's (1994) taxonomy, wherein the degree to which a strategy departed from the standard algorithm was the primary condition for its classification. The ordering of strategies along the Standard-to-Nonstandard spectrum is unaffected by the above discrepancy.

Shift in Flexibility

This section presents a summary of the number of distinct strategies used by each subject in first and second interviews to calculate sums, differences, and products of whole numbers mentally. This numerical summary provides a quantitative snapshot, which highlights two important outcomes: (1) Students generally made use of a wider variety of strategies in their second interviews; and (2) while in first interviews, the

Number of Strategies Used by Operation	1st Interview	2nd Interview
Addition	9 of 13 subjects used 1 or 2 strategies	12 of 13 subjects used 3 or more strategies
Subtraction	11 of 13 subjects used 1 or 2 strategies	12 of 13 subjects used 3 or more strategies
Multiplication	11 of 13 used 1 or 2 strategies	10 of 13 subjects used 3 or more strategies

Table 2. *Number of Strategies Used by Operation.*

MASA was used by most students for the majority of solutions they computed, in second interviews, there was most often not a dominant strategy. The numbers of strategies subjects used is summarized in Table 2. This data makes apparent that additional strategies for each of addition, subtraction, and multiplication seemed to be available and/or preferable to students in second interviews.

CONCLUSIONS

The results reported here occurred in the context of a CTE around number sensible mental math in a content course for preservice elementary teachers. These results are particularly significant since students belonging to this population are known for their lack of number sense, as well as their lack of confidence in their mathematical abilities. Despite these obstacles, the interview subjects demonstrated significantly greater number sense in their second interviews. In terms of Greeno's (1991) environmental metaphor, subjects evinced improved abilities to navigate the domain of numbers and operations. They did not merely acquire certain strategies for mental computation. Rather, they developed (at least some of) the sensibilities of skilled mental calculators.

Although useful pedagogical recommendations were found in the literature, by no means did they constitute a prescription for instruction to foster students' development of number sense. Throughout the semester, the instructional approach was modified in response to classroom events in the effort to bring theory into practice. The results were a successful integration of authentic mental math activity throughout an existing curriculum. The teaching was done with all the constraints that teachers of such courses typically have to deal with: a considerable breadth of material to cover, a single semester in which to do so, a common departmental final, and a group of students who by and large did not consider themselves "math people." It is not the case that these students became more skilled at mental computation due to additional time spent on that material, at the expense of some other piece of the curriculum. On the contrary, the result of the integration effort was that all the course material was covered, often more deeply than in previous semesters, and that students both learned the course content and developed greater number sense.

The results of this work can benefit the research community by contributing to and furthering existing literature related to mental computation and teaching for the development of number sense. Furthermore, as this work was done with preservice teachers, it adds to that specific body of literature and addresses the widespread need for instruction that contributes to the development of number sense in this group of people, for which it is so crucial.

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