A Partnership to Promote Inquiry-Based Mathematics Instruction

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Abstract

This study describes the efforts of a mathematics partnership in promoting inquiry-based mathematics instruction and the resulting impact on mathematical knowledge and classroom practices. The subjects for the study are university students and faculty, middle grades in-service teachers and pre-service teachers taking a series of inquiry-based mathematics courses, as well as general university students enrolled in a reformed finite mathematics class. A variety of measures are used in determining participants' knowledge of mathematics including objective tests, performance assessments, and portfolios. Focus groups were used to gather qualitative data from university students and from university faculty. Additional measures such as classroom observations and surveys are used to measure changes in teachers' instructional practices. This paper reports the results of changes in students' mathematical knowledge and in the instructional practices of university faculty. Implications for changes in other university mathematics courses will be discussed.

A Partnership to Promote Inquiry-Based Mathematics Instruction

Improving mathematics instruction at all levels, K-University, is of paramount importance for the educational and economic future of our country. A variety of initiatives aimed at improving mathematics instruction have been undertaken during the past twenty years at both the K-12 and university levels. Concerted efforts toward reforms have been occurring at the K-12 level since the publications of the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). At the university level, there have been efforts at reforming calculus and other mathematics courses to be more student-focused (Hastings, 1997). Such reforms have not been without debate (Silverberg, 1999). While there have been reported gains in student performances in some studies, there is still much work to be done if we are to meet our national goals for achievement in mathematics.

Improvements must be made in the mathematical content knowledge of both students and teachers of grades K-16. A variety of data sources (e.g., The Third International Mathematics and Science Study, Mullis, et.al., 2003) have concluded that many elementary and middle grades mathematics teachers do not have deep enough knowledge of mathematics to teach it in a conceptual way. Not only teachers, but many K-16 students do not exhibit the mathematical competencies needed to function effectively in today's economy. At the university level this is often shown by the number of sections of remedial mathematics courses that are taught (McCray, et.al., 2003). Thus, for many students the typical, lecture style of mathematics instruction is not always effective. For university faculty, there needs to be a greater awareness of the specific mathematical content needed to prepare future teachers, as well as knowledge of methods of instruction aimed at reaching all college students.

To address some of these issues, a partnership involving two institutions of higher education (the University of Alabama at Birmingham (UAB) and Birmingham Southern College (BSC)), nine area school districts, and a non-profit mathematics organization (the Mathematics Education Collaborative (MEC)) was formed. This partnership sought and was awarded funding by the National Science Foundation for a Math and Science Partnership, the *Greater Birmingham Mathematics Partnership* (GBMP) (Cooperative Agreement #0632522). At UAB, faculty from mathematics, education, and engineering are participants. While at BSC, faculty from mathematics and education are participants. Participants from the school districts include K-12 teachers and administrators. Among its goals, this five year project included a goal of increasing the content knowledge of pre-service and in-service teachers of mathematics in grades K-12, with a special emphasis on serving grades 5-8 teachers. It has also sought to impact university mathematics courses and instruction.

One of the partners, MEC, based out of Bellingham, WA and Portland, OR, had already established a track record of implementing inquiry-based mathematics courses for K-12 teachers. A series of nine-day courses, taught in a workshop fashion, had been developed to enhance teachers' knowledge of mathematics. The mathematics content in these courses consists of the "big mathematical ideas" of numerical reasoning, algebra, geometry, probability, and data analysis as identified in NCTM's *Principles and Standards for School Mathematics* (2000). The focus is on developing conceptual understanding of the mathematics as well as the ability to put mathematical ideas and skills to work in solving complex and relevant problems. All courses attend to the process strands of problem solving, reasoning, making connections, and communicating.

The courses model a learning environment that optimizes the learning of quality mathematics and will meet a broad range of learner needs. They allow access for those teachers who fear and/or dislike mathematics, yet challenge all participants. The courses offer teachers opportunities to struggle with complex, rich, and expandable mathematical tasks with the potential of arriving at the development of concepts that are foundational to the field of mathematics.

These courses were brought to the GBMP and first officially offered to this group during the summer of 2005 and repeated during the summers of 2006 and 2007. Course participants included K-12 teachers and pre-service teachers, as well as university faculty in mathematics, education, and engineering. These summer courses have been very successful as evidenced by participant surveys and objective data. For example, of the 143 participants in the 2005 summer course, *Patterns, Functions, and Algebraic Reasoning*, 98.6% agreed or strongly agreed that the course improved their mathematical skills and understanding. Objective measures have also shown growth in their content knowledge.

The summer courses were followed up by seeking to establish university mathematics courses that parallel the MEC/GBMP courses both in content and in methods of instruction and evaluation. This paper seeks to describe the implementation of two of these courses at the university level, as well as a related effect on a core curriculum course in finite mathematics intended for general population university students.

Method

To achieve one of our project goals, to promote inquiry-based mathematics instruction at the university level, we sought to revise two existing courses in mathematics for elementary school teachers to fit the content, pedagogy, and assessment methods of two of the MEC courses.

These two courses were re-titled, MA 313- Patterns, Functions, and Algebraic Reasoning, and MA 314- Geometric and Proportional Reasoning. These courses were to be a part of a new middle grades mathematics certification program, as well as a part of the 12 hours of mathematics required of pre-service elementary school teachers. The details of the formulation of MA 313 are given below. The reformation of MA 314 followed a similar approach.

The evidence of the effectiveness of the summer GBMP courses in increasing content knowledge in in-service and pre-service teachers supported the development of the MA 313, *Patterns, Functions, and Algebraic Reasoning*, course to be taught at the university level. This course was first taught at UAB during Fall 2005, and it has been a regular course offering since then. The audience was primarily undergraduate, early childhood and elementary education majors (ECE, ELE) taking the course as part of their 12 semester hours of mathematics coursework needed for certification. There were also a few students taking the course as prerequisites for entering the non-traditional fifth-year program in ECE/ELE. Students had previously completed at least a course in intermediate algebra as a prerequisite to taking the course. One purpose of the UAB courses was to increase students' understanding of the basis of algebra and algebraic reasoning that they may not have gleaned from the traditionally taught algebra courses taken in college and high school. Other purposes included increasing students' abilities to reason and communicate mathematically. Another purpose was to expose students to inquiry based methods of teaching and learning mathematics. The summer workshop courses (*Patterns, Functions, and Algebraic Reasoning*) were attended predominantly by inservice K-12 teachers. The purposes of the workshops were similar to the purposes for the UAB classes with the exception that students in the workshops were not taking the course for a formal university mathematics credit. The summer workshops exposed these practicing teachers to mathematical problems and experiences that they may never have experienced before.

Students taking the MA 313 course at UAB during the Fall 2005 were predominately undergraduate, ECE/ELE majors. The students were predominately typical-aged undergraduates (in their 20's) with a handful of non-traditional aged students. More or less this was a required course for students to take and thus participation was not entirely voluntary. The UAB students had from 1 to 3 college mathematics courses (taught by lecture methods) in their backgrounds prior to taking MA 313. Almost none of the students had taken a mathematics methods course and none were middle or high school math teachers (the course now has middle and high school majors as members). On the other hand, the audience for the summer courses was predominantly in-service teachers with a much greater age variation among the participants. The teacher group varied in mathematics backgrounds with a few teachers having only one mathematics course in their former academic training and others full university mathematics majors. There were K-5 teachers of mathematics along with middle and high mathematics teachers. Summer participants had from one to more than 20 years of teaching experience. For the most part, these teachers volunteered to participate in the summer workshop courses. Most were motivated to attend the workshops by their desire to improve their teaching of mathematics, as well as to enhance their content knowledge of mathematics.

Because of differences in the student populations, modifications were necessary in MA 313 to adjust the course from the summer workshops and to adjust the course from the way it had been taught by the same professor for the previous 6 years. In attempting to convert an inquiry-based, workshop course into a formal university mathematics course, a number of modifications had to be made. The first, a non-trivial, change was a modification in the time frame for the delivery of the course. The summer workshops lasted 9 consecutive days and met for 8 hours per day. Thus, there were approximately 72 contact hours in addition to hours spent at night on homework outside of the daytime work. The MA 313 course taught during the Fall 2005 at UAB met twice per week for 15 weeks, in 75 minute sessions, plus a final exam time, for about 40 contact hours. Additional homework time was spent working on tasks outside of class time. There were both advantages and disadvantages to the modified times for the university courses. The course taught during the 15 week semester had the disadvantage of a shorter time span per session than the summer workshops for teachers. Activities that were allowed to progress for up to 2.5 hours in the summer workshops had to be spread over 2 sessions (a 3 day period) in MA 313. Thus, the momentum built up in group work would be stunted by having the discontinuity of time. On the other hand, individual work on problems could be spread out over 15 weeks as opposed to the time crunch of the 9-day workshops.

A major adjustment from the summer workshops to a university mathematics course was the necessity to assign grades for the MA 313 classes. While active participation was an expectation for both groups, the assessment of individuals' performances on tasks was even more crucial in MA 313. A grading system had to be devised that discriminated between various levels of performance in MA 313. Although summer workshop participants received a 'grade' on their portfolios, there was not the pressure of assigning grades for university credit. A problem solving rubric was used to give feedback to students. This rubric was used by MEC in the summer workshops and was adapted from the Oregon Department of Education's 1995-2003 Statewide Assessment. Students completed 2 menus of mathematics tasks. A selection of 2-3 problems from each menu was assessed in detail using the rubric. Similarly, a midterm and a final performance task were assessed using the rubric. Additional measures of students' abilities were gained through classroom observations, student presentations to the class, and one-on-one conversations with students. Also, students developed a mathematics portfolio and included a reflective summary of their experiences in the course. Project evaluators had designed a rubric for assessing summer participant portfolios and this was used within the course. Final grades were assigned based on students' demonstrated proficiencies of course objectives as reflected by the artifacts above.

The content and teaching approach for MA 313 were radically different from previously taught sections of the course. Although there was an emphasis on problem solving in the former courses, the problem solving was a lot more guided by the instructor as opposed to the student-centered problem solving of the revised course. The revised course placed the students in a more active role for solving problems and for sharing their work with others. The content of the revised course was much more focused around fewer topics than previous versions of the course. Whereas, former courses had focused on problem solving topics, patterns and functions, and topics in geometry, the revised course emphasized significant extended tasks that covered fewer concepts, but dealt with these in much greater depth. There was a unifying theme of algebraic reasoning and connections between algebra and geometry.

Another major shift in the revised course from the former methods was the emphasis on students' writing and explaining their thinking. While some writing had been included in former versions of the course, it was never with the same intensity as the revised course. Students were expected to explain, using written sentences, diagrams, equations, and graphs, their solutions to all problems. There was a shift from an emphasis on just getting a correct answer to the expectation that a correct solution must include a valid justification.

A final adjustment worth mentioning was the attempt to make students take more responsibility for their own learning. The practice of responding to students' questions without directly providing an answer was intended to help students become independent thinkers. By asking students questions and listening to their responses and having them explain their attempts at solutions, students were guided toward answering their own questions. Also, collaboration with peers on problems was another way of assisting them in developing their own understandings.

During the Spring semester of 2006, the second reformed course, MA 314 *Geometric and Proportional Reasoning*, was taught. Its formulation and delivery went through a similar process as that detailed for MA 313.

These efforts to reform existing courses have led to a re-examination of a course in finite mathematics that was taught during the fall semester of 2007. This course serves as a core curriculum course for non-science/math majors. In recent years, it had been taught in a large lecture section to 300-400 students using a lecture style. One of the professors for this course had been through two of the GBMP summer courses and sought ways to incorporate this inquiry-based approach into the teaching of the finite mathematics course. Though not a total transformation from the lecture approach, the course was revised to have smaller sections and uses cooperative group problem solving. Students are placed more in charge for their own learning by working together on problem sets. There is more emphasis on students writing and making oral presentations. Feedback was gathered from students throughout the semester to determine their views on this approach to teaching.

In addition to reforming courses, the project also sought to gather data on views of higher education faculty on the influence of their participation in GBMP courses on their roles as university instructors. Focus groups of faculty from mathematics, education, and engineering were held and qualitative data was collected by project evaluators. Evaluators sought to identify factors which promote or impede the implementation of inquiry-based methods as well as positive aspects of implementing these methods.

Another part of our methodology was to observe university faculty and rate their instruction using the Reformed Teaching Observation Protocol (RTOP) (Sewada, 2002). A website describing the RTOP states, “The RTOP was developed as an observation instrument to provide a standardized means for detecting the degree to which K-20 classroom instruction in mathematics or science is reformed per the national science and mathematics standards.” (http://physicsed.buffalostate.edu/AZTEC/RTOP/RTOP_full/) The RTOP consists of five subscales: Lesson Design and Implementation, Content: Propositional Pedagogic Knowledge, Content: Procedural Pedagogic Knowledge, Classroom Culture: Communicative Interactions, and Classroom Culture: Student/teacher Relationships. Project evaluators used the RTOP in their observation of six “traditionally” taught mathematics classes and four classes taught using inquiry-based instruction. Details of these observations can be found in the results section which follows.

Results

After approximately two years of attempting to implement inquiry-based instruction in university mathematics courses, project data has begun to show positive outcomes and challenges which remain to be addressed. Findings shared below will relate to the impact of course reforms on students and instructors. MA 313 has been taught more frequently than any of the other reform efforts and most of the results below are derived from this course.

For MA 313, the translation of the nine-day summer courses/workshops into a full semester course has worked fairly well. The sequencing of course content has been adjusted to accommodate two, seventy-five minute class periods for 14 weeks as opposed to nine, eight-hour days during the summer. Of note, the actual direct contact of students with instructors is greater during the summer sessions, about 72 hours, than the full semester courses, about 40 hours. There is of course more time for out of class work over the full semester courses. Some extended tasks that may require two hours have to be split over two class days in the semester format as opposed to being completed in a single day in the summer. Nevertheless, students achievement between the two formats appear similar.

From the Fall 2005 MA 313 course, a sample of 15 course portfolios were reviewed using a rubric developed by the evaluation team to assess the summer course portfolios. The dimensions included: Problem Translation- the ability to identify key mathematics concepts involved in a problem; Mathematical Procedures- the ability to use appropriate mathematical procedures to solve a problem; Productive Disposition- the demonstration of persistence in problem solving; Inquiry and Reflection- the usage of metacognition to reflect on process used toward successful and unsuccessful attempts at a problem; and Justification and Communication- the ability to make sense of problems mathematically and to explain. Median scores on the portfolios show that students in the course were functioning at a proficient level in terms of the mathematics dimensions described in the rubric. Student performance suggests depth of understanding of the mathematics presented in the class as well as proficient ability to communicate understanding of mathematics to others.

Table 1. MA 313 Portfolio Scores Fall 2005

N = 15	Median	N Incomplete	N Emerging	N Proficient	N Expert
Problem Translation	3.00	0	4	6	5
Mathematical Procedures	3.00	0	2	9	4
Productive Disposition	3.00	1	1	9	4
Inquiry and Reflection	3.00	0	3	8	4
Justification and Communication	3.00	0	5	6	4

The evaluation team met with the MA 313 instructor on several occasions to discuss the successes and challenges he has encountered in implementing the Patterns course at the university level. In addition, he was asked to describe those challenges and successes in writing.

His reflections follow.

Students' Understanding of Mathematics. In former versions of this course, traditional measurements by quizzes and tests found the majority of students were capable of mastering basic skills and solving problems by following procedures that had been demonstrated. Course grades were typically good with the majority of students earning A's and B's. Students were generally seen as successful, yet their deep understanding of concepts and their abilities to communicate and clearly reason through solutions were not so clearly evident. Improvements in assessments and providing more engaging problems and tasks were needed.

The introduction of newer more demanding tasks helped to reveal more about students' understanding of mathematical concepts. These tasks required going beyond just finding a solution to a relatively low level problem. The problems often have multiple parts and extensions aimed at making students go deeper into the mathematics. The assessment of these tasks used a rubric that placed an emphasis on explanations and communications of mathematics by multiple means as opposed to simply rewarding an answer.

Students' responses from the beginning of the course to the end of the course showed substantial improvements among most students' conceptual understandings. In the beginning, most students had little understanding of what it meant to explain and justify their solutions. By the end of the course a majority of students could explain their solutions using a combination of words, symbols, equations, diagrams, tables, and graphs. For example, early in the course many students could identify linear patterns and generate a sequence to arrive at a specific term in the sequence. Some could even write an algebraic expression that represented a generalization of the results. However, very few students could explain why their solution was correct, nor could they connect it to a geometric interpretation of the linear relationship. By the end of the course, almost all students could make reasonable connections to the geometry of linear problems and offer some explanation of why their results were valid. Many made stronger connections than they had ever made in regular algebra courses such as intermediate algebra. The use of performance tasks rather than traditional tests allowed students to demonstrate abilities that were never formerly evident. In reflective journal writings, most remarked on their increased abilities to understand a problem and its solution. They expressed increased confidence in their mathematical abilities.

Students Abilities to Communicate Mathematically. As stated earlier, mathematical communications skills were a fundamental part of students' development and were a significant target of assessments. The rubric for scoring tasks had 5 major components: Conceptual Understanding, Processes and Strategies, Verification, Accuracy, and Communications. In truth, communication is an aspect of all 5 components. In Conceptual Understanding, students were required to translate the task and express the major mathematical concepts in words. Processes and Strategies looked for pictures, models, diagrams, and words in students' explanation of their methods. Verification required a defense of a solution. Accuracy focused on the degree to which an answer was mathematically justifiable and supported by the work.

For most students, this was initially very challenging. They had never been required or expected to go into detailed explanations. The idea of writing in a mathematics course was a completely foreign notion. Their initial results demonstrated limited awareness of mathematical communications. Through guided practice and feedback, most students dramatically improved in communication skills. They progressed from writing one or two sentence explanations, or nothing, to writing one to two page explanations for problems. They were able to combine words, with pictures, charts, and graphs to spell out their solutions.

A focus group conducted with faculty members from UAB and BSC mathematics, engineering, and education departments focused on the benefits of inquiry-based mathematics instruction and the challenges of incorporating inquiry-based mathematics instruction at the university level. Participants noted the importance of GBMP inquiry-based instruction's focus on depth of understanding rather than a breadth of knowledge, focus on exploration instead of homework and lecture, the building of self-esteem and productive disposition among students, and the value of dissonance and confusion in the face of difficult problems. All of the participants who had been part of the 2005 summer courses mentioned the difficulty they faced, both in the summer course and in the courses they taught, in trying to refrain from telling students the answers to questions and allowing them to struggle with the problems on their own. Yet, those same participants emphasized the value of that struggle as a learning tool.

When discussing how to incorporate inquiry-based instruction into the mathematics courses to be reformed as part of the GBMP, participants highlighted several challenges, including securing support and advocacy from all faculty members and outside pressure and mandates to cover a large body of content, which they believed to be counter to the principles of inquiry-based instruction. As well, focus group participants considered class size, particularly in introductory undergraduate mathematics courses, to be an obstacle to implementing inquiry-based instruction.

Project evaluators collected data on university mathematics instruction during 2005-2006 using the RTOP. Six traditionally taught class sessions and four inquiry-based class sessions were examined using the RTOP. Scores on each subscale of the RTOP range from 0 to 20 with 20 indicating a very strong correlation with national teaching standard recommendations such as those by the NCTM. Table 2 shows the results.

Table 2. RTOP Scores for UAB Mathematics Faculty 2005-2006

RTOP Categories (maximum score on each subscale is 20)	Traditional UAB Courses (n=6) Median (Range)	Revised UAB Courses (n=4) Median (Range)
Lesson Design/Implementation	1 (0 - 3)	14 (11-15)
Propositional Knowledge	3 (3 - 6)	11 (10-12)
Procedural Knowledge	2 (0 - 6)	14 (14)
Communicative Interaction	1 (0 - 3)	13 (10-15)
Student/Teacher Relationships	2 (0 - 7)	14 (12-14)

Results show a substantial difference on the RTOP between the traditionally taught classes and the inquiry-based classes. When interviewed by project evaluators, instructors in reformed courses made these comments about the benefits of inquiry-based methods: there is a focus on exploration instead of lecture; there is a building of self-esteem and productive disposition in students; students have deeper understanding of content; students show improved ability to communicate mathematical thinking; and students show improved problem-solving abilities.

Finally, we present the preliminary results from efforts at reforming a finite mathematics class during the Fall of 2007. Prior to the Fall of 2007, the course had been taught as a single section to 300-400 students in a large lecture hall. There were graduate teaching assistants to support student learning, but the primary method of instruction was lecture. In the Fall of 2007, the course was broken into smaller sections with two of these sections taught by an instructor who had been through GBMP training. Each of these two sections had about 40 students. Lecture was supplemented by group problem solving sessions in which the students took ownership of the learning. Students were randomly assigned to groups of four. The groups were given problem sets and after initial instruction were to complete the problems as a group.

Periodically, students were asked to present solutions to problems. These solutions were debated among class members and alternative solutions were sought. Students were expected to communicate their results in thorough written explanations as well as these oral presentations. Students were asked a number of times during the term to provide written feedback on the course as it progressed. Just over 50% preferred this inquiry form of learning to the traditional way of teaching. Those speaking positively cited the opportunity to work with others and have peer support during problem solving as being beneficial. They also said the course was more interesting when they were actively involved and many felt they learned the material to a greater depth. Other students preferred a more traditional form of instruction. They felt that in this form of teaching you had to "teach yourself". They view good instruction as having a professor explain each step of the solution of a problem in detail. Some did not feel group work was always an effective way to learn.

These feeling expressed by the finite math class had been echoed in focus group interviews with students taking the first reformed course, MA 313. However, students who had taken two reformed courses, MA 313 and 314, grew in their support for the method. They stated that: they learned much more about mathematics; in some ways it was more difficult than traditional method (have to think more, do more active work); in some ways it was easier than traditional method (no memorizing or cramming, no longer mysterious); it was more rewarding to find answers yourself; and they more confident about their mathematics abilities. Some were still cautious about being graded in this method of instruction and not everyone felt that they learned best during group work. Thus, there appears to be an increased comfort with the method of inquiry after taking more such courses, but not every issue is resolved.

Discussion

The results for this study to date are encouraging, yet they also leave challenges to be met. From a student learning point of view, instructors point to a greater depth of understanding of mathematical concepts using inquiry methods. While many students also, state that they feel as though they are learning more deeply, some, particularly those taking only one course, are still unsure that the method produces maximum results. These students may not recognize their mathematics growth to the extent that the instructor does. They may still believe that if they don't find the correct answer in a short time then they have not been successful. The evidence is clear that almost all students are much better at communicating their understandings in writing and orally than they were in traditionally taught courses.

Decreasing mathematics anxiety is a desired outcome of the summer courses as well as the university courses. While no formal measures of math anxiety have been used, anecdotal evidence from students' reflective writings indicate that for many teachers in the summer courses math anxiety is reduce. While they struggle with problems and experience frustration, in the nine-day format they are able to see the positive things that have happened and these out weigh the negative emotions. It is not as clear that this transformation is as dramatic in the full semester courses. Because the courses are spread over 14 weeks, there does not appear to be the intensity and the "ah-ha" that teachers experience during the short summer courses. The participants in these two groups are also quite different in their levels of maturity with the summer group mostly being in-service teachers who are typically older and more experienced with different methods of teaching. For students who take two full semester reformed courses, their anxiety does appear to lessen. Their comfort with the method appears to grow with experience.

From the perspective of university professors, the benefits of the approach surpass the drawbacks. The depth of student learning appears greater in the inquiry format. The topics are narrower in focus and time can be spent exploring problems more intensely than in courses where covering many topics is an expectation. This also points to an area of concern for those courses that are expected to deliver a lot of content that will be needed in future courses. Thus, the age old question of depth versus breadth reappears. Instructors are pleased with the level of thinking that students exhibit in the inquiry approach and find classroom exchanges energizing. Students improve in their ability to express themselves in written form. This does lead to another concern of some instructors, the issue of grading. While rubrics help to make the grading more objective, the time to assess open-ended problems is much greater than grading traditional closed-form problems. The issue of grading for process and product and how to weigh these two dimensions takes time and thought.

Finally, this study has led to some emerging ideas and questions that warrant further study. It does appear that inquiry-based instruction becomes easier over time for both instructors and students. Students who have experienced traditional instruction for twelve or more years in which they expect a skilled instructor to reveal all truths to them have to adjust to new classroom expectations. This takes more time for some than others. Another clear point is that, in order for this transition to an inquiry based approach to be successful with university students, a supportive classroom environment must be established. Instructor must invest time in initial courses using such methods to explain what they are doing in terms of their teaching method and why they are using this method. This initial investment takes away class time, but it pays off by improved efficiency and increased student morale.

Questions that remain include, can all students make the transition to a new way of learning after experiencing, and possibly being quite successful at, a traditional approach to instruction for twelve or more years? How do we deal with students who resist this method of instruction and reflect this in evaluations of instructors? How can university faculty be encouraged to and rewarded for investing time and energy into implementing such methods in courses they teach? For which courses do the benefits of depth of learning outweigh the need for breadth of coverage? With the demands of promotion and tenure for university faculty, these questions deserve consideration.

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