Analysis of Stance in Two Interactive Mathematics Lessons

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Abstract

We report an analysis of the language used by two instructors teaching two undergraduate mathematics classes that exhibited high student participation yet differed in the level of dialogical engagement. The linguistic analysis of the instructors' utterances offers an alternative lens to study the level of engagement of instructors and students in classroom interaction in undergraduate mathematics classes that complement studies that focus on more holistic aspects of the classroom interaction and their role of language on students' learning. We discuss implications for research and for faculty development.

Calls for increasing student participation in mathematics classroom from K-12 settings (e.g., National Council of Teachers of Mathematics [NCTM], 2000) have been promoted also at the tertiary level (Blair, 2006), specifically for moving from a 'teacher centered' paradigm of instruction towards a 'student centered' one. In a setting in which lecturing seems to be the dominant mode of interaction between students and instructors (Lutzer *et al.*, 2007) what mathematics instructors can do to increase participation seems a difficult task to accomplish. Moreover, as it is has been shown in the K-12 literature, it is not clear that having a high level of student participation is in itself enough to warrant student engagement with the material (Cazden, 1986; Hiebert & Wearne, 1993; Voigt, 1985). In this paper we argue that we need to refine the lenses by which we analyze classroom interaction to attend to how different instructors use language to engage or disengage students from the dialog. This type of analysis might prove useful in also devising ways to assist college instructors in changing the dynamics of classroom activity in mathematics. The main question that we sought to answer with this study was whether

the application of a linguistic framework (to be explained below) would be useful in characterizing and differentiating the language that faculty use to engage students in the classroom dialog.

The question was a natural one that emerged after noticing that in a corpus of 14 classes delivered by 8 faculty in 6 different institutions[1], student participation ranged from 3 interventions to 257 in classes that lasted in average an hour. What was more interesting is that the graduate and pre-college classes tended to have higher student participation (from 84 to 153) than the undergraduate classes, except in the case of two instructors, Instructor A, who taught a class in which the students participated 180 times, and Instructor B who taught four classes in which students participated an average of 232 times. Furthermore, although these two instructors had what in comparison to the other lesson looked very interactive classes, there were subtle differences in how the dialogue was constructed in each that we could not necessarily attribute to the different content in the task. We sought then to investigate in which way the language used by these two instructors was different. Thus we started this exploratory study to determine if the application of a linguistic framework (to be explained in the Methods section) would be useful in distinguishing between the languages that the two instructors used to engage students in the dialog in their math classes.

Given the calls for increasing student interaction in the undergraduate mathematics classroom, understanding how the language of the interaction works in this setting seems crucial to assist instructors who are interested in changing their interaction patterns in their teaching. We contend that engaging students in the dialog is a basic language function and that the analysis of

[1] The sample included three research universities, two master's comprehensive universities, and two teaching institutions (a private and a community college).

the linguistic devices that instructors use is an important contribution towards understanding how interaction can be fostered in undergraduate classrooms.

The paper is organized as follows: we start with a brief review of the literature of studies that have analyzed classroom interaction in undergraduate settings, followed by a description of the linguistic framework that guided the work. We then describe the data and the analyses we performed and present the major findings. We conclude with some recommendations for research and in particular for faculty development within mathematics.

Literature Review

Articles reporting analyses of classroom discourse in undergraduate classrooms can be separated into three categories, depending on the discipline that looks at it: science and mathematics education, higher education, and linguistics. In science and mathematics education the studies have taken a position that the social context matters for learning. Under this position, learning is both an individual and social process, and these occur co-dependently. These analyses provide rich descriptions of students' and instructors' activities in the classroom with the ultimate goal of describing the nature of learning that happens with given tasks in that particular context. Two examples are Cochran (1997) and Stephan and Rasmussen (2002). Cochran (1997) analyzed one semester of a chemistry course offered to students for whom the class was terminal. Using an ethnographic approach she collected data on the lectures and recitations to describe who did what with whom and for what purpose and analyzed the data thematically to uncover how the course shaped a view of science content, the type of knowledge students were expected to produce, and what counted as science in the course. She found that for the most part, the class promoted a view of chemistry as procedural, with students expected to repeat processes in the exams, specifically to being able to recognize answers in the multiple-choice exams; the major

narrative of what counted as 'science' in the class related to the testing curriculum and doing well in the exams. The analysis of language in this case was holistic, as the researcher looked for the content of the utterances and how they defined positions and messages (Dowling, 1998) rather than looking at specific linguistic devices that speakers used to establish their own positions.

Stephan and Rasmussen (2002) analyzed the collective generation of knowledge in a differential equations classroom that had been using a curriculum that centered on challenging tasks and support from technology. The methodology of choice was a classroom teaching experiment[2] that collected several data sources (class video recordings, students' interviews, copies of written work, researchers' journals and recordings of their meetings) that were analyzed to generate the way in which six mathematical practices got generated in the first half of the course. Those practices (predicting individual solution functions, refining and comparing individual predictions, creating and structuring a slope field as it relates to predicting, reasoning about the unknown in the equation as both a variable and a function, creating and organizing collections of solution functions, and reasoning with spaces of solution functions) referred to mathematical norms for participating in the classroom and were evidently geared to establish ways in which knowledge was being created in and shared by the community; in here the emphasis, when looking at language, is in how the community interprets meaning from utterances, texts, and other tools that are available, rather than at particular positions that the speakers take.

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[22] In a teaching experiment a researcher generates hypotheses about students learning trajectories about a mathematical notion; a task is designed to test the plausible trajectory and as the student works with the tasks the researcher can confirm or disconfirm the learning processes that happened (Steffe, 1994). Classroom teaching experiments include the instructor and are more complex, since they attempt to look not only at individual, but also at collective learning. See Cobb (2000).

Studies in higher education have also looked at classroom interaction to uncover patterns of participation that might exclude some groups from it and highlight the role that instructional practices have on the interaction patterns. The "chilly climate" hypothesis, for example refers to patterns of interaction that occur in college classrooms that prevent females or minorities from participating actively (by asking questions or offering answers) and that lead them to leave or change degrees for which they are highly qualified (Hall & Sandler, 1982; Williams, 1990). Fassinger (1995) reports that there is evidence that females in general participate at lower rates than males independently of the discipline but that there is mixed support for how much such behavior depends on the instructors' gender, although there is evidence that both males and females participate more with female instructors. Fassinger reports results of a survey of 51 classrooms in a liberal arts college in the Midwest that surveyed over 1000 students about their levels of classroom participation and some of the reasons they had for such participation level. She found that although there were differences in the factors that determined male and female participation, aspects such as student confidence, class size, and level of student-to-student interaction were more critical than gender or participation grade. She also found that there were no instructor factors that could be associated with the different participation levels of the students. In this study, classroom participation was defined as comments or questions students offered during class and it was assessed via Likert type items. In a follow-up study, Fassinger (2000) analyzed the same data set using the class, not the individual students, as the unit of analysis. She found that classes in which participation was high (measured as students offering an average of 12 or more interventions in a given class) had "more cooperative, supportive, and respectful classroom dynamics; [patterns of interaction were] more inclusive, less teacher-centered, more tolerant of student input, their members [were] more confident, and their

professors seen as more approachable and supportive" (p. 45). Studies such as these clearly attend to two important dimensions of classroom participation, namely the students' personal positions and perceptions of the value of participating as well as the role that instructors play in shaping the interactions (e.g., by generating a student centered class). In these studies, classroom participation is a proxy for classroom interaction (seen from the students' perspectives) and language is not analyzed.

Finally studies from the linguistics literature with undergraduate settings seem to be limited to analyses of academic[3] registers, both oral and written, using large corpora of data (full textbooks or collections of classroom lectures). The Michigan Corpus of Academic Spoken English, MICASE, for example, houses a searchable database of over 170 transcripts of academic speech events that occurred at the University of Michigan, indexed by speaker attributes (academic position, native speaker status, first language) and transcript attributes (event type, academic discipline, participant level, and interactivity rating). The corpus was created to describe the characteristics of contemporary academic speech—its grammar, its vocabulary, its functions and purposes, its fluencies and dis-fluencies, and to determine whether those characteristics differ depending on academic disciplines and on classes of speakers.[4] MICASE is the largest corpus of academic speech that is available for research. Studies using corpora allow for the investigations of uses given to specific words and expressions (e.g., 'point,' 'no way,' see Swales, 2001) and for making comparisons across disciplines. Although these studies are important, especially for teachers of English, it is not clear that they are useful for

[3] Academic refers to language used in post-secondary settings.

[4] Information about MICASE can be found at http://legacyweb.lsa.umich.edu/eli/micase/index.htm.

understanding classroom interaction. Analyses of classroom interaction are virtually non-existent with undergraduate populations in the linguistics literature.[5]

The different approaches taken by these three bodies of literature to studying classroom interaction attend the language but with an eye to understanding how the learner will make sense of it (either by using it or not, or by understanding mathematics or not). What is lacking from these studies is an analysis of what *instructors'* language looks like; what kind of positions they convey with their language regarding the engagement of the students in the dialog. This exploratory study contributes to filling this gap in the literature, by making salient the role of the language in such engagement.

Methods

Because it is central to our analysis, we start by describing the analytical framework that we used. Then we provide information about the classes and the segments that were chosen for analysis and how we developed and applied a coding system for the data.

The Engagement System

We use Martin and White's (2005) engagement system that suggests that interpersonal meanings are formed in the interplay of two discursive voices, *monogloss* and *heterogloss* (including contraction and expansion). Monogloss does not seek to overtly engage the audience while heterogloss is used to engage the audience at different levels. The engagement system belongs to the larger appraisal framework that has three main components: engagement, attitude, and graduation (Martin & Plum, 1997; Martin & Rose, 2003; Martin & White, 2005). This study

[5] We located seven articles that referred to classroom interaction and mathematics or science in linguistics journals. All of these occurred in the K-12 setting.

illustrates how instructors use language as a result of the interplay between these two major discursive voices.

Informed by Bakhtin's/Voloshinov's notions of "dialogism and heteroglossia," the engagement system regards all utterances as dialogic, suggesting that what is said is invariably implicated in a web of references (Martin & White, 2005, p. 93). Bakhtin (1981) elucidates that all utterances exist

> … against a backdrop of other concrete utterances on the same theme, a background made up of contradictory opinions, points of view and value judgments… pregnant with responses and objections (p. 281).

Based on this notion, engagement analysis investigates "the degree to which speakers/writers acknowledge these prior speaking,… whether they present themselves as standing with, as standing against, as undecided, or as neutral with respect to these other speakers and their value positions" (Martin & White, 2005, p. 93). Martin and White's engagement framework aims to provide a "*systematic* account of how such positionings are achieved linguistically" (p. 93, emphasis added).

Generally, *monogloss* is defined as akin to "bare assertions" in which no "dialogistic alternatives" are needed to be recognized (p. 99). It designates an inherent value of taken-for-grantedness and presupposition that allows little room for advancing a counter point. As a result, monogloss construes propositions that do not need to be brought into active rhetorical play and are therefore construed as self-evidently right and just. Monogloss text often sounds descriptive, report-like, and impersonal.

By comparison, *heterogloss*, as inherently "dialogistic locutions," overtly grounds the proposition "in the contingent, individual subjectivity of the speaker/writer" and thereby recognizes that the proposition is but one among a number of propositions available (p. 100).

We provide examples from our corpus to illustrate how each discursive voice is realized linguistically (see Figure 1). Key linguistic items are underlined to highlight the discursive effects each voice seeks to accomplish.

Monogloss

1. *So what we have to do is* compare 2006 dollars to 2006 dollars.
2. … *ask yourself* how does that procedure you use generalize.
3. So *the input here is* y.
4. *That accounts for* the fact that *they're* power series.

Heterogloss

5. For Taylor series *I think* it's fine to write $S_1(x)$, $S_2(x)$.
6. Ok, *but* we want to compare the current federal minimum wage, which is by definition 200…
7. *Why don't you* use the ten-minute rule…
8. Then how does the volume change, *if* you add an inch of radius then?
9. *I would be* very interested to see an anti derivative for e to the $-t^2$.
10. This is the part that changes *when* the point about which you expand x changes, changes.

Figure 1: Examples of monoglossic and heteroglossic discursive voices.

These 10 examples show marked difference between monoglossic and heteroglossic rhetoric. In the four examples of monogloss shown above, no dialogistic alternatives are needed to be recognized. They construe propositions that are self-evidently right and just ("ask yourself…," "So the input… is…"). The speakers therefore do not seek to engage the listeners but merely state things as they are.

By contrast, heterogloss is marked by authorial interpolation and engages the readers interpersonally. In Example 5 in Figure 1, in interpolating the authorial subjectivity "I think," the speaker, instead of construing the proposition as self-evidently right, foregrounds his opinion as confined in his very subjectivity, which can therefore be subjected to re-examination. Grounding the proposition in the contingent individual subjectivity of one speaker admits that the proposition is but one among a number of propositions available. In Example 6, by posing the statement in the countering "but," the speaker seeks to restrict the scope of dialogic possibility in

foregrounding his more assertive claim. “But” is used to counter a previous utterance to highlight the current statement as more appropriate or reliable. It therefore contracts dialogic possibility by a more assertive stance. In comparison to the more assertive claims, in Example 8, the conditional “if” renders the statement tentative and thus invites more dialogic possibilities. It opens up room for further discussions that may lead to multiple interpretations. In Example 9, the speaker, in uttering “I would be very interested,” expresses his inclination in modal term. Usually “I would be” comes with conditional phrases “if” which can rest the claim on more tentative ground. Contrary to “I am interested” which states more of a solid fact that concedes no contestation, “I would be interested” expresses more of an inclination that would stand true if some other conditions are met.

Heterogloss is composed of complex values. We can choose to heteroglossically contract or expand an argument. The difference lies in “the degree to which an utterance… actively makes allowances for dialogically alternative positions and voices (dialogic expansion), or alternatively, acts to challenge, fend off or restrict the scope of such (dialogic contraction)” (Martin & White, 2005, p. 102).

Contraction is directed toward confronting and defeating potential contrary positions in asserting or insisting, whether explicitly or implicitly (p. 116-117). Contracting then seeks to align readers to the author’s point of view (Koutsantoni, 2004, p. 164). Its two main features are *“disclaim”* and “*proclaim.*” *Disclaim*, is used mainly to reject prior utterances or alternative perspectives by denying and countering. *Proclaim*, on the other hand, is used to overtly announce agreement with the projected dialogic partner by concurring, pronouncing, and endorsing. In proclaiming, the author simultaneously designates other interpretations or

perspectives as less valid, thus contracting the argument to align the readers to his or her side (Martin & White, 2005, pp.117-127; see Figure 2).

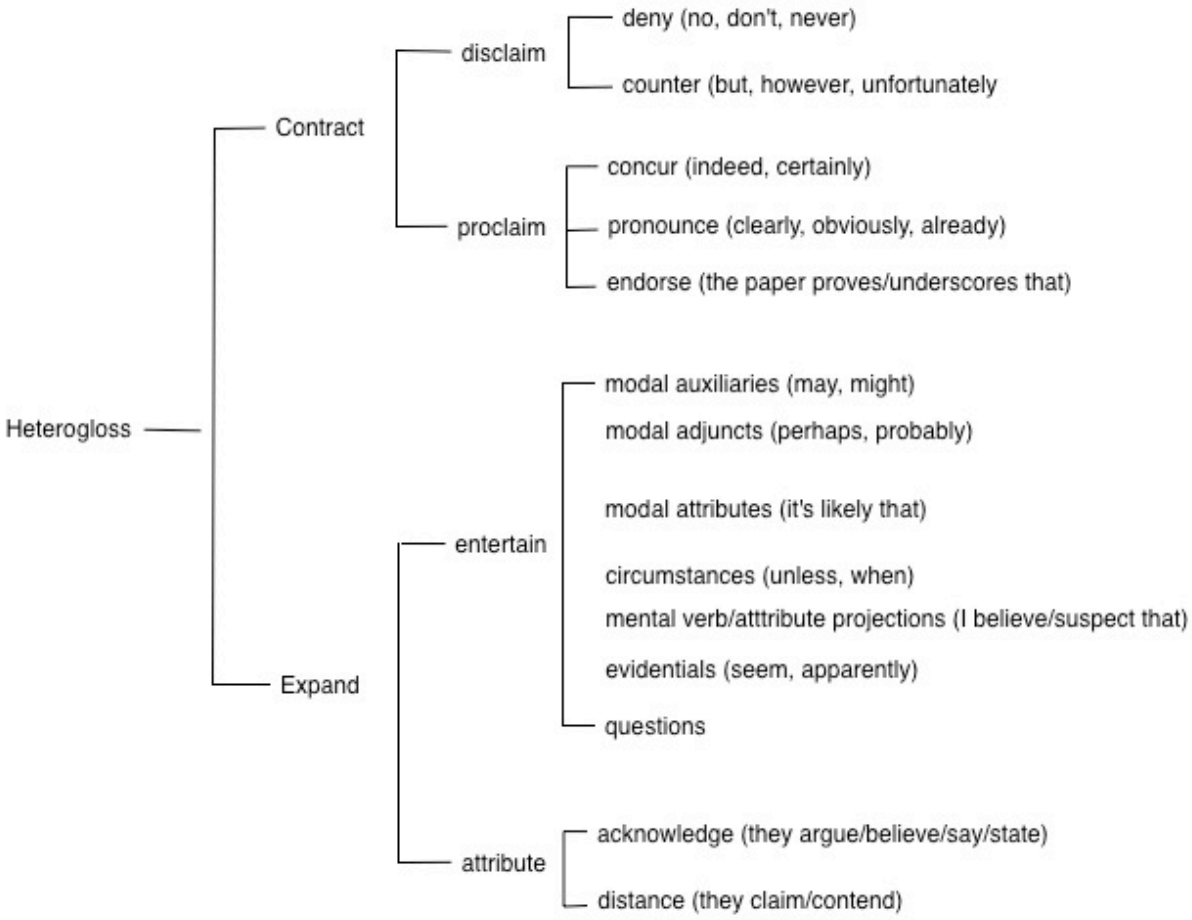

Figure 2: Engagement system: Heteroglossia (Adapted from Martin & White, 2005, p. 134)

Expansion, by contrast, concerns an authorial voice set to entertain alternatives and possibilities as claims still open to question. Its two main features are "entertain," and "attribute." *Entertain* generally softens an otherwise subjective statement by a variety of linguistic resources such as (1) modal auxiliaries (may, might), (2) modal adjuncts (perhaps, probably), (3) modal attributes (it's likely that), (4) circumstances (unless, when), (5) certain mental verb/attribute projections (I believe, I suspect that), (6) 'evidentials' (seem, apparently), and (7) questions (Martin & White, 2005, p. 105). *Attribute* is used to open up discursive or

dialogic space by referencing an external source either in acknowledging or distancing the source. With *acknowledge,* the speakers manage to stay neutral in introducing multiple perspectives; doing so enables discussing or developing key issues in the argument. With *distance* the speaker means to stand farther in the background.

The examples in Figure 3 illustrate the differences between expansion and contraction and of some of the possibilities described. The key linguistic items that set the two apart are underlined.

Heterogloss Contraction
1. disclaim-deny: *You can't* compare things that are in two different units.
2. disclaim-counter: It may not be the quickest way, *but* you can certainly do it.
3. proclaim-pronounce: *Oh I see. I see. I see* what you're saying.
4. proclaim-concur: *Yeah,* so you're *on the way, yeah*.
Hetergloss Expansion
5. entertain-modality: Well let's do it on the board and then *you can* see how your answer compares.
6. entertain-evidentials: So *it looks like* you've got one squared away.
7. entertain-possibility: So *maybe* it's not really possible to, *maybe* you need an extra -1 out in front?
8. entertain-conditional: *If* you're having trouble following Leslie's argument there or *if* you're a newcomer to the chain rule, you may try that function as a warm up…
9. entertain-circumstance: Now *when* you have, *when* you have a system of differential equations where the coefficients in that system are listed in these matrices, *then* the solution of that system is actually given by the exponential of that matrix.

Figure 3: Examples of heterogloss-contraction and heterogloss-expansion.

Blunt denial, "can't" and the contrasting connective, "but" (Examples 1 and 2), confront or defeat potential contrary positions in insisting or denying. These two forms enable the speaker to designate other interpretations or perspectives as less valid and so seek to align the listener to alternative viewpoints the speaker himself or herself endorses. Also, in proclaiming "I see what you are saying" and "you are on the way, yeah" (Examples 3 and 4), the speaker either overtly aligns himself or herself with a projected dialogic partner or announces agreement with him or her. Both disclaiming and proclaiming work toward contracting dialogic possibilities in aligning

the listeners to the claims the speaker intends to make. By contrast, expansion does not seek to align the listeners to the speaker's side. It deals with entertaining alternatives, maintaining objectivity, and introducing divergent viewpoints without overt subjective interpolation. In the examples above, "it looks like," "maybe," "can," etc., do not seek to conclude an argument; instead, these expand the argument and allow room for further discussions. Also, the conditional "if" in Example 8 and the circumstance "when" in Example 9 render the claims tentative, which sound indecisive and thus allow other dialogic possibilities.

As these examples show, this level of detail of the analysis of the language used is useful in determining what is the speaker's stance, or position, regarding the dialog: is the stance one in which the speaker wants to state facts as they are without leaving room for refutation? Is it one in which, by using more tentative language, the speaker invites others to offer their position? Does the speaker disclaim or entertain possibilities? And what kind of wording is used to accomplish those positions? And, is this analysis useful in uncover differences that otherwise could not be noticed among a variety of college instructors who seem to be successful at maintaining high classroom participation?

Sampling

We conducted a secondary analysis of classroom interaction on transcripts of nine undergraduate mathematics lessons that were collected as part of two different projects.[6] In Table 1 we present a table that summarizes the characteristics of these lessons sorted by the number of

[6] The lessons were meant to be representative of the teaching of the participating faculty. Details about the sampling for the projects can be found in Mesa (2007) and Mesa *et al.* (2007).

times the students participated in each class[7]. Eight lessons were from the same project and in seven of these lessons the level of student participation per duration of the observation was low (see last column in Table 1). This rough measure of student participation showed that there were two 'outliers,' two classes in which the students (and the teachers) were talking more per minute, relative to the other lessons in our sample. Consequently we selected these two lessons to test the applicability of the engagement system because we thought they would provide the most opportunities to analyze the instructors' language in relation to students' interactions. Common in the other lessons were long segments in the transcripts (e.g., 2 min long) in which the teachers would talk, with students intervening to ask a clarifying question or offering a comment that the instructor would address but not elaborate on or without asking for student re-elaboration. Given that the engagement system is meant to help us explain the level of engagement in the dialog we deemed these other lessons inappropriate for the analysis.

Table 1: Characteristics of the Corpus of the Classes from Which the Two Lessons were Selected.

Class Label[a]	# of Student Interventions	# of Teacher Interventions	Type of Class	Class length (min)	# Students' interventions per min.
1.1	1	2	Capstone	21	0.05
1.2	12	13	First year-Honors	46	0.26
1.3	12	13	First year	43	0.28
1.4	13	14	First year	44	0.30
1.5	18	19	First year-Honors	29	0.62
1.6	27	30	Capstone	59	0.46
1.7	32	34	First year	65	0.49
1.8	180	170	First Year	56	3.21
2.1	221	114	First Year	100	2.21

Notes: [a]. The first number identifies the project; the second number the lesson taken from that project.

[7] In the transcripts we identified whether the speaker was a male, a female, or the Instructor, who had a microphone on him or her all the time. We counted the number of times males and females spoke, and the number of times the instructors spoke.

We also had access to interviews with the faculty, regarding the goals they had for the classes, their academic background and teaching experience, and how they used textbooks for teaching. The instructor who was teaching lesson 1.8, (hereafter Instructor A) was teaching at a master's comprehensive university; the instructor teaching lesson 2.1 (hereafter Instructor B) was teaching at a research university. Both instructors were male, junior faculty with between 5 and 7 years of teaching experience. Additionally, both manifested an interest in creating classes in which students could participate more, ask questions and offer answers, and rely on others to find solutions to problems. They also indicated that these lessons were representative of their practice. In reading the transcripts and listening to the tapes, we noticed that most of the participation during the class taught by Instructor A (hereafter Class A) occurred in the first 24 minutes of the observation. Given that one of the purposes of the study was to illustrate how the engagement system could be applied, we chose to analyze segments of the same length (the first 24 minutes) in these two lessons.

Context of the Segments

Lesson A was taken from a general education requirement course that covers topics such as linear equations, linear programming, linear regression, probability, and statistics for non-math or science majors. The instructor dedicated the first part of the class to let students work in groups on a worksheet with problems of a higher complexity than those in their textbooks. The observation was done in Winter 2007 about 6 weeks into the 14 weeks course, when it was thought that most norms for classroom interaction had been established. The purpose of this lesson was to apply strategies to convert nominal to real dollars (and vice versa) for different

years. For the first 24 minutes, when most of the interaction occurred in this 80-minutes class[8], students worked in small groups solving six problems, two of which are shown in the Appendix, with the instructor going around the groups and answering questions as requested[9].

Lesson B was taken from an elective course that seeks to engage first-year non-honors students interested in math or science in learning to solve calculus mathematics problems. For each session, the instructor created a problem worksheet and after assigning the students to small groups of 3 or 4, let them work on their own, listening and intervening as needed. The lesson was recorded in October 2006 (about 5 weeks after the beginning of the term). For the first 24 minutes of this two-hours class students worked on three of the 10 assigned problems, two of which are presented in **Error! Reference source not found.**b. In both classes the problems admitted more than one solution strategy and were challenging to the students.

Analysis

The focus of the analysis was the instructors' utterances. We parsed all instructors' turns into clauses[10] and coded each clause using the categories of engagement defined previously. The second author parsed and coded both transcripts in consultation with the first author. During these consultations we refined the categorization, especially regarding 'Entertain' values. To test for consistency and reliability of the parsing and the coding, two random segments of about the same length (5 minutes) were selected for coding nine weeks after they had been originally coded. Both authors then proceeded to parse and code the segments; the second author contrasted

[8] About 20 minutes were taken for asking students to take a survey about their use of their textbook (Mesa, 2007).

[9] The analysis of the mathematical solutions for these problems is reported in Mesa and Beasley (2007).

[10] A *clause*, from the perspective of Functional Grammar, is a better unit than sentence for analysis because a clause may contain rich information that the writers or speakers use to good effect whereas sentences may have variance considering their unequal grammatical contributions to the text. Therefore, the clause is "the best basic unit of grammatical analysis of text"(Schleppegrell, 2008, p. 551).

her latest parsing with the initial one and with the parsing done by the first author. Two types of discrepancies in parsing clauses occurred: a statement could contain two instead of one clause (e.g., in 'the first thing we do is convert $3.35 into 2006 dollars' the first clause ends at 'do') or an expression would be split incorrectly, usually ignoring a subordinated clause (e.g., the clause 'I heard you say something earlier, Leslie' would be split incorrectly after 'you'). Agreement in parsing clauses ranged from 84% to 96% and thus it was deemed reliable for the purposes of this analysis. Before proceeding with the coding for the appraisal system, the two authors created a unique parsing of the clauses for both segments. The segment in Lesson A had 80 clauses (26% of the total coded clauses), whereas the segment in Lesson B had 30 clauses (41% of the total coded clauses). In coding the engagement, the main source of disagreement was in the coding by the first author of clauses that were monoglossic, which prompted for clarifying its definition. The level of agreement and consistency ranged from 70% to 96% and therefore, the coding was deemed reliable and we used the coded transcripts for the analysis.

Results

In Table 2 we present descriptive information on these two segments. In these shorter 24-minutes segments, the students participation rate in Class A was about 3 per minute and about 2 per minute in class B. Class A had 309 clauses, whereas Class B had 73. In Class A, 157 (51%) of the clauses were monoglossic whereas in class B only 25 (34%) were monoglossic. In Class A, 90 of the 147 heteroglossic clauses were expanding, whereas in Class B 34 of the 44 heteroglossic clauses were expanding. The difference in turns between both classes suggests a different level of instructor intervention in each case, with the Class B instructor speaking less than the Class A instructor.

Table 2: Descriptive Information on the Two Segments[a]

	Class A	**Class B**
Number of student turns	76	43
Number of instructor turns	76	27
Student Turn Rate (# of students' turns/min)[a]	3.11	1.76
Number of clauses	309	73
Number (percent) of Monogloss clauses	157 (51%)	25 (34%)
Heterogloss clauses	147 (48%)	44 (60%)
Contracting	57 (18%)	10 (14%)
Expanding	90 (29%)	34 (47%)
Notes: a. the segments were 24.4 minutes long		

We present now excerpts from each class in which we illustrate both the coding and the characteristics of engagement that these two instructors exhibited. The segments were chosen to be representative of the interaction in each class and to illustrate the wide range of uses observed. Each segment is followed by the analysis we conducted. Only the instructor utterances are coded and the coding is in parenthesis. We underlined text in the clause to highlight terms and expressions that assisted in the coding[11]. Bold text in the coding is used for showing the abbreviations used later on (see Figure 2).

Example 1, Class A: Using Monogloss and Contracting

1.	I: So we do want to compare old to new, right?	(**H**eterogloss **C**ontract-**pro**claim-**pro**nounce)
2.	But the old that's like really old.	(HC-**dis**claim-**counter**)
3.	We can only compare old to new	(H **E**xpand-**en**tertain: **modality)**
4.	if they're in the same units.	(HE-en: **conditional)**
5.	This is in the units of 1989 dollars I think, yeah.	(**M**onogloss)
6.	S**:** And this is in?	
7.	I: That's in units of 2006 dollars,	(M)
8.	so we can't divide them just like we can't,	(HC-**dis**claim-**deny**)
9.	we can't add like meters to miles,	(HC-**dis**-**deny**)
10.	we have to have everything in the same units.	(HE-en: modality)
11.	So what we have to do is compare 2006 dollars to 2006 dollars.	(M)
12.	So 5 45 is a 2006 dollar	(M)

[11] In the long version of this paper we include two examples from each class.

13. and what else is a 2006 dollar in this problem? (pause-5 seconds) (HE-en: **Q**uestion)
14. S**:** It would just be that.
15. I: Ok, but we want to compare the current federal minimum wage, which is by definition 200… (HC-dis-**counter**)
16. S**:** So it's going to be $5.15 then?
17. I: Yeah. (HC-pro-**concurr)**
18. S**:** Oh, so old is going to be $5.15.
19. I: Well we have to… (HE-en: modality)
20. S**:** Right?
21. I: We have two 2006 dollar values, $5.15 and $5.45, $5.45 comes from 1989 (M)
22. but it's updated. (HC-dis-counter)
23. So we're asking (M)
24. how many percent higher or lower is the current federal minimum wage than this updated value? (M)
25. Anything that comes after that is usually our old or our basis for comparison. (M)

In this example, the instructor first emphasized the inclination of doing something in "We do want to…" by proclamation, which brought the students to focus on the problem space he set forth here (line 1). Then he went on to highlight what "old" meant by countering "But" (line 2). He continued to set up a condition, "if they are in the same units," for the proposition he set forth and supplied with a "fact", "in the units of 1989 dollars" for the condition (lines 4-5). By responding to a short request from the student, the instructor was able to explain what it meant to be in the same units by using a plain statement, "That is in" followed by negatives, "can't" and modalized obligation "we have to" to direct the course of action to be taken (lines 7-9). He went on iterating what was to be done, "So what we have to do is…" and went on to offer a clue, "So 5 45 is a 2006 dollar" (lines 11-12). Then he posed a question to the students. With a short and apparently unclear response from the student, the instructor continued to explain first by countering "But" to specify the problem (line 15). When the student asked for confirmation about the amount, "So it's going to be $5.15 then?" he responded with a short "Yeah" and the student went on replacing the pronoun "it" with a specific noun, "old" (lines 17-18). The instructor then made a statement, "We have two 2006 dollar values" and countered it with "but" which refined his question that came after it (lines 21-22). The question did not expect answers

from the students but functioned more as a statement as the previous sentence. The statements he made tended toward monogloss as no interaction from the students was expected (lines 22-25).

Instructor A used a relatively high proportion of monogloss clauses to state "facts" and countering and denying values to highlight the points he made to fulfill his instructional purpose, giving information more than to engage the students. These discursive strategies, either monogloss or contracting options, resulted in interactions where dialogic space is limited. The first question he posed (line 13) did not seem to invite extended discussion with the student but led to monoglossic statements, modalized obligations, "have to" and countering "but" to offer more information to the student. The second how-question resided in another embedded clause, "So we're asking" (line 23) which again fulfilled the instructor's rhetorical purpose to offer more information than to expect real answers.

Class B: Example 1, Using Modality

1.	I: So then <u>let me</u> ask you,	(M)
2.	<u>what</u> is the purpose of setting this equal to zero in the first place?	(HE-en: Q)
3.	S: To define the zeros so you can find numbers on the different intervals. (inaudible).	
4.	I: Oh I see.	
5.	So <u>what</u> are the points where dy/dx = 0 called?	(HE-en: Q)
6.	S: (inaudible)	
7.	I: Ok. Right, right.	
8.	<u>Probably [it is] better</u> to say critical points	(HE-en: modality)
9.	because <u>there can be</u> places where y' is 0	(HE-en: modality)
10.	<u>but</u> it's neither a min nor a max, right?	(HC-dis-counter)
11.	So <u>that's one</u> approach	(M)
12.	<u>but</u> it looks like you're seeing	(HC-dis-counter)
13.	that <u>it's kind of</u> intractable here, right?	(HC-pro-pro)
14.	So then <u>you might</u> ask	(HE-en: modality)
15.	<u>can I</u> tell where this quantity's positive and negative without knowing where its zeros are?	(HE-en: Q)
16.	S**:** That's what I was trying to do. I was trying to like think about it like without the math, like, but I don't know.	

In this example the instructor used a combination of questions, modalized suggestions and possibilities to prompt the students to think. The first question here prompted for clarification of the purpose of doing something (line 2). It was followed by another question

related to the naming of "the points where dy/dx = 0" (line 6). After the questions and responses, the instructor offered suggestions by using "probably better to" with an explanation, "because there can be…" (lines 9-10). The suggestion remained a tentative claim when in both clauses we see possibility words, "probably" and modal auxiliary, "can be" (showing possibility similarly) reinforcing this tentativeness. The instructor continued with a countering "but" as a way to make the case that what was suggested was but "one approach" (lines 11-12). And the second "but" directed the discussion to the student's idea framed in tentative terms, "looks like" and "kind of" (lines 13-14). It shows that the instructor did not seem willing to interpret the student's idea subjectively; therefore, he used the tentative terms to show his interpretation as probably a way to look at it which might not do justice to the student's thinking. Seeing this, he then asked the student to think by a question that he framed in more tentative terms using "might" and "can" (lines 15-16). By showing his reluctance to commit to definitive interpretation or blunt statement, the instructor tended more to guide the students to think in questions or in offering suggestions and possible approaches to tackle the problem at stake.

Compared to students in Class A, students in Class B were asked more questions and offered more suggestions than contracting or informational statements to work on their problems. Rather than being definite the statements served the purpose of making the students take stock of what they had and what was asked or given in order to proceed. In Class A students asked questions and received more definitive explanations or information which seem to prompt them to recall and distinguish rather than to make sense; questions in this segment also were geared to highlighting the need for distinguishing between a reduced set of options. By varying discursive strategies as exemplified by these two instructors, students got to participate, respond, and

probably, think differently depending on how the instructors guided them using the discursive options they deemed appropriate.

Discussion

The purpose of this exploratory study was to see whether the application of Martin and White's (2005) linguistic framework would be useful in characterizing and differentiating the language that two college mathematics instructors used to engage students in the classroom dialog. Both instructors were committed to making their classes more participatory and therefore had created, what appeared to an observer, very interactive classrooms, with students intervening as frequently as their instructors or more.

The analysis of the instructors' language used in two segments of their classes revealed important differences in how the use of certain linguistic forms fostered or compromised the dialogic interaction between instructors and students. If one goal for instructors is to create a setting in which students are engaged, then it is necessary to pay attention to how the linguistic devices they use can act against this purpose. The study demonstrates that this type of analysis can provide evidence that there are features of the language that can be used to distinguish the level of engagement of students in different classroom settings, even if the settings appear to be highly participatory.

First, our analysis reveals that monoglossic speech was relatively frequent in both segments analyzed (51% and 34% in Class A and Class B, respectively) compared to the frequency of heteroglossic discourse observed (60% vs. 48% in Class A and Class B, respectively). This suggests that that there was a need to state facts, avoiding argumentation or discussion: certain things are, and presumably need to be used and learned as such. Because the two segments were recorded during a class period in which the students were working actively

on pre-defined problems, the observed frequency suggests that stating facts is a need for the problem solving session to be sustained, and that these two instructors were the source of a substantial number of those facts.

Second, our analysis revealed that within heteroglossic discourse entertaining and contracting forms were used to give information, assess situations, and also to seek explanations and information from the students. We saw how tentative language was used to counteract the authoritarian voice of the instructor in the setting[12].

Third, we found striking differences in how the two instructors positioned themselves with respect to their students. The two settings, differed in the level at which the instructor maintained a monoglossic voice (over half of clauses analyzed from Instructor A versus over one third of clauses analyzed from Instructor B) suggesting that Instructor A tended to maintain a more authoritarian position than Instructor B. Furthermore, within the heteroglossic voices, the two instructors expressed almost diametrically opposed attitudes in engaging students in the dialog. The instructor in Class B, in his choice of language, entertained dialogically alternative voices more frequently than the instructor in Class A. A predominant device was entertaining questions, with which the Class B instructor not only sought information or explanations from the students, but also managed to provide suggestions or hints that would assist them with their work. In contrast, the Class A instructor preferred modality, but this device was used mostly for giving information without prompting students to think or examine what they were doing.

When looking at dialogic contraction, which acts to challenge, fend off, or restrict dialogue, the two instructors also differ. The Class B instructor used the authoritarian

[12] The authoritarian voice is being recognized as a feature of mathematics, ostensibly present in textbooks (Love & Pimm, 1996; Pimm, 1987), even in those assumed to be designed to include the students in the mathematics presented (Herbel-Eisenmann, 2007).

pronouncement to assess students' progress and stress the importance of particular information that was being considered. On the other hand, the Class A instructor preferred to counter prior utterances or alternative positions, by denying a position previously recognized, with the main purpose of providing information and noting the importance of the points he made.

The analysis of the discourse of the two settings illustrate the array of linguistic devices that can be used and hints at the potential for those devices to actively include or exclude students from participating in mathematical dialogue. Thus, the analysis highlights that attention to how the language is used to position the speakers in the dialog used can reveal important elements of authority that may limit possibilities for dialogic interactions in classrooms that are expected to engage students.

Conclusion

This type of analysis opens the possibilities that go beyond what current analyses of classroom participation offers. An analysis of the linguistic strategies that instructors use might complement holistic analyses that look at how "things are" (e.g., who does what, to whom, and with what purposes, Cochran, 1995) by providing specific evidence of how the language in itself contributes to define those constructions. Similarly analysis of how socio-mathematical norms get established in the classroom (e.g., Stephan & Rasmussen, 2002) could benefit from studying how the language that instructors use might indeed be defining some of those norms, perhaps there are some forms of language that are more prominent than others or perhaps, it is precisely because of how the language is used that certain norms, and not others, get established. Our analysis also goes into more detail into lessons that would be considered highly participatory (e.g., Fassinger's (2000) definition of 12 or more interventions by the students). In our sample of undergraduate classes in all but one class there were 12 or more students' interventions, yet, our

analysis shows that the quality of those interventions is worth looking at, because even though at a surface level students might feel that they are participating, it might be that the level of engagement is such that they are just expected to agree with the instructor's position rather than to negotiate a position through the dialog. Our analysis also complements corpora analysis, by expanding the range of situations in which words are used within classroom interaction.

Our analysis seems promising in helping us understand the mechanisms by which students are actually dialogically engaged with the discourse in the classroom. We have seen that even in a seemingly highly interactive setting, there might be little room for students to include their own perspectives or voices into the dialog and that it is possible, however, to organize discourse in a way that does. Within an authoritarian discourse such as mathematics, understanding what it means to dialogically engage students and under what circumstances is somewhat urgent, given suggestions that such authoritarian voice de facto excludes some groups of students (female, low ability, minority) from participating in the mathematical discourse (Dowling, 1998) and in turn from careers in the sciences, mathematics, technology, or engineering.

Likewise, raising awareness of the role of language in sustaining dialogic engagement is an important area for professional and faculty development. Perhaps, if the goal is to create a really engaged classroom, some of these devices can be part of what instructors need to learn to teach undergraduates; an analysis of how their language affects the extent to which they invite students into the dialog seems to be an important area for consideration. While awareness about students' misconceptions has been highlighted as important for preparing future faculty (Kung & Speer, 2007; Speer *et al.*, 2005), information about the impact of language in the classroom work is also important for all instructors. We use language to deliver information and to assess

students' progress. How we use it conveys powerful messages that might exclude the students that we need to be participating in the dialog.

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Appendix

1. John had an hourly wage of $4.20 in 1970, and Carl had an hourly wage of $21.30 in 2000. Which wage had more buying power? How many percent more? Explain your reasoning.

2. The following adjustments have been made to the federal minimum wage over time:

Year	Min. wage	Year	Min. wage	Year	Min. wage
1950	.75	1974	2.00	1981	3.35
1956	1.00	1975	2.10	1990	3.80
1961	1.15	1976	2.30	1991	4.25
1967	1.40	1979	2.90	1997	5.15
1968	1.60	1980	3.10		

a. What was the minimum wage in 1989? What is the equivalent wage today? How many percent higher or lower is the current federal minimum wage than this updated 1989 value?

b. What was the minimum wage in 1968? What is the equivalent wage today? How many percent higher or lower is the current federal minimum wage than this updated 1968 value?

(a)

2. Let

$$f_1(x) = \sqrt{x},$$

$$f_2(x) = \sqrt{x + \sqrt{x}},$$

$$f_3(x) = \sqrt{x + \sqrt{x + \sqrt{x}}},$$

and so on.

(a) Where does the function in problem 1 appear in this sequence of functions?

(b) Graph f_1, f_2, f_3, f_4, and f_5 on your calculator.

(c) Let

$$y = f(x) = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}.$$

Find a "closed form" for this function; that is, find an expression for $f(x)$ that does not involve an infinite number of square roots. Here's a step-by-step plan: square both sides, then write a quadratic equation in y with coefficients that may be in terms of x. Solve this quadratic equation for y.

(d) Graph $f(x)$ along with f_1, f_2, f_3, f_4, and f_5. Stay tuned; in Math 116/146, we discuss convergence of sequences of functions.

(e) Find where $f(x)$ is increasing/decreasing and concave up/concave down.

(f) Describe $f(x)$ as the result of transforming $\sqrt{x}$ by a sequence of horizontal/vertical shifts/expansions/contractions.

3. (a) What is the volume V of a sphere in terms of its radius r?

(b) Find dV/dr.

(c) Explain the geometrical meaning of dV/dr.

(b)

Problems from the worksheet students were working on (a) Class A; (b) Class B.

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