Innovative Methodologies: the Study of Pre-service Secondary Mathematics Teachers' Knowledge

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Introduction

The presented the study is an extension to the ongoing research on secondary mathematics teachers' knowledge. This study focused on the concepts of logarithms and logarithmic functions. Several research studies have confirmed that high-school and undergraduate students have a very poor knowledge of logarithms and logarithmic functions. Possibly, one reason for students' difficulties could be the teachers' insufficient knowledge of this subject domain. As of yet, there has not been research into teachers' knowledge of logarithms. This study was an attempt to fill this gap.

The deeper understanding of teachers' knowledge, particularly subject matter knowledge and related pedagogical skills, leads towards improvement of instructional approaches for more effective teacher training. The questions discussed in this paper are: What do the designed methodologies reveal about the nature of teachers' knowledge of logarithms and logarithmic functions? To what extent are these methodological tasks effective and useful as data collection tools for research in mathematics education? Moreover, the underlying basis for the choices contributed to the researcher's design of the innovative methodologies is offered.

Research Site and Context

The reported research took place during the secondary mathematics method course, *Designs for Learning Secondary Mathematics,* offered by the Faculty of

Education at Simon Fraser University. The duration of this course was 13 weeks, with meetings once a week for four hours.

When designing the tasks used in this study, I found myself in a dual-role position: as an instructor and as a researcher. As an instructor of the secondary mathematics method course, I hoped to create engaging activities and rich learning environments, where pre-service teachers would come into contact as closely as possible, with the real life situations of a mathematics teacher. These tasks would incorporate an implicit review of the mathematical content, while explicitly focusing on pedagogical implications. As a mathematics education researcher, I tried to construct methodologies that would reveal valuable insights about pre-service teachers' mathematical and pedagogical content knowledge; in particular related to logarithms and of logarithmic functions.

The data consisted of the accumulated participants' responses gathered from their completion of the two tasks: the *Job Interview*, peer-interview conducted, transcribed and analyzed by participants; and, participants' written responses in the form of *Math Play* scenarios. Both tasks were employed as ongoing learning activities during the method course. However, this report focused only on the data collected from the Job Interviews.

Task: *the Job Interview*

Pre-service teachers were invited to assume the roles of personalities in a fictional mathematical interaction between the head of a mathematics department and an applicant for the position of substitute teacher, who would cover for a 3 week leave. Topics for teaching included logarithms and logarithmic functions.

The interviewer had to do his/her best to verify and evaluate the candidate's knowledge and understanding of the mathematical content required to be covered. The interviewer's questions were to reveal the main essence of the topic. The candidate had to do his/her best to answer the questions and to demonstrate his/her competence.

Data sources associated with this task comprised the following:

a. Interviewer's rationale for the choice of the interview questions;
b. Interviewer's anticipated answers for his/her interview questions;
c. Transcript of the interview;
d. Interviewer's evaluation of the candidate.

The *Job Interview* was assigned during the third session of the course. This session was mainly focused on the role questions play in mathematics classrooms. Students were engaged in different activities, exploring the possibility of using questions for teaching and assessment purposes. Participants were invited to reflect and discuss two following readings:

O. Hazzan and R. Zazkis, Constructing knowledge by constructing examples for mathematical concepts, *Proceedings of the 21st International Conference for the Psychology of Mathematics Education,* Vol. 4 (1997), 299–306.

Mason, J.H. (2002) Minding Your Qs and Rs: effective questioning and responding in the mathematics classroom, *Aspects of Teaching Secondary Mathematics: perspectives on practice*, pp. 248-258 Haggerty, L. (ed.) Routledge, London.

The participants were asked to complete this task in three weeks. Several content topics were provided for the participants' choices, some from the senior secondary mathematics, and others from the junior secondary mathematics level. Preservice teachers were encouraged to challenge themselves, though it is my belief that in many cases their choices were based on the 'zone of comfort' and relevant knowledge of the material they possess. Nine pairs of teachers chose to engage in activities with logarithms, and all of these participants were mathematics majors. Only their responses were selected for this study.

In the peer interview assignment, preservice teachers were invited to impersonate the head of a mathematics department and a candidate for the position of substitute teacher in a fictional job interview. They were given a choice of the mathematical topics to use as a background for the conversation. The interviewer's main goal was to verify and evaluate the candidate's knowledge and understanding of the chosen mathematical content, the candidate's - to answer the questions, to demonstrate his/her competence.

It is important to mention that teachers could write the scripts of their interviews, long before actual interviews would take place, employ and consult any possible resources. They could complete this task on campus, or at the local library, or even at home. No constrains on the materials or the location were posed. However, preservice teachers had only 3 weeks to finish this assignment.

The main purpose of this task was to analyze the interviewer, person pretended to be a head of the math department. This decision was based on the premise that the participants' questions and intentions reveal what preservice teachers think is important in teaching. Thus, teachers were asked to provide a written explanation of their choices of interview questions, elaborating on the purpose and the type of each question. They also were asked to prepare anticipated answer(s) for their interview questions, submit the transcript of the interview and the detailed evaluation of the 'candidate'.

To participate in this particular task, preservice teachers had to work in pairs. The partners had to conduct a job interview. One of them played the role of the head of a mathematics department, (referred to as an interviewer); the other impersonated a candidate for a teaching position, (referred to as an interviewee). For the purpose of this study, the attention and analysis are directed at the interviewer, his/her choices of questions and tasks, assessment strategies, and also his/her evaluation criteria of the candidate. A total of 10 participants out of 47 preservice teachers chose to engage in the activity involving logarithms.

Theoretical Considerations

> "…research methodology is not merely a matter of choosing methods and research design, … methodology is about the underlying basis for the choices that are being made…" (Goodchild & English, 2003, p.xii)

In contemporary mathematics education, one encounters different ideas, methodologies, and various approaches to investigate research questions. For example, clinical interviews and questionnaires are the most commonly used instruments for collecting data. Some others are: journaling (Liljedahl, [in press]; Flückiger, 2005), error activities (Borasi, 1996), technology based tasks (Dubinsky, 1991; Weber, 2002), and example generation tasks (Bogomonly, 2006; Rowland, Thwaites & Huckstep 2003; Zazkis & Laikin, 2007). A detailed account on a variety of research methods can be found in Goodchild & English (2003). The research in mathematics education confirms that different methodologies and approaches allow for the creation of situations that enable researchers to collect more diverse data.

In the following, I present the reader with the discussions of the research ideas from an existing body of educational research that proved to be valuable in designing, understanding, and analyzing the described research task.

Role-playing

The multidisciplinary studies of aspects of the real world in the physical and social sciences over the past century had lead to the articulation of important new conceptual perspectives and methodologies that are of value to both researchers as well as professionals in these fields. The simulation of real life problems has become one of the popular teaching methodologies in many subject areas. There were several studies that reported on the effectiveness and importance of simulation activities in language education, science education, and in education in general (Blatner, 1995, 2002). For the purpose of this study, the following discussion will be focused on findings that view role-playing as a less technologically elaborate form of simulations activities, where participants personify somebody else for a particular reason.

According to Blatner (2002), role-playing is a good inquiry approach. It possesses two distinct properties: it transforms the content from information into experience, and it exposes how the person would act when placed in other person's situation (it could be either imagined or the act of pretending). Blatner claims that role-playing is an effective method for developing the ability to think about the ways one thinks: metacognition. It is also shown that role-playing is a powerful teaching methodology. This methodology helps students to understand the nature of education. Even though the research on role-playing was conducted with drama students, it seems that this approach can provide the pre-service secondary mathematics teachers with an opportunity, which was lacking in the previously mentioned error activity, to experience the understanding of the subject matter from someone else's perspective and position.

The role-playing approach was used in both research tasks for this study. In the first task, the Job Interview, two pre-service teachers were to play roles of the head of the mathematics department and the candidate for a position of a substitute teacher. They were to discuss a particular mathematical content, and the head of the mathematics department had a goal to assess and evaluate to candidate's content knowledge; whereas, the candidate was to convince the interviewer about the expertise he or she possessed. Being placed in the role of the department head, the pre-service teacher had to come up with questions; the answers to those would be the most revealing of the candidate's subject matter proficiency. The interviewer would have to react to the received answers immediately on the spot, and if necessary, adjust their line of inquiry accordingly. This interview task allows pre-service teachers to explore and consequently expose their creativity and imagination. Their knowledge of the subject matter and pedagogy had to be verbalized and transformed into questions, prompts and activities in which the candidates were to engage.

Questioning for Interviewing in Education and Mathematics education

> We all ask questions, all the time. However, how we ask questions and how (we) reflect upon answers provided will determine what we say we 'know' and 'believe', will influence our relations with others, the world and our actions (Schostak, 2006, p.8).

In the recent years with the development of qualitative research, interviewing became one of the most frequently used methodologies in education. From one perspective, interviewing students has become a popular method of data-collection. The researcher's analysis of the responses collected in the interview creates the foundation of many studies in mathematics education. From another perspective, interviewing is used in teaching practice for the purpose of testing.

Ginsburg (1997) identifies the interview method as a powerful technique for gaining insight into a child's thinking. Through a critical analysis, he argues, "traditional research and assessment research and assessment methods involving standardized administration are often inadequate for understanding the complexities and dynamics of the child's mind" (Ginsburg, 1997, p.30). He establishes the advantage interviewing has to offer, and demonstrates the effectiveness of the interviewing.

According to Ginsburg, the clinical interview is a form of social interaction "built around actors' (an adult interviewer, and a child interviewee) ideas", "considers participants' goals", and also "comprises acts". When focusing on the interviewer, the primary concern is the art of questioning (Ginsburg, 1997, p.75). Ginsburg suggests that the beginner interviewers have to be very conscientious when selecting and preparing tasks and questions for an interview. He offers advice on the use of theoretically meaningful tasks (beginning with easy, familiar ones, and following up with the more challenging), engaging to the child (making the tasks specific, not vague or unclear), the usage of various types of problems (variety of problems, variations on the same theme), and being open-ended (giving a child freedom to respond, and finally, the allowing of an expression of personal ways of thinking).

In mathematics education, the topic of questioning is considered to be very important, and receives a great deal of attention. In *Questions and Prompts for Mathematical Thinking* (1998), Anne Watson and John Mason have put together a collection of convincing and challenging questions which are designed to draw out students' mathematical thinking. Besides a collection of questions, they present a framework for generating a wide range of mathematical questions and prompts which can be used by the teachers (for the purpose of development of their own approaches to mathematical content, and finding out more about their students' undertakings) and by the students (to make sense of mathematics and to question each other and their teachers). They illustrate how learning and teaching situations might be enhanced when using 'good' questions. The authors believe, "questions …are intended as a source of inspiration and as an aid to change" (Watson & Mason, 1998, p.3). For Watson and Mason, questioning is a social and psychological activity, where a student's experience frames her/his view about the subject. Therefore, the authors intend to explore mathematical questions as "prompts and devices for prompting students to think mathematically, and thus becoming better at learning and doing mathematics" (Watson & Mason, 1998, p.4), to help students communicate mathematics.

An interesting approach to questioning is taken by Zazkis and Hazzan (1999). These authors looked at how researchers in mathematics education choose or design their questions for interviews. Reviewing different studies and reflecting on their own, allowed the authors to identify several types of questions most commonly used: performance questions, unexpected "why" questions, "twist" questions, construction tasks, reflection questions, and "give an example" tasks. However, their study did not stop at classification of the interview tasks and questions. Zazkis & Hazzan (1999) extended their investigation to elucidate the "whys". The authors sought an explanation of rationale for the design of interview tasks. Not surprisingly, "all our interviewees have admitted that their criteria for the choice of the interview questions were implicit and hard to elicit" (Zazkis & Hazzan, 1999, p.435). However, the investigators were able to deduce several interrelated themes from the collected data: theoretical analysis, subject-matter analysis, researcher's practice and researcher's personal mathematical understanding. The theoretical analysis-based design was structured upon a particular learning theory, where participants' responses "serve as identifiers of different aspects or stages presented by a theory." The second theme for collecting information is self-explanatory - investigations into the subject-matter knowledge. The "practice rooted" design was exemplified in the report as the design of questions for the purpose of isolation and determination of the sources of observed participants' difficulties in a learning situation. And the last group of research designs centered on the researcher's personal understanding of the concepts involved, and is guided by his/her view of the important features, components, and connectivity within related topics. Although these particular findings and their influence on the design of the tasks will be discussed once again in a later section of this chapter, I wish to emphasize that this work has helped me on a personal level, to think in terms of the choices pre-service teachers make when they prepare their questions for an interview. Taking this into consideration, I required as a condition for the completion of this task, that the interviewers submit a written explanation of the reasons behind their choices of interview questions.

To create a successful teaching situation in pre-service teacher education, one might consider an interview as a special type of role-playing activity. Indeed, the traditional interview setting includes an interviewer (the teacher), an interviewee (the student), and the content that is usually prepared by the interviewer and organized into questions and tasks that are aimed to prompt an interviewee's knowledge of the content at stake. For years, the major focus of an interview activity was on the interviewee's responses, when the teacher or the researcher tried to assess the student's understanding of a particular topic. Even now, contemporary educators (Ginsburg and others) identify the interview method as a powerful technique to gain insight into thinking of others. This also resonates with the metacognitive aspect of the role-playing.

Over the last decade, the interview, as a pedagogical and as a research instrument, has attracted the attention of many educators. This began with Ginsburg's (1997) study of a child's thinking, wherein the researcher described the advantage interviewing offers over the traditional administrative tests. Following this research, some mathematics educators looked closely at the component of questioning and task creation, for the purpose of teasing out students' mathematical thinking (Watson & Mason, 1998). Watson and Mason placed value on questioning, and broadened the merits of the effective interview as a pedagogical tool. Their extended work resulted in a detailed classification of questions and prompts. Finally, they illustrated how learning and teaching situations can be enhanced for the benefit of the learners.

In aforementioned studies, the interviewer and the interviewee, (the teacher and the student, or the researcher and the participant) would have different levels of expertise in the content involved. Thus, in one way or another, the pressure of giving a satisfactory answer could affect the interviewee's responses. Such pressure could be minimized if the interviewer and the interviewee were to have similar mathematical backgrounds, and could question each other. The peer-interview would be an appropriate alternative. Another possible situation would involve self-explanatory study, where the pre-service teacher would impersonate a mathematics teacher and a student. This situation would allow self-reflection, and self-criticism. All these alterations to the reviewed, previously instructional/pedagogical practices should make learning situations of greater assistance to the learner, who is the pre-service secondary mathematics teacher, especially if the experiences are closely related to real-life ones.

Taking into account the previously discussed advantages that each of the aforementioned methodologies or approaches offered, the ideal research activity would have to consist of a combination of them. After giving a thoughtful consideration to the ideas and issues exposed in the discussed studies, the tasks finally emerged.

The Job Interview

Pre-service teachers were invited to play the roles in a fictional mathematical interaction between the head of a mathematics department and an applicant for the position of substitute teacher, who would cover for 3-weeks leave. The mathematical content for the conversation was logarithms and logarithmic functions.

Each of the participants had their own objective to accomplish. For the interviewer, the goal was to do his/her best to verify and evaluate the candidate's knowledge and understanding of the mathematical content and pedagogy required to be covered. For the candidate, the goal was to do his/her best to answer the questions to demonstrate his/her competence.

The main purpose of this task was to analyze the interviewer, the person who played the role of the head of a math department. This decision was based on the premise that the participants' questions and intentions would reveal what they think is important in teaching. Thus, participants were asked to provide a written explanation of their choices of interview questions, elaborating on the purpose and the type of each question. They were also asked to prepare anticipated answers for their interview questions, submit the transcript of the interview, and a detailed evaluation of the 'candidate'.

The interview task was designed in a way to produce further outcomes regarding pre-service teachers' knowledge that would allow me to substantiate or challenge my conjectures. By the phrase "further outcomes", I refer to the additional information revealed through the interviewer's reaction to the received answer, and his/her final evaluation of the candidate. For example, after analyzing the interviewer's intentions, questions, and anticipated answers, I could theorize that the pre-service teacher (interviewer) had strong mathematical knowledge. But later in the interview, if the interviewer received a very weak answer, containing some mathematical errors, to his/her question, and if the interviewer agreed to such an answer without further probing, it would indicate that the interviewer's knowledge was superficial. This limited his/her ability to justify the candidate's flaws or gaps in knowledge and indicated that my initial conjecture was rather premature and not valid.

Data sources associated with this task comprised the following writings:

a. Interviewer's rationale for the choice of the interview questions:

Through these data I investigated interviewer's subject matter and pedagogical content knowledge. Subject matter knowledge was based on the conceptual aspects of the interview questions, what the interviewer would consider to be the most important to target, and why. The pedagogical content knowledge is exposed by the manner of questioning, and the types of questions employed.

b. Interviewer's anticipated answers for his/her interview questions:

Through anticipated answers, I planned to learn not only if the interviewer can solve his/her own problem, but also to verify whether there was a consistency between the prepared interview questions and answers. Basically, in combination with (a), I was able to assess if the interviewer did what he or she meant, and explored what the interviewer would do if the expected and received answers were different. The variety of the anticipated answers to a particular question could indicate the scope of the interviewer's subject matter knowledge and pedagogical content knowledge (the more the merrier).

c. Transcript of the interview:

The extent of the interviewer's pedagogical content knowledge was explored upon the interviewer's reaction to the candidate's response. For instance, if the response was satisfactory, would the interviewer prompt the candidate's knowledge of logarithm and logarithmic functions further? Or, if the answer contained an error, how would the interviewer reply to it? In this case, the ignorance of misconception might be the interviewer's misconception as well.

d. Interviewer's evaluation of the candidate:

The interviewer's evaluation of the candidate was the final, and one of the most important, pieces of information. If from (c) I was able to make an assumption about the interviewer's content knowledge, then in (d) the interviewer provided his/her own reflection on what happened in the interview. The details the interviewer included in the evaluation were considered as the most important aspects of teaching and learning logarithms and logarithmic function.

Pre-service teachers' subject matter knowledge of logarithms and logarithmic functions

In this paper, I present an overview of relevant subject matter knowledge that pre-service teachers possess, by focusing on their ability to provide explanations of the meaning of logarithms and logarithmic functions. This knowledge was in evidence from the interviewers' questions, and their rationales for the selection of these questions. It is also important to mention that the pre-service teachers did not review logarithms in the class during this course. They probably did individual or small group reviews when preparing for the interview task; however, it was done without any interference on the part of an instructor.

The essence of logarithms and logarithmic functions can be explored in three major areas of knowledge:

- (area 1) logarithms as numbers
- (area 2) the operational meaning of logarithms
- (area 3) logarithms as functions (Berezovski & Zazkis, 2006, Berezovski, 2004).

At the heart of the first area of knowledge is the understanding that a logarithm is a real number, and that any real number can be presented in the form of a logarithm. An understanding of a definition of a logarithm is part of this area of knowledge. The second area focuses on the main properties of logarithms, and how they can be added or subtracted. This is otherwise known as the product and quotient laws of logarithms. And the third area encloses the knowledge of how the relationship between positive numbers and their logarithms becomes a function, and deals with the properties of logarithmic functions.

I analyzed pre-service secondary mathematics teachers' subject matter knowledge, exploring the three conceptual areas of knowledge mentioned above. Several situations are discussed in detail.

For example, the question, 'why can't we find the logarithm of a negative number?' would target the knowledge relevant to the first and/or the third areas of knowledge. Actually, this or similar questions were the most common: 8 out of 10 participants asked them in their interviews. This might indicate that interviewers identified it as problematic, or difficult for students' learning, but as important to understand. In the following episode, an interviewer, Nora, tries to assess if her candidate possessed such knowledge:

> Nora: Okay good, Okay, now moving on. As a mathematics teacher, one has to ensure that students understand the reasoning behind mathematical ideas and not just memorizing them. If a student posed the question 'why are some logarithms undefined?' how would you explain it?
>
> Candidate: I personally look to definitions as starting points, and would encourage students to examine what happens when constraints are violated. So I would have them start with a table of values of logs with negative numbers for bases, exponents, and arguments, and also examples of exponents with negative bases and exponents. And I would ideally use the question we just discussed[1] as a hook for a constructivist lesson, in which students would have to go back to the meaning of logs as they relate to exponents, and then come up with examples of different logs that are undefined, so they can see for themselves what's happening. I might even use this example to discuss continuous fractions and inverses, because it relates to that as well, or use it as an example of how to take observable relationships and translate them into a proof. So it's a very rich topic I think. I would have to structure the lessons so the students didn't get overwhelmed with too many different ideas though, uh but delving into the why's is definitely something I would want to encourage.
>
> Nora: that is definitely true, thank you.

Nora, as a department head, asked a very important question that had great potential to unravel the candidate's understanding of logarithms. It would be interesting to see how the candidate could handle it, what example would be chosen, and how the necessity of the existence of a logarithm would be established? But instead, the interviewer settled for less: a very blurry verbal description of some disorganized ideas. Did the interviewer really agree with such a response? The answer can be read from

[1] Nora's previous question:
Solve for x: $\log_3(x-4) = 1 - \log_3(x-2)$ and one of your students,
Tom answered, $x = 5$ or $x = 1$. Is his answer correct, incorrect, or partially correct?
Please explain your reasoning.

Nora's evaluation of the candidate, "Clara demonstrated she has content knowledge of logarithms and knew of the connection to exponential functions…" Another interesting question to consider might be why Nora agreed to this answer? To understand and explain it, I shall consult the interviewer's anticipated response. What did she expect to hear or to see, when the following question was asked?

> …explaining to students why $\log_b n$ only exists for $n>0$ will help students obtain understanding. It is also important to explain this idea step-by-step so that students understand the logic and don't just memorize this fact because they are frustrated by the explanation.

In this particular episode, the interviewer asked an important question that had the potential to target much deeper conceptual knowledge. Even though the answer was not well worded and revealed very little about the candidate's knowledge of a definition of a logarithm, Nora accepted it. It is questionable whether this indicated a polite response to the classmate or it meant that she possessed the same level of knowledge. To confirm, I compared it to Nora's expected answer, and confirmed that she indeed possessed a very limited knowledge of logarithms. She received a very unclear answer, and didn't ask any follow-up questions. Her personal response exposed the tendency to procedural learning. In this case, both Nora as the interviewer, and the interviewee possessed limited subject matter knowledge, particularly of logarithms as numbers (area 1).

To explore pre-service teachers' subject matter knowledge in area 2, I analyzed the interviewers' knowledge of logarithmic laws. For that matter, I chose to focus on another interview that was inventive in its purpose. The interviewer, Kurt, was trying to assess if the candidate could explain why $\log(ab) = \log(a) + \log(b)$. A knowledgeable teacher would try to establish that there exists an isomorphic connection between the product and the sum. When explaining this property to students, it would be important to mention and even illustrate the historical significance of the invention of logarithms, which allowed finding the product of numbers through addition. These are important conceptual aspects, the understanding of which could be evidence of specific subject matter knowledge.

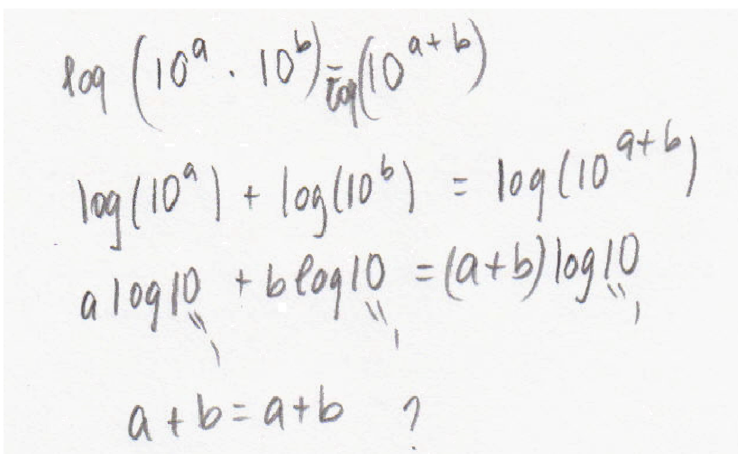
Consider the following situation:

Kurt: So the question is, you have log(ab) = log(a) + log(b) How could I explain it in terms of ….

Candidate: I see some connection between $10^a \times 10^b = 10^{a+b}$ and log(ab) = log(a) + log(b)

Kurt: What do you mean by connection?

Candidate: I mean both equalities have the same base 10, left sides of both are products, and the right sides are sums…but how can I get from one to another? Let's try to log both sides of the first equality…



Firuge 1: Kurt's Interviewee's Written Response 1

Candidate: Oops

Kurt: Maybe take a closer look at each expression

Candidate: They are inverses of each other, how would I connect them…I think I know, (writes on paper):

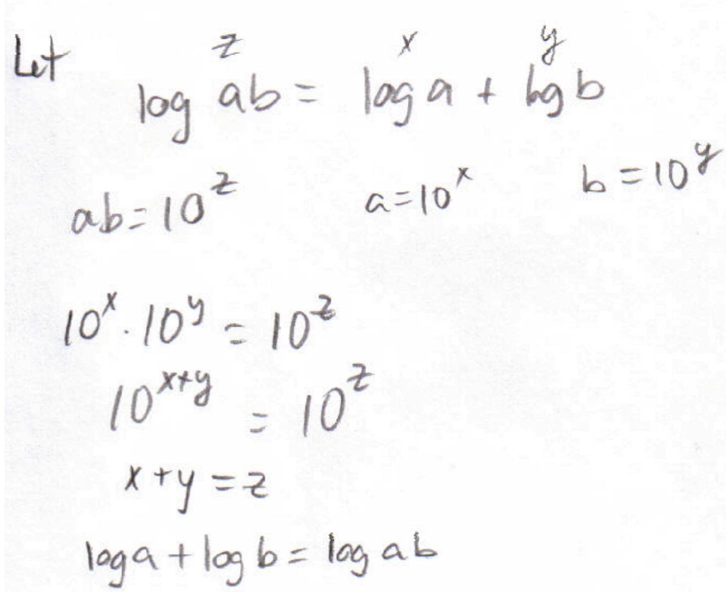


Figure 2: Kurt's Interviewee's Written Response 2

Kurt: That's right, thank you…

From this episode, one could learn that the interviewer's subject matter knowledge about the product law is formal (Fischbein, 1999), or relational (Skemp, 1987). In a way, the interviewer tried to verify if the candidate could provide a formal proof of the product law. Kurt indicated this in his rationale for the following question, "…though this question is a performance question, it should reveal a higher level of understanding. The ability to prove the identity and communicate clearly how it is done will provide information about the candidate's advanced training in mathematics and the understanding of logarithms…" The candidate's response to the answer fell right into interviewer's expectations, except for one detail. To fulfill the expectations, the candidate should be able to "discuss how the product of the exponents with the same base is the sum of the exponents and logarithms are a way of bringing multiplication down to addition…" This quote allowed me to become more precise in my analysis. Even though the wording was awkward, it was possible to sense that the interviewer tried to reach to deepest knowledge he could possibly explore. Nevertheless, Kurt settled for less. However, the intent and the interviewee's evaluation indicated that Kurt's knowledge of basic properties of logarithms went beyond instrumental and relational. It would border between relational and logical, according to Skemp (1987), or between algorithmic and formal, according to Fischbein (1999). Though the majority of pre-service teachers exhibited an understanding of logarithmic properties only at the procedural level, which indicated their limited knowledge in area 2, this particular episode exemplified an exceptional situation, where the participant moved beyond the procedural. Kurt exhibited thoughtful proficiency in the second area of knowledge.

Area 3, encloses the knowledge of how the relationship between positive numbers and their logarithms becomes a function, and deals with the properties of logarithmic functions. Seven participants included questions related to logarithmic function in their interviews. The questions ranged from how to define this function, to applications of logarithmic functions. The reader is invited to look at the treatment of the fundamental knowledge of logarithmic function offered by one of the interviewers, called Kal.

Kal: What is a logarithmic function?

This question could be addressed to the teacher and to the student. On the student level, it would probably be enough to repeat the most popular definition from the school textbook, something like, it is an inverse of the exponential function, and draw a graph symmetric in the line $y = x$, to the exponential function. The description of the main properties such as, domain, range, points of intersection, symmetries, asymptotes, etc should follow it. On the teachers' level, it could be anticipated that the interviewer knew about different representations of logarithmic functions, and how they are connected. This knowledge could be exhibited in the follow up questions or prompts that would lead the candidate to explain how logarithmic functions are used to model real life situations, and why they are appropriate for this matter. In this regard, the analysis of the entire interaction between the interviewer and the candidate seems to be most revealing.

> Candidate: Well, just to give you the definition of logarithmic function. I would say that it's the inverse of exponential function. Umm… just to give you brief history behind it, this logarithm operation was invented simply um … just to simplify long numerical operations to find the inverse of exponential functions. Can I give you an example?
>
> Kal: Sure!
>
> Candidate: …(showing work on paper) if we have the logarithm of a number x in base b, let's say $\log_b k = n$ then its inverse is the exponential function of the base b raised to the power of n equal to the number x, $b^n = x$.

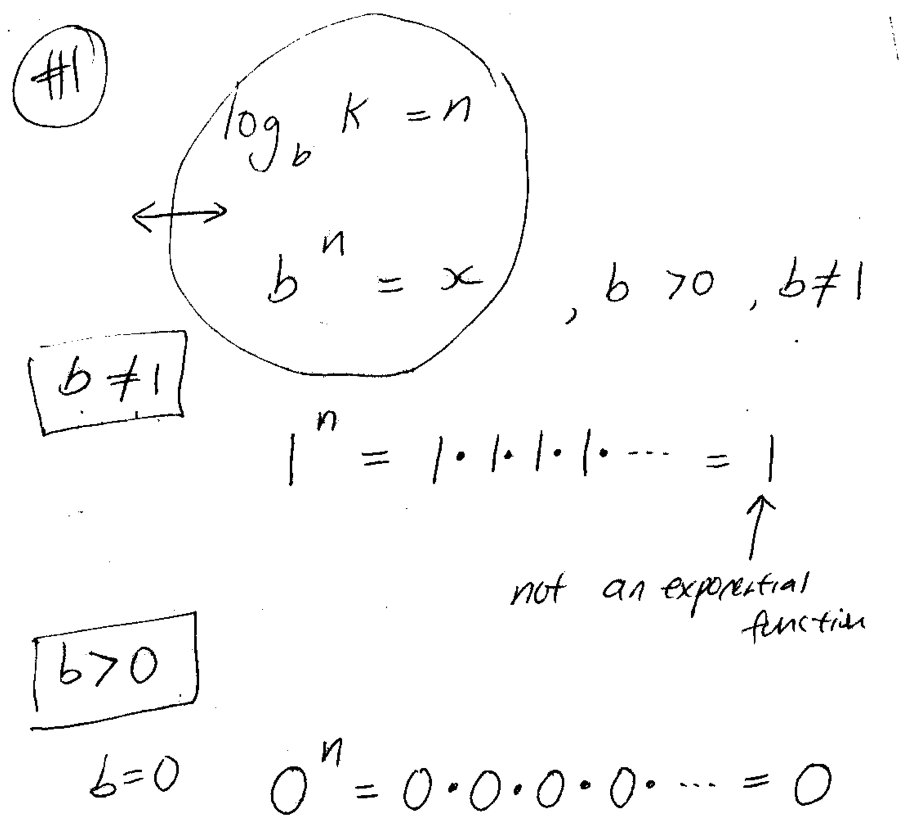


Figure 3: Kal's Interviewee's Written Response 1

> Candidate: (continued)…the logarithm is the inverse of exponential function given that the base is positive, and doesn't equal 1. The base cannot be one, because this simply means that what we are doing is, umm… if the base equals 1 that means 1 to the power of n, which means we are multiplying 1 by the number of power that we have and so on. This will always be 1. However, 1 is not an exponential function. Therefore, this

condition must apply to this definition. The second condition is that the base has to be greater than 0. And if we assume that the base equals to 0, which means we have 0 to the power of n. This, in turn, means we are multiplying 0 by how much the number of power it is raised to. This will always be 0. Once again, it is not an exponential function…Let's see if b equals a fractional number, that means if we have a fraction of half raised to the power of n,

$b<0$ b = fractional # + negation

① $\left(-\frac{1}{2}\right)^n$ = positive when pos = n

② $\left(-\frac{1}{2}\right)^n$ = neg. when neg = n

③ $\left(-\frac{1}{2}\right)^n$ = undefined. when n = fracti

Figure 4: Kal's Interviewee's Written Response 2

Candidate: (continued) Oh mind you, this has to be a fractional number and also has to be negative, because b<0. So, we have negative half raised to the power of n. If we look at an even power, that means I will have a positive value and when I have a negative n value, then I would have a negative value. Here's another case. That is in (1) and (2). (3) is that if my power is a fraction that means I cannot take the power of a negative fraction number. So this will be undefined. Given these three cases and plus the examples I've proved above, the conditions that the base must be greater than 0 and the base must not be 1 must meet in order for the definition of logarithm to be satisfied.

Kal: well, thanks for your answer…

It is evident that the candidate provided a very extensive overview of the different base exponents, and some were even incorrect, such as in what the candidate wrote (1), when n is positive, for example, let n=3, the power is negative: $\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$, which is contradictory to the statement provided. Or in (2), when n is negative, for example, let n=-1, the power is positive: $\left(-\frac{1}{2}\right)^{-2} = 4$. This contradicts the candidate's response. Even the third statement provided is incorrect, for the counter example, let n = $\frac{1}{3}$, the power will exist: $\left(-\frac{1}{2}\right)^{\frac{1}{3}} = -\left(\sqrt[3]{\frac{1}{2}}\right) = -\frac{1}{\sqrt[3]{2}}$. However, the interviewer noticed none of these. In her evaluation she wrote, "…the candidate answered my question very well, and his answer is almost the same as my anticipated answer." What became apparent from this quote is that Kal has a very limited knowledge not only of exponents, but also of logarithms and functions. Her expectations were set even lower than the requirements in high school mathematics. There was no evidence that pre-service teachers possess the knowledge of how real numbers form logarithmic functional relation. None of the interviewers asked any questions that highlight this relation; for example, "locate on the graph and compare the values $\log_{1/2}3$ and $\log_{1/2}5$". If interviewers did not question about this particular knowledge, they either did not know it themselves, or did not consider it important for a candidate to possess. The only knowledge present in the interviewers' data was a commonly used relation between exponential and logarithmic function. Data indicated that two of the interviewers did not understand this relation. Possibly, they did not know it, because they did not understand exponents in the first place, as in the aforementioned analyzed episode.

In the setting of the clinical interview, Kal did not say much, and revealed very little of what she may have known. However, the task was designed in such a way that allowed for the discovery of a deeper insight into her' knowledge, evident through her self-evaluation, implicitly provided throughout her questions, answers and evaluation. The old saying comes to mind, "who knows you better than you do?"

In general, the peer-interview task provided a view of an individual's subject matter knowledge of logarithms and logarithmic functions. Through the rationales for the selection of questions for the interview, interviewers' expected answers, and their evaluations of the candidates, I could investigate the level of the subject matter knowledge of the interviewers, who were in fact, pre-service secondary mathematics teachers, impersonating the heads of mathematics departments.

On one hand, by choosing good questions, participants revealed their awareness of possible difficulties when teaching or learning a particular content. On another, the subject matter knowledge exhibited by the pre-service teachers was generally insufficient, lacking understanding in all three areas: numerical, operational and functional meaning. This prevented them from further development of their interviews into thorough investigations of their candidates understanding and abilities.

Conclusion

One of the goals of this research was to investigate the pre-service teachers' knowledge of logarithms and logarithmic functions. This study has identified that pre-service teachers are aware of possible difficulties of teaching or learning the concepts of logarithms and logarithmic functions. An example of this is the case of the interviewer Nora, who asked one of the crucial questions: "Why are some logarithms undefined?" This question could have led to a deep, meaningful investigation of the candidate's understanding of logarithms. However, this opportunity was abandoned, because Nora's own knowledge of logarithms was insufficient to further pursue the received response.

On the whole, the pre-service teachers' subject matter knowledge was insufficient for meaningful engagement in learning about logarithms and logarithmic functions. This prevented them from developing thorough investigations of their peers' understanding and abilities. In the Job Interview task, the majority of participants, who were interviewers, simply could not explain why the situations prompted by their own questions were indeed problematic and important.

The methodological contribution that this study brings to the field of mathematics education consists of the methods used for gathering the field data. To collect data for investigating the pre-service secondary mathematics teachers' knowledge of logarithms and logarithmic functions, I designed the peer-interview task and utilized the writing of a script for a play activity. Using both tasks in this research study allowed me to collect different types of data that better informed my investigations. It is interesting to note that mainly, the findings from one task supported the findings from the other.

Moreover, in focusing on pedagogy, the study enhances the teaching of pre-service mathematics teachers by highlighting their learning through utilization of the research tasks for instructional purposes. As was pointed earlier, one of the reasons for pre-service teachers' difficulties in teaching mathematics is the lack of pedagogical practices and simulation of real teaching situations, that allow them to experience firsthand, their preparedness for the teaching of secondary mathematics. Therefore, both research tasks are a valuable addition to teacher training in mathematics education, since they serve not only as an assessment tool but also as an instructional tool that provides learners with an opportunity to engage in meaningful learning.

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