Self-Efficacy, Calibration, and Exam Performance in College Algebra and Calculus I

Joe Champion
University of Northern Colorado
joechampion@gmail.com

Self-Efficacy in Social Cognitive Theory

Albert Bandura's social cognitive theory first began as a means for explaining observational learning mechanisms by positing that a *causal triadic reciprocality* exists between individuals' behavior, environmental stimuli, and internal cognitive factors (Simon, 1999). Social cognition has since developed into a robust theory increasingly focused on explaining cognitive processes and motivational constructs such as metacognition (Schraw, 1998), self-efficacy, and self-regulation as important aspects of learners' attempts to acquire knowledge and skills (Martin, 2004). In particular, perceived *self-efficacy*, or judgments of one's ability to accomplish given performances in particular contexts (Bandura, 1997), is a particular focus of social cognitive research in mathematics education. Lightsey (1999) reports over 2500 hundred articles on the positive relationship between self-efficacy and achievement.

Social cognitive research considers self-efficacy to be a primary mediating mechanism in human cognition because self-beliefs in ability act as a filter between prior experiences and the development of abilities within a particular domain. In contrast to the often more globally defined self-beliefs associated with self-concept, Pajares and Schunk (2001), point to consistently positive effects of self-efficacy judgments on performance:

> Self-efficacy beliefs influence the choices people make and the courses of action they pursue. Individuals tend to engage in tasks about which they feel competent and confident and avoid those in which they do not. Efficacy beliefs also help determine how much effort people will expend on an activity, how long they will persevere when

> confronting obstacles, and how resilient they will be in the face of adverse situations. (p. 241)

Attributed in part to individuals' tendencies to rely heavily on self-efficacy beliefs during difficult tasks (Bandura, 1997), learners' self-efficacy judgments are often better statistical predictors of performance in academic domains than measures of ability or intelligence (Pajares & Kranzler, 1995). In fact, in path analyses of performance incorporating biographical (e.g., socio-economic status, gender), motivational, and instructional variables, self-efficacy beliefs account for the largest portion of variation in academic performance after controlling for instructional factors (Madewell & Shaughnessy, 2003). However useful self-efficacy measures may be for predicting performance, there is research evidence that strong self-efficacy beliefs themselves do not guarantee success in difficult domains such as mathematics. In particular, developing both strong and accurate self-efficacy beliefs may be the key to self-efficacy's benefits in learning mathematics.

Mathematics Self-Efficacy, and Calibration

Underscoring the complex nature of students' confidence in their mathematical abilities and performance on closely matched mathematical tasks, Chen and Zimmerman (2007) found that U.S. seventh graders reported much higher mathematics self-efficacy beliefs than sixth grade Taiwanese students, yet the U.S. students performed significantly worse than the Taiwanese students on corresponding mathematics tasks. Importantly, the U.S. participants also displayed a tendency toward more overconfidence when rating self-efficacy to complete tasks than Taiwanese students (Chen & Zimmerman, 2007). The authors suggest these cross-cultural differences in overconfidence contribute to continued underperformance by U.S. students in mathematics as compared to Taiwanese students.

Sometimes referred to as "feeling-of-knowing accuracy" (Schraw, 1995, p. 326), students' *calibration* (Pajares & Miller, 1994) in reporting self-efficacy judgments is a relatively new area for research in mathematics education with foundations in experimental psychology and reading education (Lin & Zabrucky, 1998). The tendency of students across educational levels and performance abilities toward *overconfidence*, or positively biased judgments (Schraw), has been reported in numerous studies of college students' self-efficacy for reading tasks, in particular. In their review of literature addressing calibration amongst adult readers, Lin and Zabrucky refer to this tendency as an "illusion of knowing" effect and include the following summary of findings:

> There is a tendency for adult students to generate unrealistic feelings of knowing when it comes to evaluating outcomes of learning. As can be seen in the present review, overconfidence is a common phenomenon among young adult students that may result in inadequate learning due to premature termination of cognitive processing. (p. 384)

Bandura (1997) suggests slight overconfidence in one's self-efficacy as being psychologically adaptive due to the potential benefits of overconfidence on effort and persistence. In this view, what might first appear to be poor calibration in the form of overconfidence can be reframed as a set of optimistic self-evaluations that may ultimately support taking-on challenges. However, Bandura and other calibration researchers (e.g., Pajares & Kranzler, 1995) caution against grossly inflated overconfidence, suggesting that unrealistic overconfidence can lead students to engage in self-handicapping academic behaviors (Urdan, 2004) such as avoiding studying and procrastinating.

From a quantitative perspective, there is support for calibration as a measure that contributes to statistical explanations of variation in achievement beyond the variation explained by self-efficacy judgments and prior achievement in mathematics (Pajares & Miller, 1997). Chen

(2003) found U.S. middle school students at every ability level tend to show poor calibration in the form of overconfidence, and self-efficacy and calibration provide significant and independent predictive value in a path analysis model for mathematics performance.

O'Connor (1989) supports the hypothesis that learners may grow to become more calibrated in assessing their abilities through a domain-specific developmental process. In a review of calibration research from experimental psychology in the 1960s to 1980s, O'Connor identified several factors influencing calibration: (1) familiarity with task requirements (e.g., assigning numbers to feelings of uncertainty), (2) familiarity with the topic of interest (subject matter knowledge), and (3) adequate feedback on the accuracy of prior judgments. O'Connor also identified research showing college students are well-calibrated at rating their self-efficacy to attain final grades in a course, indicating that students' may develop calibration in predicting general academic outcomes while demonstrating poor calibration in rating their self-efficacy to complete specific course-related tasks.

Through mathematics self-efficacy and calibration, social cognitive theory provides a foundation for interpreting the mathematical confidence and achievement of prospective secondary mathematics teachers in advanced mathematics. It is important to note, however, that social cognitive theorists do not generally subscribe to global models of self-efficacy and performance (Bandura, 1997). Instead, personal, social and cultural conditions are seen as important co-determinants of academic confidence, motivation, and behaviors.

The Two Pilot Studies

In preparation for a dissertation study, the author conducted two pilot studies at a mid-sized doctoral granting university in the Mountain West; the first study focused on the predictive value of mathematics self-efficacy and calibration in College Algebra (N = 128) during Fall 2007, and the second study focused extended and refined the first study into Calculus I (N = 119) during Spring 2008. The first pilot study was set within a larger study of student achievement and goal structures that incorporated balanced, random assignment of students to two instructional conditions, one of which included a classroom communication system featuring a network of graphing calculators and a classroom presentation system. Within the college algebra study, the first pilot study used a concurrent mixed methods (Creswell, 2003) design to investigate students' self-efficacy ratings, calibration, and experiences of course feedback in the four college algebra sections throughout the semester. The second pilot study utilized a post-test only with non-equivalent groups design (Creswell, 2003) to further validate and refine the measures and statistical model for the effects of self-efficacy and calibration on final exam performance in a population of students enrolled in advanced mathematics.

Quantitative results from the first pilot study confirmed many of the self-efficacy research findings that had previously been attributed to middle and secondary school students (e.g., Chen, 2003). The survey techniques used in the study were closely based on procedures used in earlier social cognitive studies of calibration (e.g., Chen, 2002; Pajares & Miller, 1994) and incorporated two measures of calibration—*accuracy*, an absolute measure of calibration, and *bias*, a directional measure of calibration—in part to compare the predictive utility of each measure. Self-efficacy, accuracy, and performance scores were converted to a five-point ordinal scale (i.e., 0 = lowest, 5 = highest), and calibration bias was expressed on a 10-point ordinal scale (e.g., -5 = underconfident, 0 = calibrated, +5 = overconfident). Descriptive statistics for the four measures are shown in Table 1.1 and suggest participating college algebra students tended to express self accuracy ratings which were moderately accurate, but consistently overconfident. Correlation analysis of the variables confirmed findings from Chen and Zimmerman (2007) that self-efficacy, mathematics performance, and calibration bias and accuracy are all significantly intercorrelated at the $\alpha = 0.01$ criterion (see Table 1.2).

Table 1.1.

Means and standard deviations for measures in the first pilot study.

(*N* = 91)	Performance		Self-Efficacy		Bias		Accuracy	
Measure	M	SD	M	SD	M	SD	M	SD
Exam 1	4.11	0.62	4.00	0.69	1.07	0.96	2.82	0.67
Exam 2	3.01	0.89	3.54	0.89	1.67	1.08	2.46	0.66
Exam 3	3.70	0.74	3.62	0.84	0.98	1.04	2.82	0.75
Final Exam	4.01	0.75	3.84	0.80	1.07	1.18	2.77	0.78
Combined	3.73	0.55	3.76	0.70	1.18	0.72	2.73	0.49

Table 1.2.

Pearson product-moment correlations (r) for measures in the first pilot study.

	Bias	Accuracy	Self-Efficacy	Performance
Bias	–			
Accuracy	-0.57	–		
Self-Efficacy	0.37	0.37	–	
Performance	-0.51	0.75	0.53	–

Data analysis in the first pilot study led to a multiple linear regression model including only self-efficacy ratings and calibration accuracy as predictors of students' performance on four in-class examinations. Analysis of the model suggested the data met the four assumptions of linear regression modeling (Osborne & Waters, 2002), including (1) linear relationships between the independent and dependent variables, (2) independence of errors, (3) normality of variables, and (4) equal variances in errors (homoscedasticity). The regression model was significant ($F(2,88)= 75.6$, $p < 0.001$) and explained a moderation percentage of the total variation in exam performance $R^2 = .63$. Standardized regression coefficients showed stronger effects due to calibration accuracy ($\beta = .638$) than those do to self-efficacy ($\beta = .294$). That is, while increasing self-efficacy judgments appears to be associated with increased mathematics performance, tendencies toward accurate assessments of confidence in one's ability to complete tasks may be even more important in explaining the exam performance of college algebra students.

The qualitative inquiry component of the first pilot study looked at college algebra students' experiences of multiple sources of feedback in technology-enriched instructional settings that included online homework, intensive graphing calculator use, course management software, and small-group in-class learning activities in the four sections taught by the two participating instructors, of which one was the researcher. In addition, one section taught by each instructor utilized a classroom graphing calculator network as a communication and presentation system during class activities. Using purposeful sampling techniques (Glesne, 2006) in conjunction with the class instructor, the qualitative investigation included data from interviews of seven students and digital artifact analyses (e.g., saved computer screenshots) as part of a holistic comparative case study (Merriam, 1998) of students experiences in the two instructional settings.

Results from the qualitative strand of the first pilot study suggest students relied heavily on performance feedback and mastery experiences in the form of quizzes, exams, and online homework as well as social comparisons to classroom peers in forming self-efficacy evaluations of their content understanding. These preliminary qualitative findings suggest a value in considering Bandura's (1997) *four sources of self-efficacy*—mastery experiences, social persuasions, vicarious experiences, and physical and emotional states—as a potential theoretical framework for exploring the relationships among self-efficacy, calibration, and performance for students enrolled in advanced mathematics courses.

The second pilot study, in Calculus I, yielded similar results to the quantitative strand of the college algebra pilot study regarding the correlations and predictive value of self-efficacy and calibration toward students' exam performance. The calculus pilot collected less data from each student (a single exam versus four) and was less controlled than the first pilot study because the cross-sectional design did not include random assignment of students to sections and the participating calculus instructors used different exams and self-efficacy instruments. In addition to a concomitant decrease in statistical power (Frankfort-Nachmias & Nachmias, 2000), it was expected that any linear regression models would be less predictive of students' performance. However, as in the first pilot study, self-efficacy and calibration accuracy effects accounted for modest independent portions of the variation in exam performance ($F(2,90)= 83.3$, $p < 0.001$), collectively explaining $R^2 = 63\%$ of the variation in calculus students' final exam performance. Effect sizes were also remarkably similar to those found in the College Algebra pilot study, with calibration accuracy displaying strong effects ($\beta = .566$) than self-efficacy ($\beta = .393$) in the linear model.

Though students' self-efficacy levels were similar across Calculus sections, both performance and calibration on the final exams varied greatly across course sections, however, suggesting future research might follow Chen's (2003) consideration of potential differences in self-efficacy and calibration by the level of difficulty in test items.

Conclusion and Future Research

The two pilot studies suggested some patterns of association between calibration, self-efficacy, and mathematics performance in advanced coursework. However, interpretation of the data analysis is limited by an assumption in multiple linear regression that independent variables should not include measurement error (Frankfort-Nachmais & Nachmias, 2000). So, although reliability coefficients of self-efficacy and calibration measures are typically moderate to strong (O'Connor, 1987; Pajares & Miller, 1997), the fact that each measure includes self-reports of *latent* psychological variables suggests that structural equation modeling is more appropriate. Moreover, the theoretical support for directional relationships among the constructs, such structural equation modeling is particularly well-suited for the study of self-efficacy in mathematics (Pajares & Kranzler, 1995).

In summary, the two pilot studies reported in this paper inform future research into the relationships between self-efficacy, calibration and exam performance in advanced mathematics courses in four important ways. First, the procedures in the pilot studies help to refine the data collection instructions and to establish the feasibility of the data collection and analysis procedures at mid-sized universities. Second, the regression findings from both studies suggested a independent effects of self-efficacy and calibration on performance, with calibration in the form of accuracy perhaps exhibiting stronger effects. Third, the qualitative inquiry pointed to Bandura's (1997) conception of the four sources of self-efficacy as a conceptual tool for investigating relationships between self-efficacy, calibration, and performance. Finally, the data analysis suggested the potential value for structural equation modeling techniques in future research into the self-efficacy of students. The author's dissertation addresses each of these findings.

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