WHAT IS MATHEMATICS? STUDENT AND FACULTY VIEWS

Jacqueline M. Dewar
Loyola Marymount University

Abstract. In this short paper we report the results of a preliminary study that explored two questions: (1) How undergraduate STEM students' understandings of mathematics compare to an expert view of mathematics, and (2) whether a single course can enhance future teachers' views of mathematics. Written responses to the question *"What is mathematics?"* from 55 STEM students, 7 future teachers, and 16 mathematics faculty revealed that most students see mathematics as being the study of a list of topics (primarily numbers) and applications. On the other hand, for faculty, mathematics encompasses pattern, proof, logic, abstraction, and generalization in addition to applications. Hardly any students (initially) considered mathematics to involve abstraction or generalization. The responses gathered from the 7 future teachers before and after a particular course along with additional evidence from the study indicate that a single course can nudge future teachers toward a more expert view of mathematics.

1. Introduction and Background

A small study conducted at a medium-sized comprehensive university examined and compared student and faculty views of mathematics. The specific questions the research explored were: (1) How undergraduate STEM students' understandings of mathematics compare to an expert understanding of mathematics; (2) whether a single course can move future teachers' views toward those held by experts; and (3) whether changed views were stable or would revert.

Schwab (1964) argued the importance of undergraduates, and especially future teachers, learning underlying structures and principles of their majors. More recently Huber and Hutchings (2005) describe case studies of scholars from a variety of disciplines examining key questions about students' understanding of their subject matter areas while Riordan and Roth (2005) address teaching disciplinary-specific practice. Researchers in mathematics education (Schoenfeld, 1985) and in K-12 education (Alexander, 2003) have studied and described novice and expert approaches to mathematics and other subjects.

2. Significance of the Question

Mathematics faculty would be expected to have an intrinsic interest in what views undergraduate mathematics majors hold of their discipline by graduation. Accreditation and program assessment requirements certainly invite and encourage departments to investigate students' understanding of their discipline. Furthermore, the importance of this question for future teachers can hardly be exaggerated, since what views they hold will influence their choices about what content they teach and how they approach it, especially in the face of State Standards and "No Child Left Behind."

3. Methodology

The study first examined descriptions of "mathematics" from 55 undergraduate STEM majors (sophomores, juniors and seniors) and compared their descriptions to those of 16 mathematics faculty at the same institution. The descriptions were obtained from a survey that contained the following item:

Briefly describe what you think mathematics is….

Later the study gathered similar descriptions from 7 future math teachers (2 pre-service secondary and 5 pre-service elementary) at the beginning and end of a somewhat unusual course that addressed both mathematical content and gender equity issues in mathematics through the examination of the lives and contributions of women mathematicians from Hypatia to Emmy Noether. The 78 written responses were analyzed using grounded theory with open coding (Glaser, 1992). Additional evidence included a reflective writing assignment by the future teachers at the end of the course, and interviews conducted with three of the pre-service elementary teachers about 18 months later.

The six categories that emerged from the data are contained in Table 1 along with an example of a response for each category.

Table 1 Emergent categories of responses to *What is Mathematics?* with examples

Category	Example
Numbers (including computation)	*the study of numbers*
Listing of topics (could include numbers)	*algebra, pictures, numbers, everything encompasses math*
Applications	*a "mental gymnastic" that helps you solve real-world and theoretical problems*
Pattern/Proof and/or Logic	*the search for and the study of patterns*
Structure/Abstraction/Generalization	*the analysis of abstract systems*
Other	*a language* or *everything is mathematics*

Some responses were assigned to two or more categories, for example, *the study of numbers and their application to real life* would be coded as **N** and **A.**

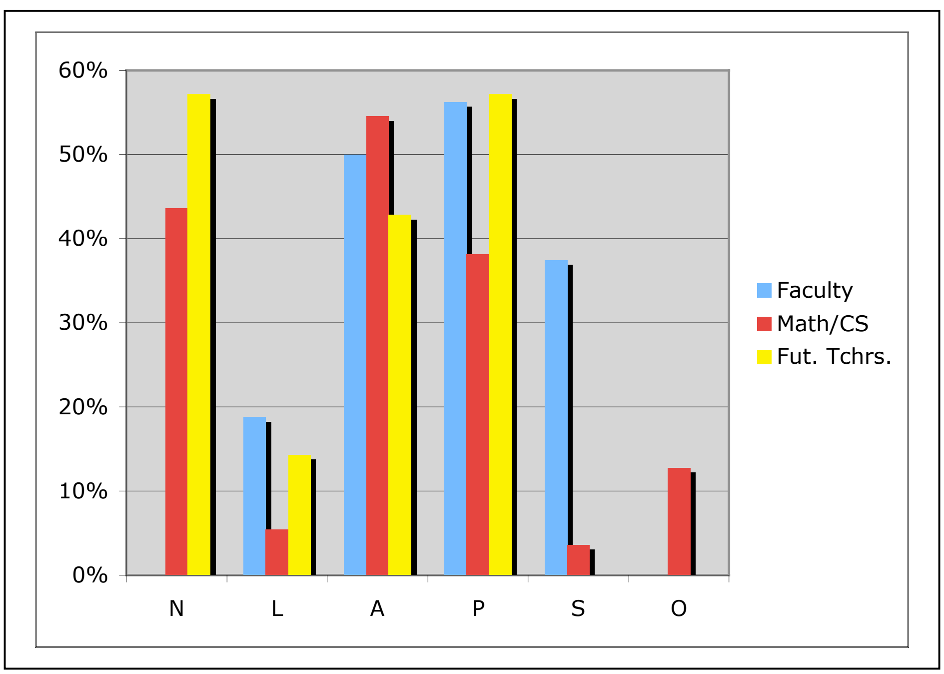
After completing the coding, the researcher observed that each of the emergent categories aligned with one of the four bases (content boundaries; skills and habits employed by practitioners; modes of inquiry; and purposes or outcomes for the disciplinary work) that Schwab (1964) described for classifying disciplines. Specifically, both **N**umbers and **L**ists of Topics are delineating content boundaries, while the **P**attern/Proofs/Logic category describes modes of mathematical inquiry, in particular, how mathematicians formulate and determine the truth of conjectures.

Structure/Abstraction/Generalization corresponds to skills or habits of mind practiced by mathematicians and the Applications category relates a purpose for doing mathematics. This alignment provides evidence of the appropriateness of these emergent categories.

4. Results

Figure 1 gives the comparisons of the responses by Faculty (in blue), Math/CS students (in red), and Future Teachers (in yellow) at the beginning of the course on "women and mathematics." Across the bottom are the categories N for Number, L for a Listing of topics, A for Applications, P for Pattern/Proof/Logic, S for Structure/Abstraction/Generalization, and O for Other.

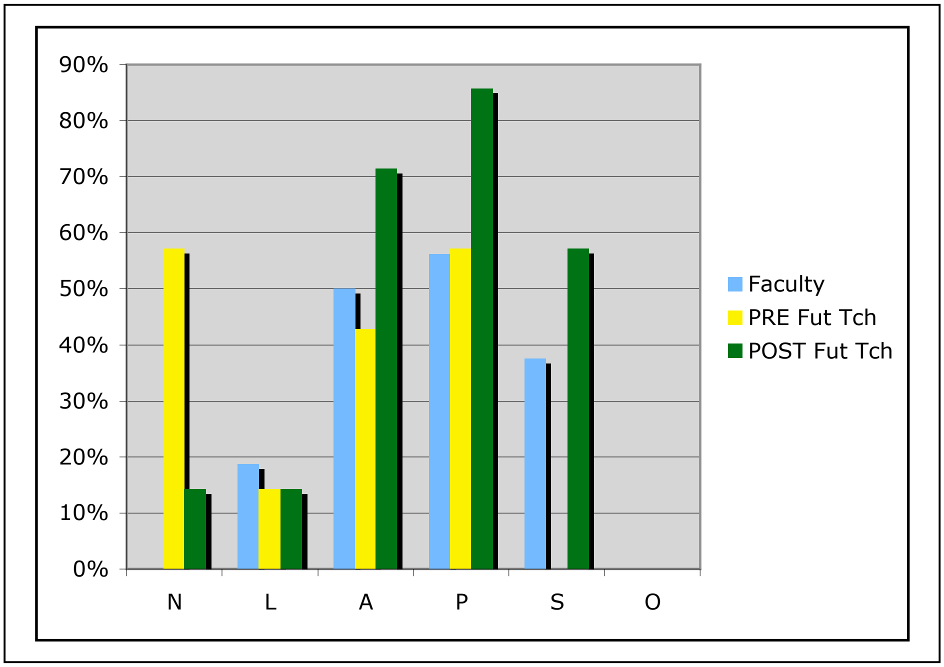
Figure 1 *What is Mathematics?* responses:
Faculty (blue), Math/Computer Science students (red), Future teachers (yellow)
N – Number, L – Listing of Topics, A – Applications, P – Pattern/Proof/Logic,
S – Structure/Abstraction/Generalization, O - Other



The results show that student and faculty views of mathematics differed considerably. One striking difference occurred with respect to the role of "Number" in defining the discipline. There were no faculty responses in the Number category, while that category tied for highest for future teachers and was second highest for math and computer science majors. Another striking difference occurred relative to Structure/Abstraction/Generalization. Faculty were almost alone in seeing mathematics involving structure, abstraction and generalization – important skills for developing new mathematics and for seeing commonalities across various mathematical domains. It is troubling that no future teacher response fell into that category. On the other hand, a higher percentage of future teachers initially identified patterns, proof, or logic as part of mathematics than math or computer science majors did.

Figure 2 contains the responses of the future teachers at the end of the "women and mathematics" course (POST - dark green). Comparing these to their responses at the beginning of the course (PRE - yellow), we see these future teachers shifted substantially away from a view of mathematics being about numbers and began to see that structure and abstraction are involved. (It may be worth noting the one student who maintained the view of mathematics being "about numbers" was not a native English speaker.) Comparing the future teachers POST responses (dark green) to the faculty (blue), we see that distributions are much more similar in shape. The dark green bars are taller because the students' responses at the end of the course were richer, that is each student touch on more aspects of mathematics in his or her response. (Future teachers POST averaged 2.3 categories per response, while faculty averaged 1.6 categories per response).

Figure 2 ***What is Mathematics?*** **responses:**
Faculty (blue), Future teachers PRE (yellow), Future teachers POST (yellow)
N – Number, L – Listing of Topics, A – Applications, P – Pattern/Proof/Logic,
S – Structure/Abstraction/Generalization, O - Other



There is additional evidence that something happened in that semester to change the seven future teachers' ideas about math. In a final reflection they were asked whether/how their view of mathematics had changed as a result of the course. Their comments indicated that they were more aware of the role of patterns in mathematics and the desirability of understanding underlying reasons. Eighteen months after the course, follow-up interviews with three of the pre-service elementary teachers indicated that two of the three students still held their richer views.

5. Conclusions, Limitations, and Directions for Future Investigation

In order to draw further conclusions regarding what might have promoted the “changes” in the future teachers’ views of mathematics, it is necessary to inspect the course more closely. This course examined the lives and work of nine women mathematicians from the 4th (Hypatia) to the 20th (Emmy Noether) centuries. Its goals were:

- To provide students an opportunity to experience “doing mathematics” in a supportive and cooperative environment and to encourage students to be more aware of their own mathematical thinking
- To investigate current gender issues related to women’s skills and participation in mathematics from elementary school through graduate school and women’s access to and participation in math-related careers
- To develop expertise in addressing equity issues related to K-12 mathematics education

Each mathematical topic discussed had some connection to the work of the nine women mathematicians, thus providing a broad list of topics that included number games and puzzles, conic sections, algebra, elementary number theory, functions, limits (calculus), geometry, infinite sequences, and some abstract algebra topics. These topics provided many opportunities to search for and study patterns and then to attempt to provide justification that the patterns did (or did not) continue indefinitely. As a result there was a great emphasis on mathematical epistemology, in particular, how mathematicians use inductive reasoning to discover followed by deductive reasoning to prove. Thus it appears that students’ views can be nudged toward more expert views by a single course, when an *intentional* effort to do so is made, and that these enhanced views may persist to some extent over time. The main implication to be drawn for teaching is that clarity of expectations and intentions very likely plays an important role in student outcomes.

This preliminary study was limited both in scope and by the small number of future teachers enrolled in the course that seemed to promote change in views about mathematics. The investigation provided a number of questions for further research, such as: What specific aspects of the course content or pedagogy contributed to the changed views of mathematics? How will future teachers' views of mathematics influence their classroom instruction? How do students form views of what mathematics is all about?

References

Alexander, P. (2003). "The development of expertise: the journey from acclimation to proficiency." *Educational Researcher*, 32(8), 10-14.

Glaser, B. (1992). *Basics of grounded theory analysis*. Mill Valley, CA: Sociology Press.

Huber, M. and P. Hutchings. (2005) *The advancement of learning: Building the teaching commons*. San Francisco, CA: Jossey-Bass.

Riordan, T. and J. Roth (eds.). (2005). *Disciplines as frameworks for student learning: Teaching the practice of the disciplines*. Sterling, VA: Stylus Publishing.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Schwab, J. (1964). "Structure of the disciplines." In G. W. Ford and L. Pugno (Eds.), *The structure of knowledge and the curriculum*. Skokie, IL: Rand McNally.