RUME 2008 Conference Proceedings (On-line) Paper Submission

Preliminary Research Report (Jarvis, Lavicza, & Buteau)

Title

Computer Algebra Systems (CAS) in University Mathematics Instruction: A Preliminary Research Report Investigating CAS Technology Use and Sustainability

Abstract

The use of Computer Algebra Systems (CAS) is becoming increasingly important and widespread in mathematics research and teaching at the university level. Notwithstanding, there exists very little in the way of formalized support presently in place to assist: (i) university mathematics instructors who wish to move forward in the area of technology for teaching; and, (ii) university mathematics departments that wish to sustain the use of CAS-based software in their program courses over time . Furthermore, in contrast to the large body of research focusing on technology usage which exists at the secondary school level, there is a definite lack of parallel research at the post-secondary level. In this paper, we will highlight some of the findings of one of the author’s recent doctoral study on CAS use in university instruction; briefly describe the development of a technology-rich, Canadian university mathematics program; and, finally, provide a preliminary report of our ongoing, international research study in Canada/UK.

ICMI Study 17 (Hanoi, Vietnam)

It was in Hanoi, Vietnam in December 2006 that Dr. Seymour Papert addressed the international group of researchers attending the International Commission on Mathematical Instruction (ICMI) Study # 17 with these powerful and challenging opening remarks:

> Although the computer hardware and software options have been present for decades, we have still not seen a major shift in pedagogy within our education systems such as was widely predicted. . . . We need to dedicate perhaps 10% of our individual energy and working lives to the exploration of new ways of teaching—of reconceptualizing how it is that we teach and students learn mathematics at all levels. (Papert, 2006)

Dr. Papert is widely known for his research and innovations in the areas of Artificial Intelligence, the development of Logo for early childhood instruction, his work with Piaget in the 1960s around constructivism, his teaching and research at MIT, and his most recent initiative, the "$100-dollar laptop." The entire ICMI study conference, featuring working groups, keynote presentations, and computer lab sessions, focused on technology use within mathematics education at all levels. The ICMI Study 17 conference title, *Digital Technologies in Mathematics Education: Rethinking the Terrain*, bore direct reference to the very first ICMI Study held in 1985 which also dealt with the potential effectiveness of technology usage in mathematics education. Our research project is, at least in part, a shared response to Papert's 21st-century challenge to rethink the *status quo* mathematics pedagogy found in universities worldwide, and to facilitate, via an international research study, a positive move forward with regard to digital technologies in the undergraduate mathematics curriculum.

Lavicza Doctoral Study (2004-08)

For his doctoral thesis research Lavicza designed a two-phase international study that aimed at answering several questions. Firstly, *To what extent and manner are Computer Algebra Systems currently used in university mathematics departments?* At the school level, a number of large-scale studies have been conducted nationally and internationally to provide benchmarks for other studies and to document changes occurring in this area over time. Similarly, Lavicza desired to provide a partial overview of the current use of CAS in universities. Secondly, *What mathematical and pedagogic beliefs and conceptions do mathematicians hold with regard to CAS, and how do they envision the role of CAS in university-level mathematics education?* School level studies suggest that teachers' conceptions, beliefs, attitudes, and motivations (of mathematics, mathematics teaching, and technology) are important factors influencing the integration of technology (Ruthven & Hennessy, 2002). Thus, it was important to examine these factors at the university level as well. Thirdly, *To what extent do nationally-situated teaching traditions, frequently based on unarticulated assumptions, influence mathematicians' conceptions of, and motivation for, using CAS?* School level studies also indicate that social and cultural factors play an important role in integrating technology into mathematics teaching and learning (Hennessy et al., 2005). Therefore, the design of the study followed an international comparative approach to be able to account for cultural influences as well.

Lavicza's study design followed a two-phase mixed methods approach (Johnson & Onwuegbuzie, 2004). The qualitative phase of the study comprised interviews, class observations, and a review of curriculum materials of 22 mathematicians in Hungary (HU), the United Kingdom (UK), and the United States (US). Findings of this phase were incorporated into the development of an on-line questionnaire that was sent to 4,500 mathematicians in the participating countries. The questionnaire included sections dealing with mathematicians' personal and academic characteristics; mathematicians' views on the role of CAS in mathematical literacy, teaching and learning; the factors hindering CAS integration in teaching; the resources available for CAS-assisted teaching; the description of current CAS use; and, optional space for written responses. Overall, 1103 mathematicians responded (average 24.62% response rate) to the questionnaire, which constituted a surprisingly high response rate according to the web-survey literature. In addition to responses to closed questionnaire items, mathematicians wrote an approximate total of 150 pages for the optional open questions and sent approximately 600 e-mails, many of which included relevant comments. Furthermore, 297 mathematicians volunteered to participate in future technology-related studies.

The high response rate and the generally positive feedback suggest that mathematicians are interested in learning about technology applications in mathematics teaching and many of them are open to discuss educational issues. In addition, changes in higher education during the past decade such as the increased enrolment in universities, the lower student interest in STEM[1] subjects, and difficulties in school-level education resulted in a decline in mathematical preparedness of students entering universities. Furthermore, the emergence of new technologies available for teaching opened new perspectives and intensified demands for the changes in teaching practices. The observed weaknesses in students' mathematical preparedness and the availability of technology prompted numerous mathematicians to experiment with innovative teaching practices and to question their more traditional pedagogy. Moreover, in many cases the integration of technology into undergraduate teaching is seen as a way to revitalize teaching and to assist students in raising their level of mathematical understanding. Although university-level mathematics teaching is undergoing considerable changes and is in need of assistance, little attention has been paid to teaching issues at this level by the educational research community. In particular, little is known about the current extent of technology use and mathematicians' practices in university teaching.

[1] STEM = Science, Technology, Engineering, and Mathematics

Figure 1 shows that approximately 67% of participants indicated that they use CAS for their own *research* at least on an occasional basis.

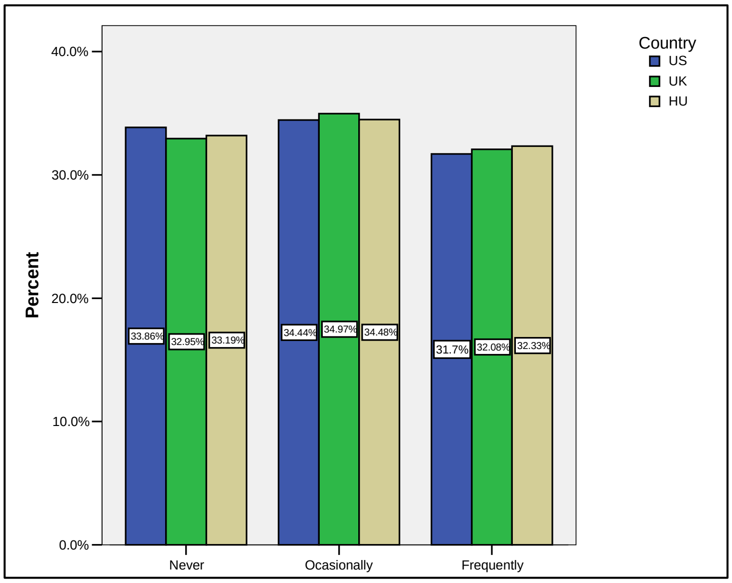

Figure 1: CAS use in mathematical research

This percentage is considerably high even when considering that mathematicians who have some kind of connection to CAS were likely among those responding to the questionnaire. After accounting for this possible bias it can be suggested that every third, or more likely, every second mathematician uses CAS in their own research. Thus, there are a large number of mathematicians who have acquired strong working knowledge of at least one mathematical software title and this knowledge can be readily utilized for CAS-assisted teaching. Proficiency in the use of a software package offers an advantage to mathematicians over teachers as they often don't require initial training for software before beginning to use it in their teaching.

Figure 2 shows the distribution of CAS use in university *teaching* among countries.

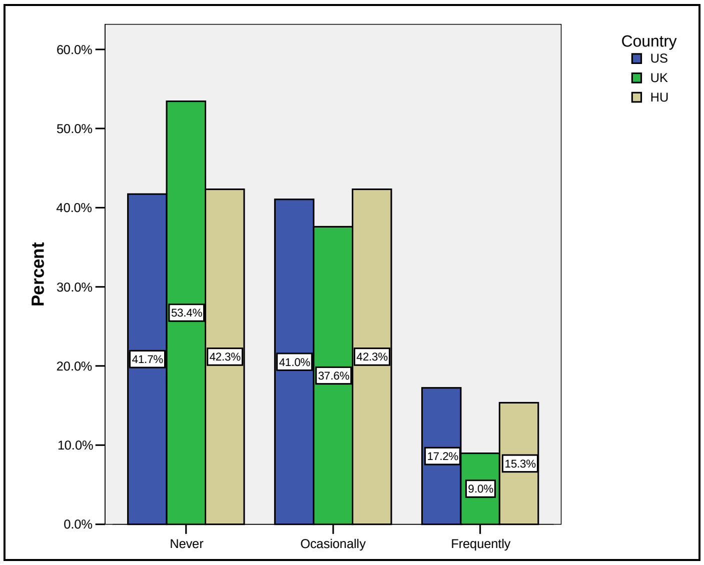

Figure 2: CAS use in mathematics teaching

Fifty-five percent of the participating mathematicians reported that they utilize CAS for their teaching at least on an occasional basis. Comparing this result to school level studies it can be implied that technology use at universities is substantially higher than those measured (5-10%) at the school level (Gonzales et al., 2004). This result also indicates that mathematicians have already accumulated an immense expertise and knowledge in technology-assisted teaching, although it is only sparsely documented. Unlike teachers, mathematicians are not required to follow pre-existing curricula. Therefore, this instructional freedom, together with considerable knowledge of mathematics and software, means that there may be impressive instructional innovations already existing at the university level which simply need to be documented.

Responses indicate that mathematicians view the role of technology, particularly CAS, positively in terms of mathematical literacy and in university curricula. They agree that proficiency in CAS use is beneficial for students' future studies and career, and they suggest that CAS will eventually become an integral part of the undergraduate mathematics curricula. This finding reinforces results in the first phase of the study suggesting that mathematicians are a more internationally mobile group than school teachers and that their thinking and beliefs are less culturally-based than those of teachers. It should also be mentioned that the study revealed that there is a high percentage of foreign-born mathematicians working in UK and US mathematics departments and numerous Hungarian mathematicians studied and worked outside of Hungary at some point during their career.

Based on his study findings, Lavicza asserts that it would be advisable for educational researchers to pay closer attention to mathematicians' technology-assisted teaching through documenting and researching these practices and innovations. This could significantly contribute to not only advancement in research and practice at universities, but also, by extension, at the school level. Further, he believes that educational researchers would be well-advised to focus their attention on university level research. In particular, mathematicians are becoming more open and attentive to educational research and so opportunities are becoming increasingly available for research at this level. This situation creates a good opportunity for mathematicians and mathematics educators to jointly engage in discussions about teaching issues at the university level and this will likely contribute to students' learning at all levels of education.

Brock *Mathematics Integrated with Computers & Applications Program* (MICA)

As far back as in the 1970's, Brock University located in St. Catharine's, Ontario, Canada, had begun using software such as *Maple* and *Minitab* in its large service enrolment Calculus and Applied Statistics courses (Auer et al., 1982). Beginning in the 1990's, Brock faculty teaching core mathematics courses were invited to attend open laboratory sessions of service courses wherein they could observe students using the software, with a particular focus on the types of questions being asked and the level of student engagement. By the mid-1990's, the number of courses integrating technology had grown significantly at Brock and these changes had become, by-in-large, accepted by the members of the Mathematics Department. By the end of that decade, Brock faculty, in an attempt to also address declining student enrolment, had moved towards integrating even more technologies into their program and this resulted in the creation of a core undergraduate mathematics program entitled, *Mathematics Integrated with Computers and Applications* (MICA) (Ben-El-Mechaiekh et al., 2007).

Two of MICA's main guiding principles are: to encourage creativity and intellectual independence among undergraduates, and to develop mathematical concepts hand-in-hand with computers and applications. Traditional courses, such as Calculus and Algebra were revised under the MICA principles, and now feature a technology component. In addition, three project-based courses called MICA I, II, III were introduced, all of which were designed to integrate the above-mentioned principles. Within these MICA courses, students are regularly exposed to complex situations that are not clearly defined; problems with multiple solutions; and programming environments requiring the precise input of mathematical details and for which abstraction, normally exclusively required for proofs, is necessary. For example, students learn to explore and simulate mathematics concepts by using an interactive interface which they design and implement themselves. Students also complete an original final project on a topic that they individually select themselves, thus increasing the level of engagement and interest in the curriculum. The final project may be an exploration, an application, or might also be a teaching project as commonly selected by prospective teachers. According to Buteau and Muller (2006), the latter a founding member of MICA and career-long advocate of technology-rich instruction, students have responded to the MICA courses at Brock in very positive ways:

> We have found that the approaches, activities, and experiences in the MICA courses are able to harness the students' motivations thereby empowering them to become their own mediators in the development of their mathematical knowledge and understanding.

> Students develop their understanding and knowledge of mathematical concepts using, progressively and recurrently during their undergraduate studies, diverse mathematics technologies, in many diverse ways.

The following figure (Ben-El-Mechaiekh et al., 2007) demonstrates the significant gains in student enrolment trends at Brock University, a result that could be at least in part attributable to the technology-based curriculum changes.

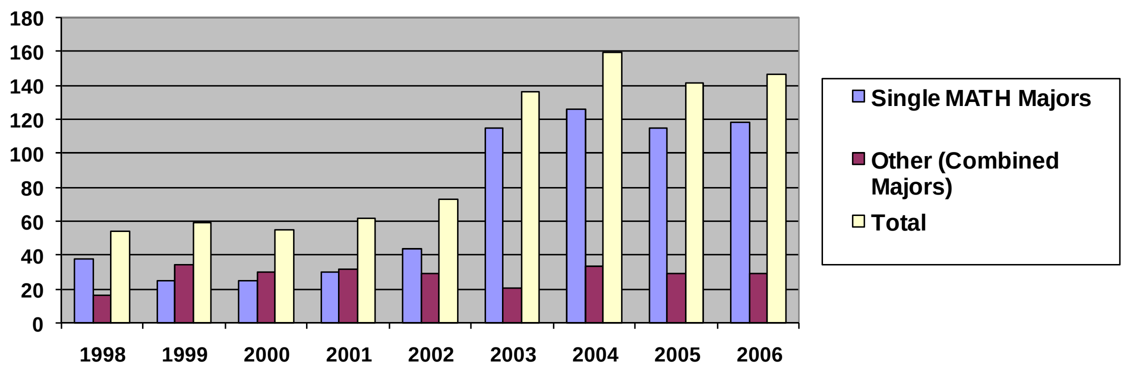

Figure 3: Enrolment trends at Brock University (1998-2006)

The MICA program at Brock University represents one successful approach to departmental, or systematic, change in undergraduate mathematics curriculum development. As recounted in their paper (Buteau & Muller, 2006), this thirty-year process has not transpired without its share of difficulties and obstacles alongside the successes and innovations. For this reason, Brock will serve as one example of an important case study for departmental reform within our research project agenda, the results of which will hopefully provide useful insights into the complex reality of bringing about such significant changes at the university level.

Research Project Overview (2008-10)

A growing number of international studies have shown that CAS-based instruction has the potential to positively affect the teaching and learning of mathematics at various levels of the education system, even though this has not been widely realized in schools and institutions

(Artigue, 2002; Bossé & Nandakumar, 2004; Kendal & Stacey, 2002; Lavicza, 2006; Meagher, 2001; Pierce & Stacey, 2004; Steinke et al., 2004). However, remarkably little research has been done relating to the support of mathematicians using CAS-based instruction at the university level. The overarching objective of our ongoing research project is to examine, via a mixed-methods approach, issues associated with the implementation of CAS-based technology in undergraduate mathematics courses. More specifically, we wish to document what is currently being done with regard to the use of CAS-based technology in universities, and then to make recommendations for supporting individual instructors and mathematics departments who are wanting to move towards a technology-rich approach to teaching mathematics.

Throughout 2007-08, with financial aid from the Social Sciences and Humanities Research Council of Canada (SSHRC)[2] we have been able to use the International Opportunities Fund Development Grant (IOFDG) award to finance the initial planning stages of our project. This process has involved planning meetings in both countries (Canada and England); the hosting of, and attendance at, two international computer algebra and interactive geometry systems conferences (Lavicza's CADGME[3] in Pécs, Hungary; Jarvis' CCADGME[4] in North Bay, Canada); and, consultations with international research experts in this field (i.e., technology in mathematics instruction at the post-secondary education level).

Building on Lavicza's doctoral work, and drawing upon Buteau's insights into the MICA program at Brock (i.e., one of the projected case studies for our work), we now move forward with an international research project. Our participation in RUME 2008 in San Diego has served to offer yet another important point of contact with mathematics education researchers, so as to refine our research questions, methods, and agenda. We look forward to completing and

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[2] We gratefully acknowledge SSHRC as our sponsor for this research project: http://www.sshrc-crsh.gc.ca/
[3] CADGME details/resources can be found at the following link: http://matserv.pmmf.hu/cadgme/index
[4] CCADGME details/resources can be found at the following link: http://www.nipissingu.ca/ccadgme/index.htm

reporting on this study in 2008-10, and thereby providing recommendations and relevant resources for the mathematics education and research communities.

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