Calculus students' perceptions of graphing calculators and play: Am I 'doing math'?

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Abstract
This paper reports on a mixed methods study designed to give voice to the student sin the ongoing debate of the use of graphing calculators in calculus. Close attention is given to the students' perceptions of their mathematical and affective experiences when problem solving in order to answer the following questions: 1)how do calculus students use their graphing calcualtors to engage in playful mathematical activities? and 2) how do calculus students perceive their use of the graphing calculator fits with their perceptions of what it means to 'do mathematics'? The data indicates that these students' actions are very much aligned with what mathematicians would define as mathematical problem solving (Polya, 1945; Schoenfeld, 1992). However, these actions do not coincide with the students perceptions of what it means to 'do math' in school.

Introduction

Throughout history many different tools have impacted the ways that we 'do mathematics'. The pencil and paper, the compass, the textbook, and the computer are all examples of technologies that have greatly changed both the mathematics that can be done and often times the ways that it is done (Kaput, 1995). In the last 30 years the most widely debated tool in mathematics education has been the graphing calculator (Ellington, 2003). Until recently most of the research on graphing calculator use in mathematics education has either been quantitative in nature, focusing on student achievement and attitude, or qualitative focusing on the teaching and learning of a particular mathematical topic (Burrill et al, 2003; Choi-Koh, 2003; Ellington, 2003; Forester & Mueller, 2002; Smith & Shotsberger, 1997, for example). In addition, there is a growing body of research on how students are adapting graphing calculator technology to their mathematical learning (Artigue, 2002; Drijvers, 2000; Guinn and Trouche, 1999). However, little, if any of the research addresses how or why students choose to use graphing calculators in private situations. In addition, none of the graphing calculator literature has focused on the students' voices and their perceptions of graphing calculator use. As we learn more about the effects that the adoption of graphing calculator technology seems to be having on the assessment, attitude, and learning of particular topics it becomes apparent that we need to know more about what students are actually doing with this tool in independent situations and how that use impacts their views of the mathematics that is done.

Few studies have looked specifically at how and why students use graphing calculators in particular ways, the exceptions are studies by Doerr and Zangor (2000) and Goos et al. (2003). These studies were both classroom based studies that aimed to understand the different roles that the graphing calculator takes on within a classroom community. Doerr and Zangor (2000) conducted an observational case study of two precalcululs classes and their teacher. Within this case study they considered how the classroom as a community shaped the ways in which technology was used. It was determined the within the context of the class the graphing calculator was used by the teacher and students' in five different modes: as a computational tool, transformational tool, data collection and analysis tool, visualizing tool, and checking tool. Goos et al. (2003) conducted a longitudinal study of 5 secondary classrooms and how graphing calculators and their peripheral devices were used as a tool that was integral to the learning environment in the context of these classrooms. They theorize that when technology like the graphing calculator is used in relation to teaching and learning interactions there are four roles it may take on which they characterize with the metaphors 'master', 'servant', 'partner', and 'extension of self'. Each of these studies has added to the knowledge of the complex role that the graphing calculator plays in the mathematics that his produced and shared in the context of the classroom. However, none of the previous work in this area has attended to the ways in which students use their graphing calculators in independent situations and their reasons for doing so. This paper reports on part of a larger study that aimed to attend to this gap in the research (McCulloch, 2007).

The larger study in which this is situated was a mixed methods study of 111 calculus students. It was found that when calculus students are working in independent situations that their reasons for using a graphing calculator fall into four different categories: to change the cognitive demand of a task, to check their work, to save time, and to engage in playful mathematical activities. An analysis of students' problem solving methods revealed that some of the most powerful activities that the students engaged in were those that involved using the graphing calculator as a tool to play with their mathematical ideas. These actions were defined as 'playful mathematical activities' and were often described by the student as 'playing around'. It is these types of actions and the students' perceptions of them that are the focus of this paper. Specifically, this paper addresses the following research questions:

- How and why do calculus students use their graphing calculators to engage playful mathematical activities?
- How do calculus students perceive their use of the graphing calculator fits with their perceptions of what it means to 'do mathematics' in school?

Theoretical Perspective

The framework for this study draws on research in mathematics education and cognitive science. From a mathematics education perspective I am concentrating on how students use the graphing calculator and why. It ends up that for these students the graphing calculator often provides a means for changing the cognitive demand of a problem and in turn allows for more playful mathematical activity, something that has not been explored in the graphing calculator literature (McCulloch, 2007). To comprehend the depth of the mathematics that the students were engaged in I drew on literature on problem solving (Polya, 1945; Schoenfeld, 1992, for example). In order to better understand the importance of being able to engage in playful mathematical activities for these students it was essential to turn to the research on play in both cognitive science and mathematics education (Davis, 1996; Dewey, 1916; Holten et al., 2001; Piaget, 1962; Steffe & Wiegel, 1994). However, first and foremost, in order to better understand the students' full mathematical experience it was necessary to draw on research on affect, specifically *local* affect (Goldin, 2000).

Unlike describing mathematical experiences, the idea of looking at affect is relatively new and thus warrants a bit of clarification. With respect to affect, I focus specifically on the notion of *local* affect. Local affect is defined as "the rapidly changing (and possibly very subtle) states of feeling that occur during problem solving – emotional states, with all their nuances" (Goldin, 2000, p. 210). Pilot studies indicated that students sometimes have very strong feelings not only about the mathematics that they are engaging with, but often about graphing calculator use (McCulloch, 2005). These feelings might be related to whether or not a graphing calculator is available or even to a solution that they have produced using the calculator. Either way, these feelings are sometimes intense and seem to impact their mathematical experiences.

Goldin (2000) has pointed out that students use emotions to provide useful information, to facilitate monitoring and to evoke heuristic processes. He suggests that affect is not inessential, but critical to the structure of competencies that account for success or failure in problem solving. An example of an affective pathway follows:

> In an (idealized) model, the initial feelings are of curiosity. If the problem has significant depth for the solver, a sense of puzzlement will follow, as it proves impossible to satisfy the curiosity quickly. Puzzlement does not in itself have unpleasant overtones – but bewilderment, the next state in the sequence, may. The latter can include disorientation, a sense of having "lost the thread of the argument" of being "at sea" in the problem…If independent problem solving

> continues, a lack of perceived progress may result in frustration, where the negative affect becomes more powerful and more intrusive. This is associated with the occurrence of an impasse. However, there is still the possibility that a new approach will move the solver back to the sequence of predominately positive affect. Encouragement can be followed by pleasure as the problem begins to yield, by elation as major insights occur, and by satisfaction with the sense of a problem well solved and with learning that has occurred (p. 211).

This idealized model illustrates how local affect might influence the heuristics employed by a problem solver. In the context of this study it is important to consider how the availability of a tool like the graphing calculator might further influence an affective pathway like the one described above. For example, if a student is facing feelings of bewilderment or disorientation it is possible that the introduction of a useful tool might invoke feelings that are of a more positive sequence. When considering the role of a tool like the graphing calculator when students engage in playful mathematical activities the only way that the full experience can be understood is by attending to the affective dimensions as well.

Play in Mathematics

A lot has been published about the role of play in learning over the last 100 years (Holton et al., 2001). Dewey (1916) wrote about the natural engagement in play when people, of any age, come into contact with new materials. He stated that at this first stage of contact the student must engage in trial and error type activities in order to get to know the material. Similarly, Piaget (1962) suggested that children expand their understanding of the world and themselves by engaging in play. According to Piaget, children collect bits of information about an object as they interact with that object. This information is then assimilated into their already existing knowledge to expand the child's knowledge of that particular object. Though Piaget wrote extensively on the importance of play in the cognitive development of young children, according to him play was not as important as children mature. Though some of histories greatest educational theorists have suggested that play is an important aspect of cognitive development, they did not consider its role in learning and doing mathematics.

More recently researchers such as Steffe and Wiegel (1994), Davis (1996) and Holton et al. (2001) took the idea of play a bit further and considered its value with respect to mathematics. In discussing the role of play in doing mathematics Davis (1996) says:

> Put simply, play is not so much an activity as it is an acceptance of uncertainty and a willingness to move. Play is thus the antithesis of the modern ideals of certainty, predictability, and linear progress. But it is not an abandonment of our quest for structure, order, pattern, and comprehensibility. Quite the opposite, these are the ends of play.But these ends are revealed only in the playing, for play is not simply random activity. Rather, by opening the door to the as yet inexperienced, to the possible, play reveals what is not yet known as it simultaneously offers space to support learning (p. 222).

Like Dewey and Piaget, Davis suggests that play is a way of making order in one's world, in this case one's mathematical world. "The acceptance of uncertainty and a willingness to move" is an action that has both affective and mathematical implications. The feelings that accompany uncertainty are often powerful feelings, possibly resulting in frustration and defeat or possibly in curiosity. A student that is frustrated and feeling defeated will likely stop working on the mathematical task at hand. In contrast, a student that is curious might make a move to explore the task. The data from this study indicate that the presence of a tool like the graphing calculator might possibly make the difference between accepting defeat and the willingness to make a move.

Steffe and Wiegel (1994) have also discussed the importance of mathematical play. They classify mathematical play as "independent mathematical activity with a playful orientation (p. 131)". Independent mathematical activities are those that are initiated by the students themselves. They operate in spontaneous ways that are not suggested by others. The term 'playful orientation' is taken from Piaget's notion of play, meaning the mathematical activities that the students engage in are done so for pleasure. This notion of play being necessarily pleasurable is different from the mathematical play described by Davis. Both suggest that through play students make sense of their world, even their mathematical world, but for Steffe and Wiegel students engage in this type of play because it is pleasurable while for Davis play is not necessarily a pleasurable activity, but one that can occur in the context of frustration and confusion.

Though Steffe and Wiegel and Davis have provided detailed discussions of the importance of play in mathematics, neither formally defines the mathematical activities involved. Holton et al. (2001) wrote on the importance of play in mathematics and in doing so put forth a formal definition for mathematical play. For Holton et al. formal mathematical play is a bit more complex than accepting uncertainty or even engaging in mathematical activity because it is fun. They define mathematical play in the following way:

> By mathematical play we mean that part of the process used to solve mathematical problems, which involves both experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion. Mathematical play involves pushing the limits of the situation and following thoughts and ideas where ever they may lead. Hence there are no obvious short-term goals for mathematical play; it is designed to allow complete freedom on the part of the solver to wander over the mathematical landscape. However, there is a long-term goal and that is the solution of the problem at hand (p. 403).

This formal definition differs from the previous discussion in that there is no mention of the affective dimension that may be involved in this type of activity. The feelings that are associated with the engagement in mathematics and might influence one's mathematical decisions are not taken into account. Whether those are feelings associated with the fun of play as described by Steffe and Wiegel or feelings like frustration or confusion as suggested by Davis, it is likely that the affective dimension is what needs to be attended to in order to identify the motivation for engaging in mathematical play.

Particular to this study, the focus will be on the students' perceptions of mathematical play and the role of the graphing calculator when they engage in such activities. The following sections will include the design of the study, examples of mathematical play within the context of graphing calculator use, students perceptions of what mathematical play is and why is important to them, and a discussion of students perspectives of how mathematical play with graphing calculators conflicts with their beliefs about what it means to 'do math', specifically school math.

Methods

This study was designed to capture, in as much detail as possible, the powerful ways in which calculus students use the graphing calculator and their perceptions of the 'pay-offs' that they associate with its use. It was essential to pay close attention to both the mathematical and affective 'pay-offs' to construct a complete narrative of the students experiences and perceptions. The data described here comes from both survey data (n = 111) and six in-depth case studies chosen from the survey participants. The surveys were made up of both likert scale and open ended items designed to attend to students' modes of use, reasons for use, and perceptions whether or not they are helped or hindered by use of the graphing calculator in private problem solving situations. The case study students participated in an additional back ground interview, task-based interview, and a stimulated response reflection interview. During the task-based interviews the students were given four non-routine tasks to complete with the graphing calculator of their choice available to use if they chose to do so. In these interviews both the students written work and the graphing calculator screen were video taped. Within three days after the task-based interview each student participated in a stimulated response reflection interview. In this interview they were shown side-by-side video of their work, both written and graphing calculator, in real time and were asked to talk about their actions, their decisions, and their feelings as they worked through the tasks. The reflection interviews were also video taped. All data was then transcribed and shared with the students for member checks.

The task-based interview and the reflection interview were analyzed using both a deductive and an inductive coding scheme. The purpose of the initial coding was to get a feel for the data, to identify how the students used their calculators on these particular tasks, and what triggered them to do so. The codes for calculator use as a tool were adapted from a study on graphing calculator use in the context of the classroom (Doerr & Zangor, 2001). The codes for triggers were developed during pilot studies (McCulloch, 2005). Once deductive coding was complete the data was reassembled and reevaluated to look for emerging codes relating to graphing calculator use and the problem solving experience. Throughout the analysis the results of the cases and the survey data were constantly compared.

Findings and Implications

Mathematical play in the context of graphing calculator use: An example

An excellent example of mathematical play in the context of graphing calculator use comes from Rudy and his work on the task: For what values of x is $-2 < |k - x| < 5$. Rudy did not understand the task initially. His long-term goal was to find a solution for the problem, but his short-term goal was not obvious. He used the graphing calculator to explore the mathematics and had very creative ideas about what might help him to do so. Rudy's work on the task appears below.

Rudy read this task for quite a while. He asked, "Can k be anything?" To which the researcher replied it could be any real number. Rudy then wrote down *7 – x =* and picked up his calculator. He entered $y1 = 7 - x$, graphed it, and quickly went to the table. He scrolled up and down the table of values between $x = 2$ and $x = 9$ and paused for a moment. Next he returned to the $Y =$ screen and changed the function from $y1 = 7 - x$ to $y1 = 6 - x$ and went back to the table. On the table he scrolled between $x = 1$ and $x = 8$. Then he wrote on his paper, "Depending on *k* there are 6 numbers that make *x* greater than -2 and less than 5."

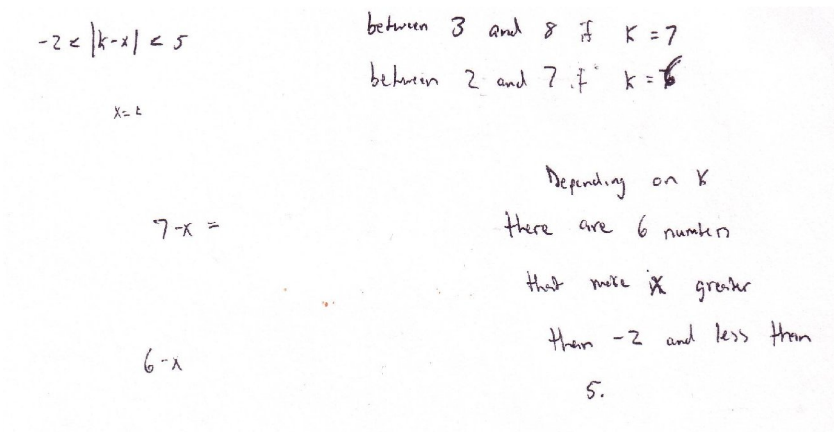

Figure 1: Rudy's written work

Rudy was unsure where to begin on the problem so he picked a simpler problem to think about. With a simpler problem in mind he then used his graphing calculator to explore what solutions might work. He noticed that there were six integral solutions when he chose *k* to be 7. When the same was true for $k = 6$ he realized he had found a pattern. The interviewer asked Rudy about his work on this problem. The conversation follows:

I: So on this particular problem, let's see what you did here…you went to the calculator a few times…you decided to use 7…

Rudy: I went to the table.

I: Ok, and what were you looking for on the table?

Rudy: On the table here I was looking for the numbers that were between, that was less than five and bigger than negative 2. I saw that there were six numbers, three, four, five, six, seven, and eight. And it's

	the same for, there are six numbers…depending on the k, that's what I noticed.
I:	What made you decide to try this on the calculator?
Rudy:	At first I didn't understand it. I was trying to figure out what they meant. I was just playing around.

Though he didn't meet the long-term goal of solving the problem, he did take an interesting stroll over the landscape. He chose a simpler problem to work on and used the table mode of the graphing calculator to test some numbers while trying to build an understanding of the role of the inequalities and the absolute value in the task. Rudy's short-term goal was not well defined. When he was exploring the table it was not in the hope of solving the task, but in hope of better understanding the task. When I asked him if he could have solved this problem without his graphing calculator he said, "Well, maybe if I had understood it from the beginning." However, it was through play on the graphing calculator that he began to build an understanding of the inequality in the task and thus the task itself.

It is likely that Rudy would not have attempted this problem at all if he did not have a graphing calculator available. The availability of the graphing calculator changed the situation from one in which likely no mathematics would have taken place to one in which somewhat sophisticated mathematics was engaged in. Most importantly, was that Rudy's self described "playing around" with the graphing calculator provided him with an easily accessible way to engage in the mathematics when he was facing uncertainty.

Students' perceptions of 'playing around'

The graphing calculator is a relatively small piece of technology. It is similar in size to many hand-held gaming systems. Students often report that they have stored games on their graphing calculators and use them to play when they are bored in class. With that in mind, it is probably not surprising that students refer to the graphing calculator as a toy. However, a closer look at what the students were often referring to when they spoke about playing around on their graphing calculator revealed that they were not referring to playing programmed games, but actually engaging in playful mathematical activities.

All of the students in the study referred to engaging in play when using their graphing calculators at least once. For these students play in mathematics is what you do when: (1) you don't know what to do; or (2) you are curious about something. Rudy's work on the absolute value inequality task above is an excellent example of a student engaging in what he called play because he didn't know what else to do. Like Rudy, Melissa explained that having her graphing calculator available when she didn't know what to do was important to her. When she spoke about how she solved a particular task she pointed out that she often plays using her graphing calculator.

I: What made you go to the calculator on this one?

Melissa: I didn't know what to do.

I: Do you do that a lot?

Melissa: Yeah, you just play around with the calculator.

I: Do you find playing around to be a helpful thing?

Melissa: Yeah.

I: How often do you do that?

Melissa: When I'm stumped. (laugh)

I: Any time you're stumped?

Melissa: Yeah.

In contrast to Rudy and Melissa's examples above, is Enoch's response to the task: Give an example of a function for which $|f(x)| = f(|x|)$. Enoch came up with two functions that would make the statement true ($f(x) = x^2$ and $f(x) = x^3$) and checked them by graphing them by graphing both $|f(x)|$ and $f(|x|)$ on the same graphing calculator screen and making sure that they produced the same graph. Once he was sure that he had found two correct solutions he started to use his graphing calculator to try other, more complicated functions. When he reflected on his work on this task the following day he said: "I was thinking, like, ok, where do I start…first I was thinking about the simplest function to do. After that I started thinking that I didn't want to do it the same way everyone else did…I wanted to try to do it differently, so started playin' around." Enoch was curious about other solutions so he engaged in some playful mathematical activity to assuage his curiosity.

Rudy, Melissa and Enoch all referred to their graphing calculators as being powerful ways to explore situations in which they were either uncertain or curious. They even used the language of "play" to describe their behaviors. They all suggested that having the graphing calculator makes exploring using a graph or a table an easy thing to do. It is possible that they would be less likely to engage in playful mathematical activities if they did not have a graphing calculator available to them to create the graphs and tables that they deem so helpful.

The role of the graphing calculator in mathematical play

Students often use toy metaphors to describe their graphing calculators. One student in the study even went as far as calling it a Game Boy[1]. The use of a toy metaphor for this tool might suggest that the ways in which students interact with this tool when they are engaging in playful mathematical activities is very simplistic, when in fact they are quite complex. A close

[1] A Game Boy is a hand-held gaming system produced by Nintendo.

look at the ways in which students interact with the graphing calculator in playful situations reveals that they do so in two different ways. They either interact with the graphing calculator as if it is part of the game they are playing or they interact with it as if it is a playmate.

For students that interact with the graphing calculator as if it is a playmate the graphing calculator is seen as an ‘other’, sometimes even a more experienced other. In this case the student has anthromorphized the technology, meaning it is viewed almost as having human characteristics (Geiger, 2007). This is most evident when the student reflects on the work and gives credit for any mathematical progress to the graphing calculator. Enoch consistently used his graphing calculator in this way. In his first interview when he was asked to talk about how he typically uses his graphing calculator he said, “It’s like a little toy. You share your experience with it, experiment with it and things like that.” This type of graphing calculator use is consistent with what Goos et al (2003) describe as ‘technology as a partner’.

Not all students view their graphing calculator as a partner in their play. Instead some view the calculator as part of the game, part of their repertoire of tools to use that they are using to engage in the play. Listening closely to the reflections on their work, students with this view of the graphing calculator recognize that decisions that they make about when and how to use the calculator is part of the problem solving process. Unlike students who view the graphing calculator as a playmate, these students accept credit for all mathematical progress in the problem solving process. The student and calculator are a single identity working together in playful activities. This type of graphing calculator use is consistent with what Goos et al (2003) describe as ‘technology as extension of self’.

Discussion

Studies on graphing calculator use have neglected to take into consideration students' voices. When we listen to the students voices we hear that they value being able to use the graphing calculator to engage in playful mathematical activities. In this study they have shared their perspectives of what it means to 'play around', why they value being able to so, and the role of their graphing calculators in such activities. The following section will compare their perceptions of play and its place in school math to the literature.

Rudy described his work on the absolute value inequality task as 'playing around'. However, comparing his work to the definition of mathematical play provided by Holton et al. reveals that Rudy's play is not consistent with their definition. Rudy did "experiment" and he was "creative" in his generation of ideas. It could be argued that he did "follow any ideas to some sort of conclusion". However, one of the requirements that Holton et al. puts forth for an activity to be considered mathematical play is that "there are no short-term goals". Rudy did have a short-term goal; it was to build an understanding of the task itself. Furthermore, I believe that there are always short-term goals when one engages in any type of playful mathematical activity. Whether it is to build an understanding of a task, to try a special case, or even to test an idea to see if it works in another situation, there is always some sort of short-term goal. I think the difference is that the short-term goal does not necessarily have to be directly in line with the long-term goal of finding a solution to the problem at hand.

Steffe and Weigel described mathematical play as independent mathematical activity that has a distinctive playful orientation. Rudy was clear in his reflection that he was frustrated when he was working on this task and his engagement in what he called play was to help him work through his frustration, it certainly did not have a playful orientation. He did not engage in the playful activity because it was fun, he did it to move himself from what could have been a very negative affective sequence into a more productive one. In contrast, if we look carefully at Enoch's work on the task in which he was asked to find a function for which $|f(x)| = f(|x|)$ it is much more in line with Steffe and Weigel's description. He had already found a solution to the problem when he started to play around and look for further solutions. He said he did this because he was curious. The smile on his face as he reflected on the situation suggests that this activity did take on a playful orientation.

Holton et al. have provided a definition for mathematical play that is very helpful for making explicit the actions that should be considered mathematical play, however it is devoid of any reference to the role of local affect. Furthermore, it is limiting in that it suggests that to be engaged in play one must not have any predetermined short-term goals. Steffe and Weigel have provided the necessary affective component to the definition, but in suggesting that it must be carried out purely for pleasure the definition does not consider the full range of the possible affective sequences that might take place before one experiences such pleasure.

Listening to the students perceptions of what constitutes play and what they gain from such experiences might prove helpful in refining the existing definitions for mathematical play. Based on the work of Steffe and Weigel and the students that participated in this study, I would like to suggest a modification to the Holton et al. definition for mathematical play. This modified definition capitalizes on the explicit description of mathematical activity that Holton et al. provides while also attending to the role of local affect. The modified definition follows:

> Mathematical play is the process used to solve mathematical problems, which involves experimentation and creativity to generate ideas, and using the formal rules of mathematics to follow any ideas to some sort of a conclusion. Mathematical play involves pushing the limits of the situation and following thoughts and ideas where ever they may lead. Mathematical play is designed to allow complete freedom on the part of the solver to wander over the mathematical landscape. Mathematical play is motivated by one's desire to move along a productive affective pathway and one's hope that this pathway will lead to experiencing pleasure.

It is my hope that this modified definition will maintain the essence of what Holton et al. believed mathematical play to be, while making it more easily identifiable due to the fact that it is not limited to situations in which there are no obvious short-term goals. Finally, and most importantly, it makes explicit the importance that affect plays in the problem solving process.

Analysis of the ways in which calculus students used their graphing calculators when they were engaged in mathematical play revealed that they used them in two different ways, as a playmate and as part of the game. These two roles were consistent with the metaphors defined by Goos et al. (2003) for the ways in which technological tools mediate learning, as a 'partner' and as an 'extension of self' respectively. The use of the graphing calculator as a playmate in mathematical play is particularly interesting. For example, Enoch pointed out that he thinks of his graphing calculator as someone that he shares his experiences with. The notion of the graphing calculator, or any other technological tool, as a partner changes the way we have to think about problem solving in independent situations. If a student uses a technological tool as a partner in an independent situation, should the situation still be considered independent?

Geiger (2007) has extended the work done in Goos et al. (2003) concerning the metaphor of technology as a partner. He has suggested that technological tools can play the role of an "almost peer with expertise that can be drawn on in the same way as other members of a community" (p. 247). In this framework, independent situations like those described by students using their graphing calculators as playmates would no longer be considered independent situations. The ways in which students in this study that viewed their graphing calculator as a partner described their use was consistent with Geiger's notion of technology as a 'quasi-peer'. Geiger has suggested that if technology is viewed as a 'quasi-peer' then Vygotsky's notion of the Zone of Proximal Development (ZPD) can be extended to include technology an a member of the learning community. The notion of technology as a member of the learning community is a very different and powerful idea. Given the results of this study I believe that it is one that warrants further research.

Even though the students use graphing calculators for things that we might find natural, they are doing so in a way that hides from them the very fact that they are doing mathematics. There were many instances throughout the data collection process that I witnessed students using their graphing calculators to engage in mathematical play and eventually successfully solve a problem. However, when they spoke about their calculator use they did not recognize their work as 'doing mathematics'. As a matter of fact, some explicitly stated that they had avoided doing any mathematics. For example, when reflecting on their use of the graphing calculator to engage in mathematical play they said:

> "I got this answer from the calculator. I couldn't, like, get it on my own. I don't know."
>
> "I s'pose I used the calculator as a scapegoat. I mean I used it like, so I could, basically I got the answer from the calculator. I couldn't get it on my own…it's a bad think if I used it like a scapegoat."
>
> "What I did here [points to the calculator] doesn't count."

However, about those same problems they also said:

> "I wouldn't have even tried if I didn't have my calculator."
>
> "If I didn't have my calculator I would have stopped there…where I got stuck."

The students did not seem to recognize their play with the graphing calculator as 'doing mathematics' when in truth actions like these are actually more similar to the work that mathematicians do than the rote repetition of algorithms on paper that they seem to associate with 'doing mathematics' in school (Polya, 1945; Schoenfeld, 1992). This is problematic. If students perceive the very actions that allow them to take risks and make moves when they don't know what else to do or are curious about something are not valued as ‘doing math’, then it is unlikely that they will be interested in continuing their mathematical studies. When in fact, it is the students that are willing to take risks and are taking action when they are curious that we need to continue because that is what moves scientific fields forward.

Conclusion

The results of this study suggest that calculus students do use their graphing calculators in very powerful ways when they are engaged in solving non-routine problems in private situations. Specifically, if we listen to the students’ voices we hear that they value using the graphing calculator in ways very similar to the ways that mathematicians use tools of all types, to engage in playful mathematical activities. If we listen more closely we also hear that students are conflicted by their use of the graphing calculator. This conflict is a result of their perceptions of the actions on the graphing calculator differing greatly from their perceptions of what it means to ‘do math’ in school. Though the data indicates that their actions are actually very much aligned with what mathematicians would define as mathematical problem solving (Polya, 1945; Schoenfeld, 1992), they do not perceive that these actions are consistent with what it means to ‘do math’.

In the ongoing conversation about if and how tools such as the graphing calculator should be incorporated into mathematics courses we need to listen to these students’ voices and consider how we can make explicit the deep understanding of the underlying mathematics that is often involved in using the tool in sophisticated ways. If students are aware that when they use the tool to engage in playful mathematical activities that they are actually ‘doing mathematics’ they might be less hesitant to engage in such activities.

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