Prospective Secondary Mathematics Teachers' Conceptions of Rational Numbers

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It is apparent in our modern society that students leaving our schools are expected to hold a certain degree of foundational knowledge. This includes the abilities to read and interpret a large range of texts. A crucial point is made by Siemon (2002):

> In an analysis of commonly encountered texts, that is, texts that at least one member of a household might need to, want to, or have to deal with on a daily, weekly, monthly or annual basis, approximately 90% were identified as requiring some degree of quantitative and/or spatial reasoning. Of these texts, the mathematical knowledge most commonly required was some understanding of rational number and proportional reasoning, that is, fractions, decimals, percent, ratio, and proportion.

Siemon (2002) suggests that while students are initially familiar with fraction names such as half and quarter, this does not necessarily imply that they understand the conceptual relationships of rational numbers. Students may simply use these terms to "describe and/or enumerate well-known objects (Siemon, 2002)."

Prospective mathematics teachers must possess a conceptual understanding of rational numbers if they are to effectively impart this knowledge to their students. Literature in mathematics education suggests possible deficiencies in students' understanding of rational numbers (Post et al, 1982). However, an especially disturbing suggestion is a lack of proficiency among teachers (Harel & Behr, 1995). This is alarming because of teachers' responsibilities for their students' knowledge. Furthermore, the results of a study conducted on prospective and in-service teachers in Israel revealed that both groups' incorrect responses on a diagnostic questionnaire mirrored mistakes made by students, as had been previously reported in the literature (Klein & Tirosh, 1997). Such similarities may imply that teachers are passing their lack of knowledge, and/or misconceptions, onto their students.

Harel and Behr (1995) explored the rational number understanding of college students and pre-service elementary school teachers. They stated that, "it was found that many in this population possess the very same limiting concepts identified in the research with children. Further, similar results were found even with in-service elementary school teachers." The issues of both student and teacher misconceptions motivated this research with prospective high school teachers.

As stated, research has suggested students' difficulties applying concepts related to rational numbers. Post (1982) suggests this may be caused by school programs' tendency to emphasize procedural skills and the computational aspects of mathematics. Schools across the country appear to be more concerned with students' ability to produce the correct answer than whether they are correctly approaching the problem. That is, teachers are not properly developing in their students the foundational knowledge that is crucial for success. The National Assessment of Educational Progress (NAEP), conducted in 1972-73 and 1977-78, found that students appear to be learning mathematical skills at a rote manipulation level and do not understand the concepts underlying the computation (Post et al., 1982). Not only is this practice ineffective, but it may also prevent students from developing a deeper understanding of the material. It is interesting to note that most prospective teachers today were in elementary school shortly after these NAEP findings were reported and the work of Post et. Al (1982) was published.

This study explored the rational number comprehension of future secondary school mathematics teachers. As previously mentioned, numerous studies have researched rational number comprehension among future elementary school teachers. Since it is understood that future high school mathematics teachers with at least a Bachelors degree (or near completion) should be capable of demonstrating a high competency of the subject matter, this study explored whether similar misconceptions are present within this group.

Methods

Participants and Survey

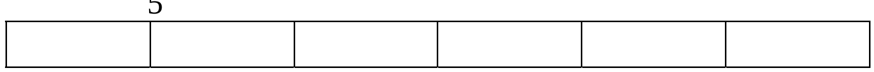
A survey (see Appendix A) based on some of the most common misconceptions presented in the literature was designed for this study. The goal of the survey was to explore connections between representations and common misconceptions.

The survey was administered to four undergraduates at a large, predominantly undergraduate university in the west. The participants were chosen by responding to an e-mail seeking volunteers from those students who planned to become junior high or high school mathematics teachers. At the time of the research Ashley was a senior, while Brad, Caitlin, and Dylan were juniors. All of the participants had completed the calculus track, as well as some form of an introductory course in linear algebra and differential equations. Furthermore, all of the participants have begun taking upper division math classes. Each participant completed a videotaped interview during which they completed the survey and responded to researchers' questions about their problem solving. The goal of the researchers' questions was to have participants make their problem solving processes clear and justify their work on the survey.

Coding

After developing and administering the survey the researchers categorized the problems based on the type of knowledge that could be utilized for solution. For initial coding it was determined if problem could potentially being solved using procedural knowledge. For this coding a procedure was defined as computation based on an algorithm. After the survey was given, this definition of procedure was further distinguished from a similar notion of the participant's past knowledge. This implied the participant simply recalled a rule of mathematics or made a statement to the effect of "because I just know". Once an agreement was made on these distinctions, each remaining problem, which could not be solved using procedural or past knowledge was categorized as requiring participants' conceptual knowledge. The researchers determined that conceptual knowledge was necessary to solve problems 3, 4, 17, 19, and 20, while the remaining problems could be solved using procedural or past knowledge. The questions asking for definitions were not categorized in this manner (problems 1 and 2) because the researchers believe definitions are not typically formed by relying on conceptual or procedural knowledge, but are fundamental building blocks that form the basis of a student's knowledge. Table 1 includes examples of this coding with explanations.

Table 1
Example of Problem Categorization

Solution Type	Problem and Explanation
Procedural (P)	#10. Let $a > b > c > d$ for the following questions. Insert the correct inequality and explain your reasoning: $\frac{a}{b}$ ___ $\frac{a}{c}$. It is possible to plug in numbers to discover the answer to this problem using a guess and check procedure.
Past Knowledge (PK)	#5. Insert the correct inequality: .5 ____ .6 It is possible to simply "just know" the answer without being able to provide an explanation.
Conceptual (C)	#3. Shade $\frac{2}{5}$ of the rectangle below:  There is no apparent procedure that can be used to shade the correct portion of the rectangle. The participant must have a conceptual understanding of part vs. whole to reach the right answer.

Based on the surveys and interviews each problem attempted by the participants was coded with regards to whether it was answered correctly, on which attempt it was answered correctly, and which type of knowledge was relied on during the solution attempt. The following is a key to the notation used in the coding of each problem:

- 'Y' indicates that the problem was answered correctly.
- 'N' indicates that the problem was not answered correctly.
- 'Ø' indicates that the participant did not answer the problem.
- 'C' indicates that the participant's attempted solution relied on conceptual knowledge.
- 'P' indicates that the participant's attempted solution relied on procedural knowledge.
- 'PK' indicates that the participant's solution relied on his or her past knowledge.

Note that it is possible for a participant to approach a problem by relying on more than one type of knowledge and when this occurred both are specified. Also note that problem 10 was scored as correct in the a>b>c>d>0 case since only one participant considered other cases.

Results

Individual

Table 2 shows the aggregate data from the survey including the misconception or representation of rational numbers addressed by each question (MMB/DMS stands for Multiplication Makes Bigger / Division Makes Smaller); whether the question was answered correctly and the type(s) of knowledge each participant relied on to solve the problem. If a problem was solved correctly on the second attempt it is noted in parentheses. The final four rows of the table include the percentage of questions answered correctly by each participant on the first attempt and the frequency of participants attempted solution strategies.

Table 2
Individual Results

Problem	Representation/ Misconception	Ashley	Brad	Caitlin	Dylan
1	Definition	Y	N	N	N
2	Definition	Y	Y	Y	N
3	Part-Whole	Y - C	Y - C	Y - P	Y - C
4	Part-Whole	Y - C	N - C	N - C	N - C
5 a)	Fraction v. Decimal	Y - PK	Y - PK	Y - C	Y - C
5 b)	Fraction v. Decimal	Y - C	Y - C	Y - C	Y - C
6	MMB/DMS	Y - P	Y - P	Y - C	Y - P
7	MMB/DMS	Y - P	Y - P	Y - C	Y - P
8	Multiplication	Y(2) - P	Y(2) - P	Y - P	Y - P
9	Multiplication	Y -P	Y - P	Y - P	Y - P
10 a)	Abstract Comparison	Y - C & P	Y - P	Y - C	Y - C & P
10 b)	Abstract Comparison	Y - P	Y - P	Y - C & P	Y - P
10 c)	Abstract Comparison	Y - P	Y - P	Y - C & P	Y - P
10 d)	Abstract Comparison	Y - P	Y - P	N - P	Y - P
11	Longer = Larger	Y - P	Y - P	Y - C	Y - C
12 a)	Density/Order	Y - P	Y - P	Y - C	Y - C
12 b)	Density/Order	Y - P	Y - P	Y - C	Y - C
13	Density/Order	Y - P	Y - P	Y - P	Y - P
14	Repeating Decimals	N - C	Y(2) - PK	Y - P & PK	N - C
15 a)	Ratio/Proportion	Y - P	Y - C	Y - C	Y - C
15 b)	Ratio/Proportion	Y - P	N - P	Y - P	Y - P
16	Ratio/Proportion	N - P	Y(2) - C & P	Y - C	Y - C
17	Function/Operator	Y - P	N - C	N - P	N - P
19	Function/Operator	Y - P	Y - P	Y - C	Y - P
20	Part-Whole	N - C	Y - C	Y - C	Y - C
21 a)	MMB/DMS	N - C & P	N - C & P	N - C	N - C
21 b)	MMB/DMS	N - C	N - P	Y - C	Ø
Total Score (First Attempt)		77.8%	66.7%	81.5%	74.1%
Procedural Solution Attempt		66.7%	63%	35.7%	48%
Conceptual Solution Attempt		29.6%	29.6%	60.7%	52%
Past Knowledge Solution Attempt		3.7%	7.4%	3.6%	0%

As stated previously the researchers determined that problems 3, 4, 17, 19 and 20 required conceptual knowledge for solution. Aggregately the participants provided correct solutions for 41.7% of these problems. However participants provided correct solutions for 83.8% of the problems that they could rely on procedural or past knowledge. Comparing these percentages and the percentages of attempted solution strategies raises the question as to whether these mathematics majors have come to rely heavily on procedural skills and past knowledge.

Definitions

The first question asked participants to define the set of rational numbers. Ashley was the only participant to get the definition correct, which is shown in Figure 1.

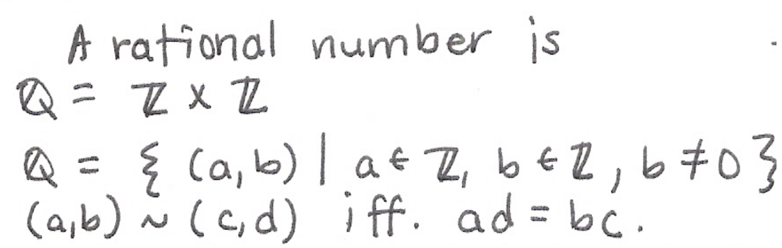


Figure 1. Ashley's Definition of the Rational Numbers

Two of the participants who did not correctly define the set of rational numbers omitted the necessary condition that the denominator is non-zero in their responses. However, Dylan gave the definition in Figure 2 which omits the fact that rational numbers are either expressed as repeating or terminating decimals.

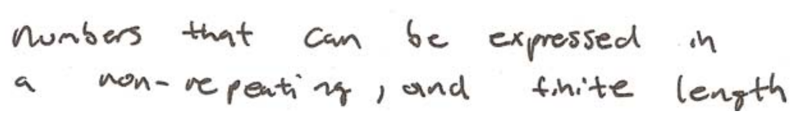


Figure 2. Dylan's Work from Question 1

The second question asked participants to describe the difference between rational and irrational numbers, which required participants to provide the definition of an irrational number. Three participants provided a correct definition similar to Brad's in Figure 3.

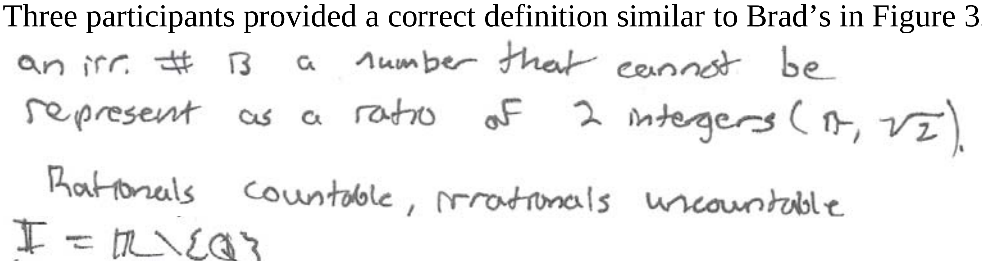


Figure 3. Brad's Work from Question 2

Dylan provided the only incorrect answer, which was based on an initial incorrect definition of rational numbers and is shown in Figure 4.

Figure 4. Dylan's Work from Question 2

Ideally educators could assume that late in their undergraduate careers mathematics majors who intend to be secondary teachers would have little problem defining rational and irrational numbers. Aggregately these prospective teachers were only able to produce correct definitions 50% of time.

Definitions vs. Repeating Decimals

There appeared to be a significant inconsistency with the participants' definitions of rational numbers and answers to some survey questions. For example, Dylan defined rational numbers as "numbers that can be expressed in a non-repeating, and finite length." When he later answered question 14, which asked if $0.\overline{4}$ can be written as a fraction, he used $0.\overline{3}$ to devise an answer. He reasoned that since $0.\overline{3}$ could be written as $\frac{1}{3}$, that $0.\overline{4}$ can also be written as a fraction, but was unable to produce the equivalent fraction. There was no indication that he realized that this reasoning contradicted his earlier definition of rational numbers. Similarly, Ashley correctly defined rational numbers yet later stated that $0.\overline{4}$ could not be written as a fraction since it was a repeating decimal.

The severity of this disconnect is amplified when it is pointed out that possibly the first definition of rational numbers children encounter describes them as numbers whose decimal form either repeats or terminates. Only later do they learn a more formal definition. One may expect that such concepts would be deeply ingrained in the minds of upper division math majors.

However these results may suggest otherwise, as three of the four participants began this question with the thought process that since the decimal was repeating, and non-terminating, it was not possible to express this number as a fraction. Dylan and Brad soon connected this number to their familiarity of $\frac{1}{3}$, which changed their opinion although they were unable to justify their reasoning. Only Caitlin recognized immediately that this repeating decimal could be written as a fraction.

Abstract Comparisons

It is indicated that participants had general difficulty with questions related to their understanding of part-whole relationships. Participants had trouble differentiating between the fraction as one entity and the fraction as a relationship between numerator and denominator. A participant who possesses a conceptual understanding of rational numbers should be able to quickly answer parts of question 10, which asked participants to compare the magnitudes of general rational numbers, as shown in Table 1. Part one addressed the fundamental part-whole relationship, comparing two rational numbers with the same numerator but different denominators. The third dealt with the relationship between reciprocals.

In Question 10 the participants all made a decisive shift from finding the answer conceptually to plugging in numbers by the time they reached the second part of the question. Only Caitlin considered multiple cases and she was the most persistent, as illustrated in Figure 5, but eventually gave in to using specific numerical examples. None of these upper division mathematics majors were discouraged from using a guess and check approach and provided answers based on specific cases.

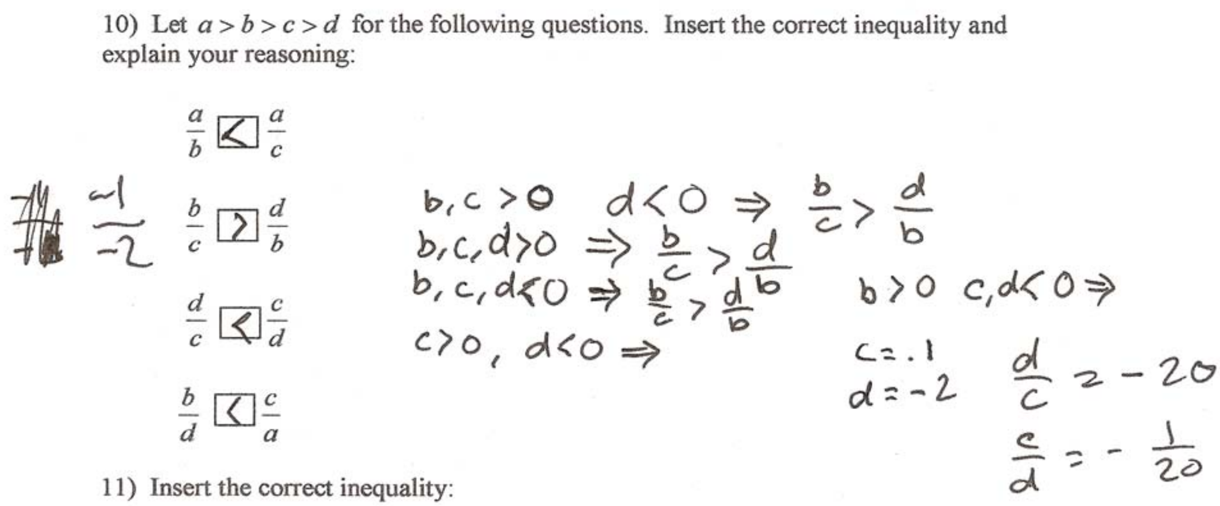


Figure 5. Caitlin's work on Problem 10

Question 19 also addressed the notion of a fraction as one entity versus a relationship between numerator and denominator. This question asked how long it takes Sam to walk 2 meters at $\frac{2}{3}\,{}^{m}\!/\!{}_{s}$. If one notices that the fraction is in fact $\frac{2m}{3s}$, the answer of 3 seconds can be produced immediately. Caitlin was the only participant to recognize this. The other participants relied on some mode of procedure, including writing equations and solving by canceling units. Dylan in particular set up an equation that read: $\frac{2}{3}x = 2$. He is apparently considering $\frac{2}{3}$ as a whole entity versus the relationship of 2 meters for every 3 seconds. This question is an example of the participants' tendency to favor using equations and procedures over relying on conceptual knowledge. While procedures are an important aspect of mathematics, there is a possibility that the conceptual understanding surrounding the procedures is missing among these prospective teachers.

Operators

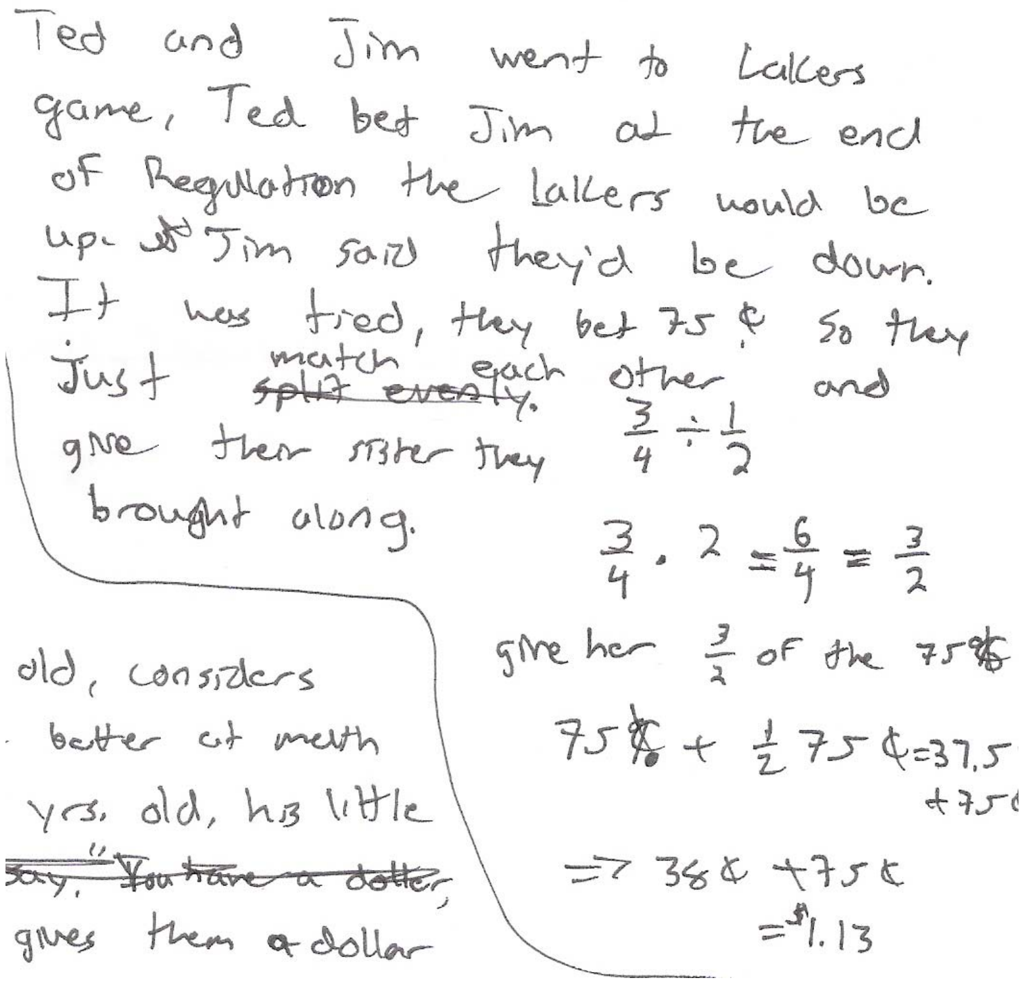
Question 17 also explored participants' conceptual understanding of rational numbers by asking if $\frac{5}{3}$ represents $\frac{1}{3}$ of a segment of length 5 or 5 parts that each has length $\frac{1}{3}$ of the whole. Participants possessing conceptual understanding should have recognized both of these as possibilities. However, Ashley was the only participant to do so. Brad, Caitlin, and Dylan, who relied on the wording of the question instead of conceptual knowledge, failed to connect the symbolic representation, $\frac{5}{3}$, to both of the aforementioned interpretations.

It is interesting to note that Brad and Dylan both understood that $\frac{5}{3}$ represents 5 segments of length $\frac{1}{3}$, but said that it did not represent $\frac{1}{3}$ a segment of length 5. That is, both viewed the question as $5\times\frac{1}{3}$ but not as $5\div 3$. Furthermore, only two of the participants drew a picture to accompany this problem, and only one of these participants did so without being prompted. Even still, none of the participants were able to correctly draw a picture modeling the two possible interpretations. Specifically, this disconnect appears to have hindered the attempts made by Caitlin and Dylan to visualize the question. This suggests that Brad, Caitlin, and Dylan lack some conceptual understanding of what it is that the symbolic form $\frac{a}{b}$ represents, which in turn may cause these participants to rely on procedural knowledge.

Ashley's response was classified as procedural as she was unable to demonstrate a truly conceptual understanding of the question. While she was able to recognize the key word "of" in the problem as representing multiplication, she was unable to move away from the procedural explanation that since $\frac{1}{3} \times 5 = \frac{5}{3}$ and $5 \times \frac{1}{3} = \frac{5}{3}$, the two possibilities must be equal. A conceptual answer would have involved the use of a diagram or verbal description of the two representations. Since none of the participants were able to correctly provide such a response, they did not demonstrate a conceptual understanding of a rational number as an operator.

Word Problems

Question 20 asked participants to create word problems for two specific division problems and proved to be difficult for the participants. None of the participants were able to write a word problem that modeled $\frac{3}{4} \div \frac{1}{2}$. Caitlin wrote, "if you run one mile, then how many times will you run $\frac{2}{3}$ of a mile" and correctly modeled $1 \div \frac{2}{3}$. This was the only correct problem posed by the participants for question 20. While rational number division is frequently encountered, the participants' struggle to create word problems suggests that the development of the conceptual idea of rational number division may have been bypassed in favor of procedural skills when this topic was learned. An example of the typical process for these participants can be seen through Brad's work in Figure 6.


Ted and Jim went to Lakers game, Ted bet Jim at the end of Regulation the lakers would be up. Jim said they'd be down. It was tied, they bet 75 ¢ So they Just match each other ~~split evenly.~~ and give their sister they brought along.

$\frac{3}{4} \div \frac{1}{2}$

$\frac{3}{4} \cdot 2 = \frac{6}{4} = \frac{3}{2}$

give her $\frac{3}{2}$ of the 75¢

75¢ + $\frac{1}{2}$ 75 ¢ = 37.5 + 75¢

=> 38¢ + 75¢

= $1.13

old, considers
better at math
yrs. old, his little
~~say, "You have a dollar,"~~
gives them a dollar

Figure 6. Brad's work on Problem 21

Although participants were very familiar with the invert and multiply technique they typically started by creating a word problem for $\frac{3}{4} \times \frac{1}{2}$. It was as if the participants did not even notice the division sign. This may be due to the heavy dependence on procedural knowledge and also because participants usually evaluate the division of rational numbers using multiplication.

After the participants wrote their initial word problem, they attempted to plug in the numbers to demonstrate that their answer worked. It was not until this point that participants recognized their errors and proceeded to edit their word problems in an attempt to illustrate

$\frac{3}{4} \div \frac{1}{2}$. Ashley and Brad simply replaced phrases such as “split” and “half” with “match” and “double”, respectively. This may demonstrate that the algorithm of inverting and multiplying has become so engrained that participants were unable to move away from it.

The inability of participants to correctly create a word problem for rational number division seems to imply that participants depend predominately on procedural knowledge, rather than conceptual knowledge. This is further supported by the fact that once participants realized their error they were still unable to modify their scenario to correctly model the division problem, despite subtle guidance. It could be argued that participants exhibited signs of holding the misconception referred to as “multiplication makes bigger and division makes smaller”, which might explain why they immediately saw the problem as multiplying by $\frac{1}{2}$, since this would result in an answer smaller than the original factors.

Discussion

As explored in the results, it appears that these participants are more adept at solving problems that allowed them to rely on procedural and past knowledge rather than problems that require the use of conceptual knowledge. While a participant may have defined rational numbers accurately, this did not always correlate to their overall conceptual understanding. In fact, the definition seemed to play no connective role in solving later questions in the survey. Participants also struggled to quickly recognize relationships between abstract rational numbers. They relied instead on the use of specific examples rather than making general conjectures based on part-whole and numerator vs. denominator relationships.

In addition, the participants had difficulty visualizing several of the questions, which was evidenced by their inability to create an appropriate diagram. This may be due to an absence of perceptual variability in lessons when participants were first exposed to rational numbers. Perceptual variability enhances learning by exposing children to a concept in a variety of physical contexts (Post et al., 1982). If participants did not receive instruction that allowed them several opportunities to make concrete connections with rational numbers through the use of physical models, it may account for participants' difficulties in visualizing questions.

Furthermore, the participants frequently failed to demonstrate comprehension of what it is that the symbolic notation of a rational number represents. This was especially evident in questions involving the form $\frac{a}{b}$, but was also present in questions utilizing decimal notation. Due to these shortcomings participants were repeatedly unable to move away from purely methodical explanations, revealing their strong confidence in their procedural knowledge. This demonstrates that misconceptions involving rational numbers may not only be rooted in their failure to produce correct answers, but also in their failure to correctly analyze certain questions Again, an absence of variability may lend an explanation for this tendency. Mathematical variability involves exposing the student to the new idea by incorporating a number of different conceptual perspectives, which for rational numbers include the following representations: part/whole relationships, decimal, quotient, operator, and ratio (Post et al., 1982). If students were not exposed to the concept in a variety of forms, and were not held accountable for manipulating rational numbers using all of these forms, then they will experience difficulty when they are not allowed to rely on their favored form. For example, if students have been able to convert rational numbers to part-whole representations whenever they are multiplying, but are now required to perform this operation in decimal notation they may not be able to do so, even though they may be perfectly capable of performing this task in the part-whole representation.

Participants tended to consistently favor the use of step-by-step procedures as opposed to reasoning during their attempted problem solving. When logic was required, most struggled to explain, depict, or solve the question correctly, as was shown by the overall scores. There may be a few different explanations for this. Perhaps there exists a notion that using an equation or a procedure appears more eloquent and insinuates a higher level of intelligence. This could be an affect brought on by the learning styles of our classrooms. Yet, one would hope that by the junior and senior years of a mathematics major that the underlying conceptual knowledge would be a given. Note that while approaching certain problems conceptually was not necessarily their initial choice, most still had trouble responding to the subtle conceptual guidance and probing done by the researchers. This further illustrates the seriousness of their inability to move away from procedural explanations.

Someone may also ask if such problems arise from a lack of exposure to these concepts or a lack of consistent use. One could argue, however, that when knowledge is accompanied with context and a depth of meaning, the learner is more likely to "own" this knowledge and retain it for a longer period of time. A major hypothesis of the Rational Number Project, which is a comprehensive program for research on rational number learning, is that the ability to translate between the several representations of rational numbers makes the ideas more meaningful to learners (Behr et al., 1981). If students did not receive this exposure, then they would have a more difficult time making meaningful connections to the material, which might account for their reliance on procedural attempts over conceptual explanations. A final possibility is that the participants simply became impatient with the problems that required more in depth reasoning and concluded that concrete examples were adequate. This might parallel the "instant results" mentality our current society has adopted. However, the number of hours that math majors dedicate to meticulously perfecting proofs would imply that this mentality has been forgone. Furthermore, one may counter argue that if sufficient conceptual knowledge is present, the process of determining the answer should not cause such a high level of frustration. This might in turn suggest that participants do not have as deep of a conceptual understanding as some might expect. No matter where the roots of these difficulties lie, the issue remains that these disparities do exist.

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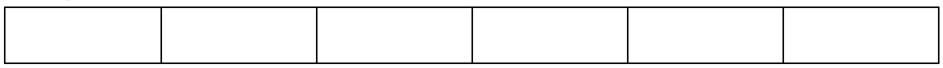
Appendix A – Survey

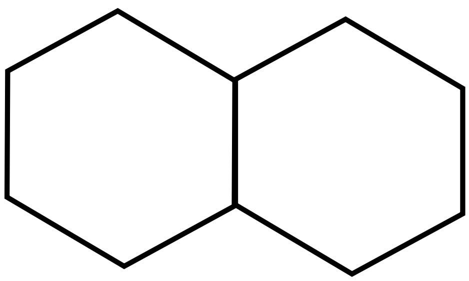
1) Define the rational numbers:

2) Describe the difference between rational and irrational numbers.

3) Shade $\frac{2}{5}$ of the rectangle below:



4) Using the diagram below, with two hexagons as the unit, shade $\frac{1}{6}$ of $\frac{5}{2}$.



5) Insert the correct inequality:

$0.5 \ \square \ 0.6$

$\frac{1}{5} \ \square \ \frac{1}{6}$

6) Explain whether or not it is possible for the product of two real numbers to be smaller than:
   a. One of the original factors.
   b. Both of the original factors.

7) Is it possible for $\frac{a}{b} = c$ where $c > a$. Why or why not?

8) $0.1 \times 0.1 = \square$

9) $\frac{1}{20} \times \frac{1}{20} = \square$

10) Let $a > b > c > d$ for the following questions. Insert the correct inequality and explain your reasoning:

$\frac{a}{b} \square \frac{a}{c}$ $\frac{d}{c} \square \frac{c}{d}$

$\frac{b}{c} \square \frac{d}{b}$ $\frac{b}{d} \square \frac{c}{a}$

11) Insert the correct inequality: .538 $\square$ .62

12) Write two numbers between:

a. 0.2 and 0.21

b. $\frac{1}{4}$ and $\frac{1}{5}$

13) Arrange in order from smallest to largest.

$\frac{9}{4}$, 25%, 0.3, $2\frac{1}{2}$, 0.295, $1.1\overline{9}$

14) Can $0.\overline{4}$ be written as a fraction? Why or why not?

15) Complete the following statements:

a) If 3 feet = 1 yard, then 7 feet = ? yard

b) If 32 ounces = 1 quart, then 6.7 quarts = ? ounces

16) In a lemonade punch, the ratio of lemonade to soda pop is 2:3. If there are 24 gallons of punch, how much lemonade is needed?

17) Is the operator $\frac{5}{3}$ equivalent to one third of a segment of length five or five segments of length one third? Explain.

18) Sam walks $\frac{2}{3}$ m/s. How long does it take him to walk 2 m?

19) Use a diagram to illustrate how many thirds are in 1½.

20) Create and solve a word problem for the following statements:

a. $\frac{3}{4} \div \frac{1}{2}$ b. $1 \div \frac{2}{3}$