Connecting Beliefs and Missed Opportunities: An Opportunity for Graduate Student Instructors' Reflection on Teaching

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Abstract

There is an expressed need for structured reflection on Graduate Student Instructor (GSI) teaching practice (Austin, 2002). Aligned with Schoenfeld's theory of teaching (1998, 1999) and a framework suggested in Arcavi and Schoenfeld (2008), this study provides a model for such reflection in which we identify GSI beliefs from interviews, observe GSI/student interactions in video data, and draw connections between the professed beliefs and decision-making in interactions with students. Using one illuminating example, we illustrate how these beliefs shaped the interactions in such a way that leads to "missed opportunities" – instances in which there is an observable disconnect between the mathematical issues discussed among a group of students before and during interaction with a GSI. Highlighting the connection between beliefs and missed opportunities is meant to be a useful space for reflection to improve teaching practices.

Introduction

The lack of systematic professional development opportunities and feedback from mentor faculty in graduate students' academic experience is a cause for concern (e.g. Austin, 2002). Austin's study suggests that Graduate Student Instructors' (GSIs) expressed interests in structured, systematic, and ongoing discussion and reflection on their teaching, with both peers and faculty advisors, are both missing from their current academic life and would be appreciated (e.g. page 105). Further, mounting concerns about the general quality and purpose of undergraduate education have incited reforms of varying scale and degree, many of which are not practical or possible for all universities to adopt (e.g. Barr & Tagg, 1995; Fairweather, 1996).

It is well established that teachers' beliefs are a key factor in shaping teaching practice (e.g. Aguirre & Speer, 2000; Calderhead, 1996; Leder, Pehkonen, & Torner, 2002; McLeod & McLeod, 2002; Speer, 2008; Stipek, Givven, Salmon, MacGyvers, 2001; Reviews include Fang, 1996; Kagan, 1992; Pajares, 1992; Thompson, 1992). Further, research has shown that examining teacher beliefs is an important aspect of professional development programs, specifically in understanding how teachers engage with and learn from professional development activities (e.g. Franke et al., 1998, Simon, 1997; Wiemers, 1990; Wilson, 1990, as cited in Speer, 2008). However, research on teachers' beliefs has yet to overcome some limitations. First, there is no consensus on the definition of beliefs, which confounds the findings and the discussion within the field around this construct (for review see Pajares, 1992). In addition, Speer (2008) found that most studies that attempted to link beliefs and practices have focused on broad characterizations of *both* beliefs *and* practices. She argued that coarse-grained analysis is "unlikely to do justice to the complex, contextually dependent acts of teaching" (224). Utilizing some of the common trends in defining 'belief,' as well as an appropriate grain-size lens on beliefs and practices, this paper will propose the use of 'missed opportunities' as a means for examining the complex interplay between a GSI's beliefs and practices, with potential implications for professional development.

Beliefs

While different researchers advocate various definitions of 'belief,' (Pajares, 1992), there are some general trends in the common definitions that we have adopted in order to discuss the construct in this paper. First, beliefs must be perceived true by a particular GSI for a particular context, when in reality they may be either true or false. For example, a GSI may believe that a particular group of students works through the material quickly and does not struggle with mathematical tasks, while in reality they struggle with tasks often.

Second, beliefs are *beliefs about*, or have a specific focus (Pajares, 1992; Rokeach, 1968). While the subject of the belief may be of varying grain size – from broad categories of beliefs such as 'beliefs about mathematics' to more refined categories such as 'beliefs about one particular group of students,' for this paper the focus is on beliefs that are specific both in content and context. For example, a belief about one particular group of students' way of working together on one particular problem.

The focus on beliefs stems from the fact that a teacher filters his/her resources based on his/her set of beliefs. Pajares (1992) argues that how a teacher decides what resources to use and when to use them is influenced by what teachers believe is important and plausible (Pajares, 1992). Speer (2008), drawing on prior research, makes the explicit connection that "Among [teachers' knowledge… goals and various social and contextual] factors, researchers have found beliefs to be a significant influence on teachers' use of cognitive (and other) resources," (p. 221).

For purposes of this research, resources fall into two broad categories: resources that the GSI brings to an interaction (such as prior experience with various students, prior knowledge of student struggles) and artifacts that a GSI 'inherits' when entering an interaction (such as board work, group's progress on an activity). It is most important to note that we are *not* including a GSI's beliefs, themselves, as a resource. For reasons previously stated, the beliefs may act as a 'filter' through which a GSI chooses to use some available resources, and not others. For example, a GSI may have many available resources from which he could draw in an interaction with students, but if he believed that some of those resources were not relevant then there was no reason for him to use those particular resources.

Speer's (2008) Recommendation for Grain Size

Speer (2008) reports that professional development programs which focus on "small, meaningful aspects of practice" have found success in helping teachers develop reform-oriented practices (p. 219). It is her recommendation, then, that research in this area should focus on a small and meaningful aspect of practice, in context, while considering 'collections of beliefs.'

Speer defined a collection of beliefs as "a small set of related beliefs that, in combination, describe a GSI's perspective on a particular topic" (Speer, 2008; 235). In other words, these are sets of beliefs that are themselves related not only because they are held by a particular GSI, but also because of their collective utility for explaining one of his practices. Her recommendation for this unit of analysis is based on the observation that beliefs are context specific and interconnected; separating beliefs would remove their explanatory power for a particular choice of action. For example, after seeing a group of students' solution, a GSI may be influenced by his belief about what an appropriate mathematics solution should be, his belief about the group's dynamic, and his beliefs about the particular students. These beliefs form a collection of beliefs which offer explanation for the particular practice of approaching this group after seeing their solution.

One may argue that another collection of beliefs is the set of a GSI's 'beliefs about mathematics.' However, using Speer's definition of the construct, this may not be considered a 'collection of beliefs' because it is not context specific, and the category of beliefs is far too broad to have explanatory power for the specific use of a practice. Appropriate 'collections' are also reminiscent of ‘belief bundles’ (Aguirre & Speer, 2000) in that they are context specific and connect beliefs from many aspects of a teacher’s superset of beliefs.

“If, as prior research indicates, beliefs play a role in determining practice, it follows that understanding connections between beliefs and specific practice will contribute to work in educational reform,” (Speer, 2008; 224). Aligned with Schoenfeld’s theory of teaching (1998; 1999), and the push for more opportunities for reflection on teaching practices (e.g. Austin, 2002; Friedberg, 2001; Swan, in preparation), we aim to illustrate how a particular GSI’s beliefs contribute to decision-making in interaction with students.

In order to focus this discussion and tackle a reasonably sized piece of the struggle to improve undergraduate mathematics education, this paper focuses on the relationship between beliefs and practices during instances of “missed opportunities,” or situations in which students’ ‘opportunity to learn’ some mathematics was not seized. Broadly, educational researchers consider ‘opportunity to learn’ a well-documented construct that links teaching and learning (Hiebert & Grouws, 2007). While ‘opportunity to learn’ is not entirely determined by a teacher’s actions (e.g. Stein, 2007), teaching plays a “major role in shaping students’ learning opportunities,” as evidenced by types of tasks posed to students, kinds of questions asked of students, responses teachers are willing to accept, etc… (Hiebert & Grouws, 2007).

Missed opportunities are characterized by a noteworthy disconnect between the mathematical issues raised by a group of students *before* a GSI engages in an interaction with them, and *during* a subsequent interaction with a GSI. In particular, in this paper we look for those “missed opportunities” that happened as a direct result of guided interaction with, or the decision-making of, a GSI.

Research Questions

In our attempt to illustrate how different 'collections of beliefs' for one GSI may have influenced his decision making (in interactions with groups of students) in cases where "missed opportunities" occur, we consider questions such as:

(1) How can 'missed opportunities' shed light on the interaction between the GSI's beliefs and his practice?[1]

(2) Is "Missed Opportunity" a lens through which structured professional development activities for GSIs may be viewed, such that they can use this construct to explore how their beliefs are related to their practice?

Methods

Data Collection and Selection

Data used in this study was collected as part of a larger project studying first-year calculus students at a large university (Adiredja et al., 2008), and includes video of students working in groups in an intensive calculus discussion section[2], interviews with a GSI after each class meeting and before and after the semester, survey data from enrolled students, and interviews with the students. For purposes of this investigation, we focused specifically on selected video episodes and GSI interviews.

Interviews. A semi-structured interview protocol was used to interview the GSI regarding his practices prior to the start of the semester and after the semester was over. We also invited the GSI to reflect on his day of teaching after each section meeting. The reflection usually started with an open-ended question about the events of the day. These conversations were not

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[1] We are *not* (1) making claims about the frequency of one particular GSI 'missing opportunities' nor (2) asking/encouraging GSIs to change their beliefs.

[2] Outside of lecture, students meet for 5 additional hours to work on problems and discuss homework questions. This discussion section is modeled after the math workshops (Fullilove & Treisman, 1990).

specifically designed to focus on the GSI's beliefs, but some of the beliefs emerged during the discussion.

Video selection. Video episodes of students working together in groups were selected on the basis of the occurrence of a missed opportunity, as previously defined. Researchers looked for a missed opportunity in episodes that were previously identified for other purposes of the larger study, and those that were specifically mentioned by the GSI in interviews.

Looking to Arcavi & Schoenfeld (2008), we understand the importance of viewing classroom video to uncover teacher beliefs and goals, and agree that the video cannot stand alone. While they acknowledge that "…an action may be consistent with certain goals or beliefs…(even if they cannot be proven 'right')…" (291), we can go one step further, with respect to 'accuracy,' by identifying GSI beliefs directly from interviews, instead of simply inferring them from watching his practice.

Preliminary identification of beliefs. From these interviews, researchers preliminarily extracted some beliefs that the GSI mentioned at several times during the semester. These beliefs are the ones from which we selected a collection that we argue to be relevant to a particular choice of practice.

Analysis Methods

In establishing the connection between collection of beliefs and specific practice exhibited in a video episode, we focus on the practice that leads to a missed opportunity, or *actual practice*. As previously mentioned, a set of resources can be used in different ways leading to different practices, and the manner in which such resources are interpreted is dependent in large part on the GSI's beliefs (Speer, 2008; Pajares, 1992). Figure 1 shows the proposed connection between the different constructs.

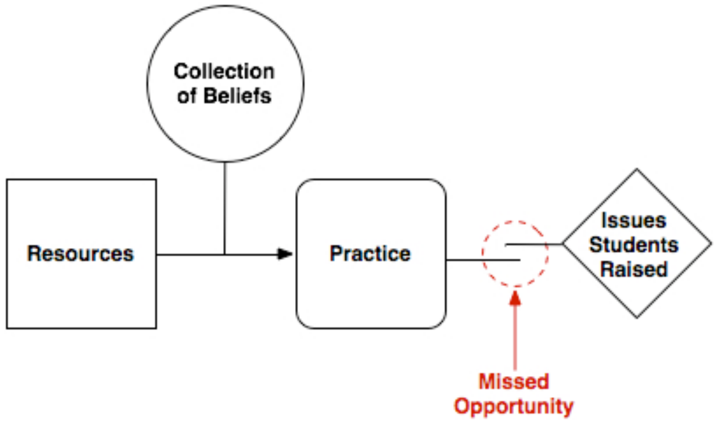

Figure 1. *Framework to analyze missed opportunity in student-GSI interaction*

To unpack some of the beliefs that may be at play, researchers asked the question, "Suppose the GSI used all the resources available to him, and he also had the students' best interest in mind – What beliefs could influence his choice of this particular practice?" (adapted from Arcavi & Schoenfeld, 2008). This allows for a more complete consideration of relevant beliefs because attention was moved from viewing the missed opportunity as a GSI's mistake, to viewing it as a by-product of the GSI's best intention. It is important to note that we are not trying to exhaust all the beliefs that could be at play in an interaction, but simply to highlight some of the beliefs that can provide plausible argument for the GSI's choice.

Once we have analyzed the interplay between beliefs, resources, and practice for the particular interaction that leads to a missed opportunity, researchers proposed a different choice of practice, from the GSI's repertoire of practice, which the GSI could have made in the same situation. Note that these *alternate practice(s)* may or may not lead to a missed opportunity, but do provide avenues to address some or all the issues that students raised. In our example, we focus on one alternate practice because we have evidence of the GSI's interaction with a different group of students on the same mathematical task. In reality there could be more than one alternate practice, and each would likely involve a different collection of beliefs.

It is important to emphasize that these alternate collections of beliefs and practices were selected from the particular GSI's repertoire of beliefs and practices, and not a broad database of "ideal" beliefs and practices that *any* GSI may hold or choose. Figure 2 is an overview of the connections between different practices and collections of beliefs.

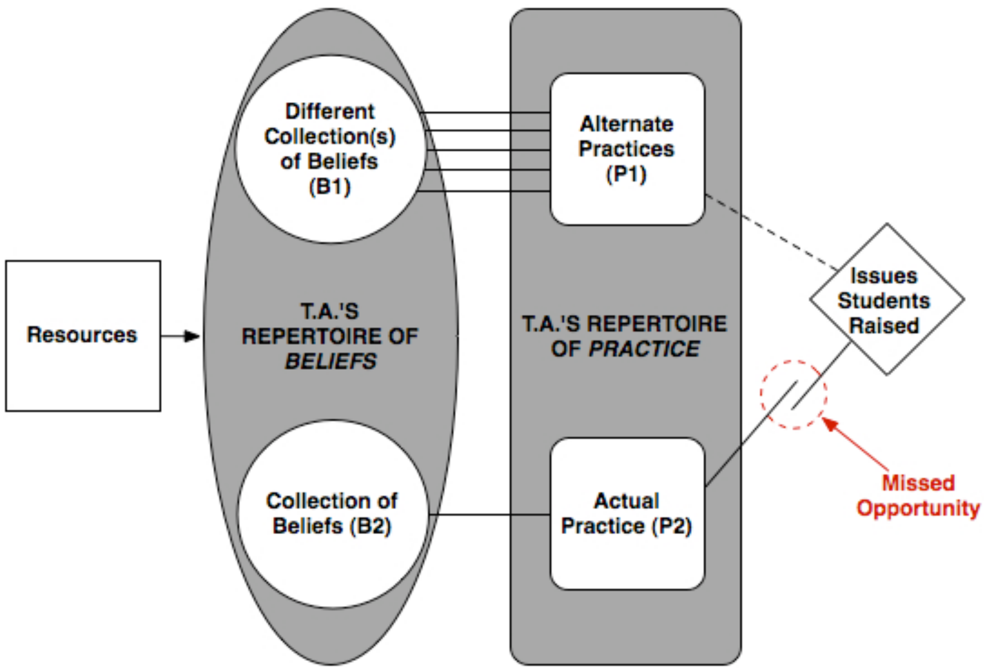

Figure 2. *Representation of proposed connection between different practices and collection of beliefs, with respect to resources and mathematical issues raised by students*

Analysis

Overview of the Episode

We will observe a missed opportunity for group 2, Amelia, Nanette and Priya,[3] as they attempted to complete the following task: $\lim_{x\to\infty} x - \sqrt{x}$ (problem 3, of 18 on a worksheet). Group 2

[3] All pseudonyms

struggled with three mathematical issues: the necessity for multiplying by the binomial conjugate, which ‘power of x’ to divide all terms by, and the meaning of 1/0 in a limit. As they struggled, their GSI, (Zack) had been discussing the same task with another group (group 1). Figure 3 shows the layout of the classroom and where the different groups were located. Group 2 watched the other students work on a chalkboard for almost half of the time they spent on this task. Group 1 struggled with the task for almost double the time group 2 spent on the task, though the aforementioned issues were successfully resolved for group 1, with Zack’s help. Group 2 was unable to resolve their issues, abandoned their partial solution, and copied the work from group 1’s chalkboard. Some time passed, and Zack approached group 2, confirming their solution, still unaware that they had copied their solution. Group 2’s progress on the task is displayed in figure 4. It is worth noting that both solution paths would lead to the correct answer, but the students decided to abandon and erase what they did on the left when they copied group 1’s solution. In the next section, we present transcripts of the segments during which each of the mathematical issues arose for group 2, as well as the subsequent GSI interaction.

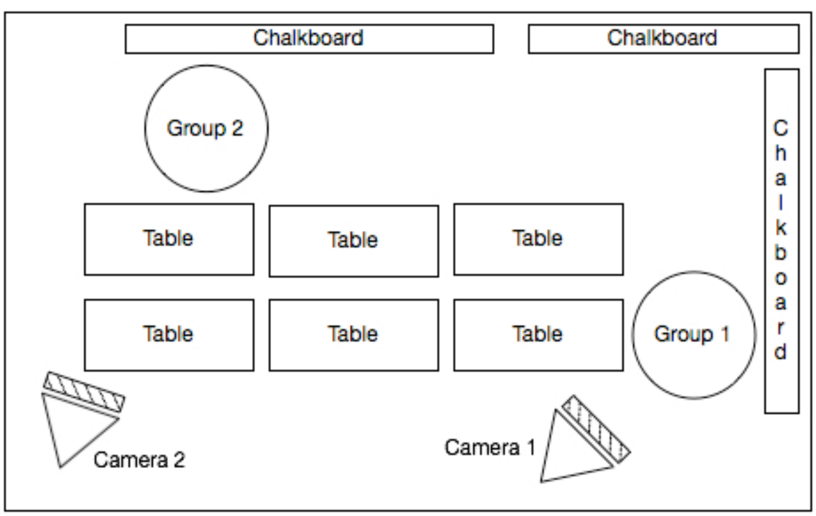

Figure 3. *Classroom layout of groups and camera locations*

$$\lim_{x\to\infty}\frac{x-\sqrt{x}}{1}\cdot\left(\frac{x+\sqrt{x}}{x+\sqrt{x}}\right)$$

$$\lim_{x\to\infty}\frac{x^2-x}{x+\sqrt{x}}$$

$$\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{x}{x^2}}{\frac{x}{x^2}+\frac{\sqrt{x}}{x^2}}$$

$$\lim_{x\to\infty}\frac{1-\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^{3/2}}}$$

$$\lim_{x\to\infty}\frac{x-\sqrt{x}}{1}\cdot\left(\frac{x+\sqrt{x}}{x+\sqrt{x}}\right)$$

$$\lim_{x\to\infty}\frac{x^2-x}{x+\sqrt{x}}$$

$$\lim_{x\to\infty}\frac{\frac{x^2}{x}-\frac{x}{x}}{\frac{x}{x}+\frac{\sqrt{x}}{x}}$$

$$\lim_{x\to\infty}\frac{x-1}{1+\frac{1}{\sqrt{x}}}=\infty$$

Figure 4. *Student board work prior to copying (left) and after copying (right).*

The Missed Opportunity

We argue that the missed opportunity is identified by a disconnect between three mathematical issues brought up by the students, and limited discussion with the GSI. The following pieces of transcript will provide evidence for the three mathematical issues that were not addressed during their interaction with Zack.

Students' mathematical issues

Issue #1: Why multiply by binomial conjugate?

20 A: I'm just trying to, I'm just trying to figure out why we did that [*pointing at conjugate*]. We can't do anything to there [*pointing at* $\frac{x-\sqrt{x}}{1}$]?

21 A: So from here we divide everything by x squared. I just don't get why we couldn't just divide everything here by x *[pointing at the same part]*

22 N: Yeah and make these to the [*inaudible*]

23 P: Probably because it doesn't have a denominator.

24 N: Oh, [*inaudible*]

25 A: Infinity minus root infinity. I don't know why we had to do (any) more steps. That would just be infinity. Uh- I don't know [*Priya is looking at group 1 who is talking to Zack and has infinity as the final answer*].

26 P: The thing is you cannot treat infinity like a number

27 A: Okay.

28 P: And by doing infinity minus root infinity, you're treating it like a number.

29 A: Okay, so you('ve got to) make it more complicated in order to make it count? [*laugh*] Whatever, whaaaa -

Lines 21, 25 and 29 show Amelia's struggle with the necessity of multiplying by the conjugate. She seemed to believe that the group could have divided by x without multiplying by the conjugate. Nanette agreed with Amelia (line 22), while Priya attempted to explain why it was necessary (lines 23, 26 and 28). At the end, Amelia still displayed frustration (line 29).

Issue #2: What to divide by - x or x^2?

38 P: Where did they get that? They divided by x [*looking at group 1, who are finishing their discussion with Zack on this problem*].
39 N: Yeah
40 A: But did they [group 1] do it right? [*inaudible*] Because that's what we did the first time. Did we have it wrong the first time or was it fine?
41 P: No no no no no no. They divided by x instead of x squared [*pointing at their work, Amelia turns to look at group 1*].
42 A: [*Amelia facing her group*] Up there? But that'd be stupid because they [Zack] said to use the largest exponent [*turning back to look at group 1*].
43 N: Is it largest exponent at the denomina/tor?
44 A: / [*Still facing the other group*] Did they do it right? Cus he checked them… Huh *[to Nanette]*?
45 N: [*inaudible*] the largest exponent that's on the denominator?/Not the whole?
46 A: /Nah, just the largest exponent.

This segment shows how the work of group 1 started to influence group 2 (line 41). This instigated a discussion about which 'largest power' they needed to divide by. Zack had taught them to divide by the largest exponent overall, while a member of group 1 decided to divide by the largest exponent on the denominator. Nanette tried to understand what the other group did, but Amelia persisted with the way Zack had taught them (line 46). The group seemed confused because Zack had checked and confirmed group 1's solution (line 44). We will see later that this issue persisted even after group 2 finished the problem.

Issue #3: What is 1/0 in a limit?

49 A: I don't think that cus then you don't get infinity then you get.. one... We did something wrong, we just don't know what.

50 P: We get one over zero *[within a limit]*.
51 A: Yeah. that's what's wrong. [*Amelia erases last line of their solution*]. Something's wrong-
52 P: Oh wait, one over zero is infinity-
53 A: No it ain't. One over zero is undefined.
54 P: Let's just (announce) everything infinity.
55 A: Nooot exactly.

Priya noticed that the group got 1/0 once they calculated the last line of their solution, before copying (see Figure 3 left) (line 50) and Amelia concluded that that was the mistake (line 51). Although Priya suggested that 1/0 is infinity, Amelia claimed that 1/0 is undefined (line 53). Priya recognized Amelia's argument and suggested that they call everything infinity.

Interaction with Zack

77 Z: Let's see how these are going
78 A: Three, four and five.
79 Z: Alright, good. Four… good…Five…
80 A: This part and then that (part)
81 Z: Also.. very good. [*inaudible*] x inside, perfect. One half, great!
82 Z: So the radicals are going a little bit-
83 A: Yeah, but they're [*inaudible*]
84 Z: They're very annoying. They only get more annoying the further you go in calculus.
85 A: Peachy.

Zack's involvement consisted of affirming group 2's solution (lines 79, 81) and briefly discussing how "annoying" radicals are (lines 82, 84). Given Zack's response, a clear disconnect between the mathematical issues brought up by students and those raised by the GSI was demonstrated. This disconnect persisted after the GSI left the group, as we see below.

86 P: Do you know why and how I got to divide by x-squared?
87 A: Do I know how to divide by what?
88 P: (Divide by x-squared)
89 A: Cuz if you divide by x-squared you get messed up. You can ask him if you want.
90 P:(Nah)

Interplay of Resources, Collections of Beliefs, and Practices

In this section, discussion of the interplay between beliefs and practice is presented. The analysis begins with the resources available to Zack and the way they were utilized in the *actual* *practice* with group 2 [which led to the missed opportunity]. Analysis of an *alternate practice* follows. This alternate practice is the practice that Zack chose when interacting with group 1.

Actual Practice: Confirming solution as presented. Figure 5 gives an overview of the analysis of the practice of confirming solution as presented. Some of resources that were available to the GSI include artifacts like the student board work (Figure 4, right) and the group's progress on a worksheet, while Zack's resources that he brings to the interaction include prior experiences that he had with group 2, and prior experience he had with other groups working on this problem. For example, Zack had just interacted with group 1, and had a lengthy discussion about the task.

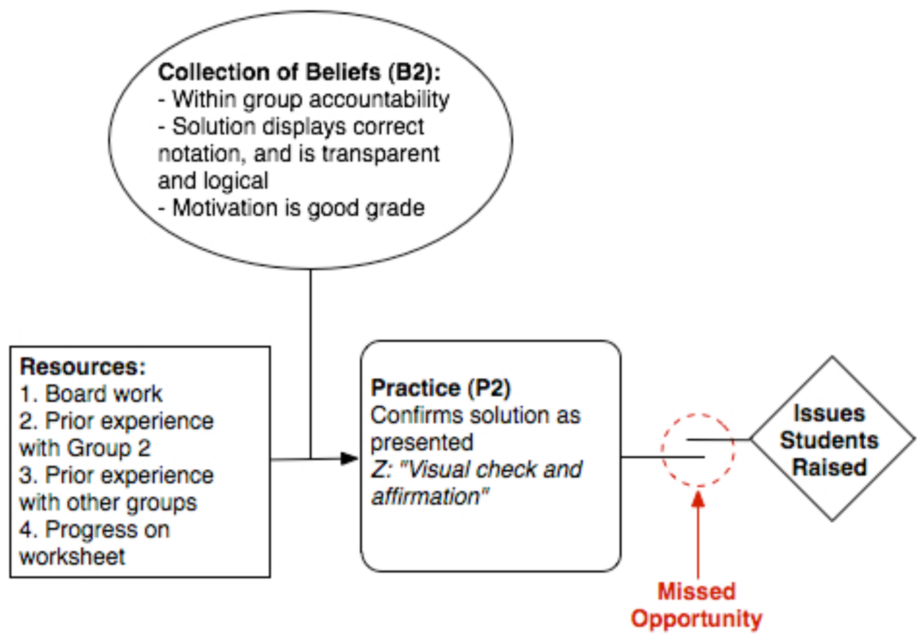

Figure 5. *Interplay of resources, one collection of beliefs, and practice of confirming solution*

A collection of Zack's beliefs that would be consistent with this choice of practice include his belief about students in group 2 – namely, their within group accountability and motivation – and his belief about the characteristics of correct solutions.

Zack believes that group 2 works well together. When one of them does not understand something, the others stop and explain. They also do not move on unless everyone is in agreement with their solution. Though this is not what happened in this particular video episode, Zack repeatedly expressed in interviews that these students were accountable to one another. This is an example of a belief that he perceived to be true, which in reality was sometimes false. Because the students came to a correct solution, and his belief would indicate that this group would have explained to one another if there was confusion, Zack had no reason to stop and ask for further explanation.

Zack’s belief about how a mathematical solution should be presented is also consistent with this choice of practice. He believes that a solution should be transparent such that students do not need to rely on a reader to make inferences. It also should use correct notation and follow a logical order. This was what he saw on the board - the students’ work was “perfect” [as they copied what he had discussed with the other group] - and it did not occur to Zack to question it.

Finally, students in group 2 were not engineering or mathematics majors. Zack believes that getting a good grade is the motivation for these students. Therefore, provided that they wrote up their solution perfectly and had internally explained and resolved any issues that might have come up, it was safe to assume that this would be sufficient for them to do well on exams.

Alternate Practice: Asking for explanation or justification. The collection of beliefs that is consistent with this alternate choice of practice includes beliefs about students in group 1 and about classroom norms. Figure 6 gives an overview of the analysis.

Zack believes that group 1 was his “fast group” - the group who worked quickly, “didn’t get stuck” as often, and was able to make a “cognitive jump.” By cognitive jump, Zack meant that if he pushed them, they would be able to make connections between concepts and ideas in mathematics. Given the fact that he spent a considerable amount of time resolving issues around the limit task with group 1, and knowing that group 1 is usually much faster than group 2, Zack may have considered probing group 2's solution, even though it appeared correct.

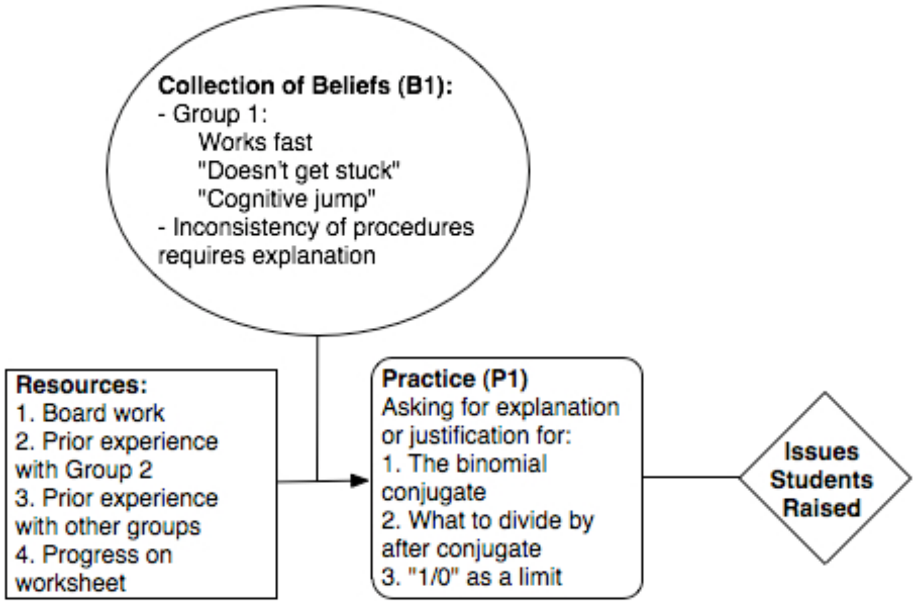

Figure 6. *Interplay of resources, one collection of beliefs, and practice of asking for explanation or justification*

The GSI also holds many beliefs about classroom norms, one of which is relevant to this alternate practice. That is, if a student's procedure or method of solving a task is different from the method discussed in class, s/he should explain her/his method to the group. In interaction with group 1, Zack noticed that a group member used a differing method, so he took steps to ensure that the particular group member explained his strategy to the others. As group 2 copied the work from group 1's board, it stands to reason that Zack would expect the same explanation within group 1 - in other words, he should ask for the student to explain her differing solution path. Had he done this alternate practice, which he demonstrated with group 1, elements of this missed opportunity may have been addressed and resolved.

Summary of Analysis. Different collections of beliefs provide a plausible explanation for different choices of practice. A collection that includes Zack's belief about group 2's motivation and within group accountability, and an appropriate mathematics solution is consistent with the choice of practice of confirming solution as presented. On the other hand, a different collection that includes his beliefs about strengths of students in group 1 and consistency for choosing procedures is more consistent with the practice of asking for justification and explanation. While the first practice led to a missed opportunity, the alternate one may have addressed some of the issues brought up by students. Locating and examining this 'missed opportunity' elucidates the complex relationship between Zack's large set of beliefs and his specific choices of practice for this task.

Validation: Post-Semester Video Clip Interviews

After having analyzed this and other instances of missed opportunities for Zack, researchers were able to utilize further data collected in the original study (Adiredja et al., 2008) in an attempt to 'validate' these findings. During video clip interviews (as discussed in Speer 2001, 2005), Zack was shown this particular episode, parsed into several phases, and asked to comment on the purpose of the task, what happened at different points of the interaction, and how his actions related to the students. His comments at different points of the interaction align with the collections of beliefs used to explain the interaction (see Table 1).

Though it was not the intention of the researchers to study 'missed opportunities' at the time of data collection, Zack also provides evidence for such a classification during the video-clip interview. After watching the entire episode, the following exchange takes place:

> Interviewer: How is what you were doing related to what the students did, if at all?
>
> Z: Right here, not much. It's just a visual check and affirmation.

As Zack indicates, an opportunity to address the problematic mathematics for the group is not seized here, due to his decision to use the practice of 'visual check and affirmation.'

Table 1. *Consistencies and Support for Collection of Beliefs*

	Zack's comments from video clip interview	**Corresponding belief from collection of beliefs**
Support for Collection of Beliefs related to *Actual Practice(s)*	Z: "[Group 2], as a group, was very good about internal discussion. Meaning that they were more likely than an average group to ask each other questions and they were more likely than an average group to accept input."	Students in Group 2 feel accountable for one another's understanding of the mathematics
	After observing the confusion and copying: Z: "I had no reason to think that there was… when I see work that is correct, I have no reason to think, no why … I don't know the thought process that went into it. And as long as I see accurate work, I can assume that a thought process went into it, and I'm not concerned."	An ideal solution will (1) be transparent to a reader, (2) follow a logical progression of thought, and (3) use acceptable notation/style.
Support for Collection of Beliefs related to *Alternate Practice(s)*	Z: "[Group 1] really didn't ask so much, but then they really didn't get stuck as much… as most of the other groups. And even they were probably also just more comfortable…"	–Group 1 is able to work quickly through worksheet tasks. –Group 1 doesn't get stuck as much as other groups.
	After observing the confusion and copying: Z: "What do I see?... I see largely copying down the way [a member of group 1] had done it [dividing by x]…What bothers me about this is, first of all, is that they undid the x-squared, which means that they didn't see how they could get to the right answer…"	A student needs to share with his/her group his/her way of solving a problem, especially when it is different from the method covered in lecture or discussion

Discussion and Implications

Using video data, the disconnect between group 2's struggle with mathematics, and their subsequent interaction with Zack – the missed opportunity – is evident. Most importantly, though, one can use this "missed opportunity" as the lens through which relationships between different beliefs and practices can be examined at the most appropriate level of detail (Speer, 2008). Specifically, one collection of beliefs gives a plausible explanation for Zack's actual choice of practice, allowing group 2 to move on without explanation. Another collection of beliefs that Zack holds would provide plausible explanation for an alternate choice of practice, questioning the group's correct solution, in the same scenario. Therefore, the missed opportunity afforded us a means for identifying how these specific collections of beliefs may have interacted to influence Zack's decision to allow group 2 to move on without explanation.

A well defined, 'narrowing' procedure, such as identifying missed opportunities, is necessary to carry out this analysis in a meaningful way, because broad categories of beliefs and general statements of practice shed very little light on how specific collections of beliefs influence practices in context. Missed opportunities afford researchers and practitioners the opportunity to consider both the *actual* outcome, and an *alternate* outcome, had the GSI made a different choice of practice within his set of typical practices. This construct also does not force researchers to isolate a practice or set of practices a priori, and then seek to explain them, nor does it require researchers to conjecture a series of possible practices that are not grounded in a specific video episode before examining their relationship to beliefs. In other words, missed opportunities allow a discussion of beliefs and practices to be grounded in a specific video episode of a particular GSI.

Many professional development programs dedicate significant amounts of time and resources to explaining and advocating "ideal practices" to GSIs, with the intention of improving their instruction. However, Zack was a passionate, well-intentioned GSI whose practices often resonated with his math department's suggested "ideal practices," (such as questioning and using group work), and there are *still* isolated instances in which Zack's choices of practice resulted in missed opportunities that could have been avoided. This analysis invites a discussion of what may be done within professional development programs and math departments, in addition to simply advocating ideal practices, in order to improve undergraduate instruction.

Considering the call for structured professional development for GSIs, and the need to support the use of “ideal practices” and reflection, we propose that missed opportunities may provide a useful lens through which GSIs may explore the relationship between their beliefs and their practices. Existing research advocates asking GSIs to think about their beliefs and practices, and how the two interact, but this is very general activity, and it may be hard to focus a discussion as such. Using a construct such as missed opportunity may allow GSIs to engage with a very tangible and approachable scheme for watching video of themselves or others, and think about the interplay between their beliefs and practices. Future work will explore GSIs’ use of this construct in examining their own teaching, and how ‘missed opportunities’ may provide meaningful structure for professional development activities.

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