

Tertiary Students' Informal Conceptions of Margin of Error and Confidence

Estrella M.S. Johnson, Portland State University
Sonya Redmond, Portland State University
Joanna Bartlo, Portland State University
Jennifer Noll, Portland State University

Introduction

In recent years the demand for statistics education has seen a dramatic increase. In addition to academic and professional needs (Ben-Zvi & Garfield, 2004; Luzter, Rodi, Kirkman & Maxwell, 2005; National Council for Teachers of Mathematics, 2000; Ben-Zvi & Garfield, 2004), statistical knowledge is also necessary for making intelligent personal and professional decisions, and for equal participation in society (Ben-Zvi & Garfield, 2004; Gal, 2004).

One of the most commonly used statistical tools designed to help inform personal and professional decisions is public opinion polls; however, these polls are routinely interpreted in unintended ways (Saldanha, 2004; Thompson & Liu, 2005). Our research examines tertiary students' understandings of published polling results, and in particular, the ways in which tertiary students rely on colloquial meanings of margin of error and confidence to make sense of polling results.

Background

There are two statistically equivalent ways of thinking about margin of error. From a *population centered* view, one can imagine the margin of error creating an interval centered at the population parameter. Through repeated sampling a sampling distribution is generated, and the percentage of sample statistics captured by the interval centered at the population parameter is defined as the *confidence level*. From a *sample centered* view we can alternatively imagine the margin of error creating an interval

centered at a sample statistic. Through repeated sampling, a collection of intervals is generated wherein a percentage of intervals equal to the confidence level capture the population parameter. It should be noted that a sample centered view of margin of error and confidence is based on a long-term relative frequency perspective.

From either perspective, a coherent statistical understanding of margin of error requires an understanding of confidence, repeated sampling, sample variability, and sampling distributions - concepts that are themselves complex and not easily understood or coordinated (Saldanha & Thompson, 2007; 2002; Thompson & Liu, 2005). The research literature reports on difficulties both high school statistics teachers (Thompson & Liu, 2005) and mathematics and statistics graduate teaching assistants (Noll, 2007) have had with these concepts. However, little has been said with regard to tertiary students' understandings. Our study aims to fill this gap by reporting tertiary students' informal and intuitive understandings of margin of error and confidence in the context of a published political polling scenario.

The Study

As part of our initial data collection, surveys were given to sixty-four students who were enrolled in either an introduction to statistics course or one of the first two courses of a three-course mathematics for elementary teachers sequence. During our preliminary analysis of students' responses it became evident that students were not necessarily using statistics terminology in its intended way, and moreover, that many students appeared to equate a small reported margin of error with a high level of "confidence". An interview task designed to uncover students' ideas about margin of error and confidence in relation to these observations were developed, and follow up

interviews were conducted with eight of the mathematics for elementary students.

Students' responses to the margin of error interview scenario form the basis of this preliminary report.

The follow-up interviews were video taped, and responses to the margin of error scenario transcribed verbatim. Our analysis of the margin of error interview data included an independent review and open coding by each of the first two authors. Further discussion and analysis of the video transcripts spurred the negotiation of three research questions:

- 1) What intuitions and informal understandings of margin of error are held by tertiary students, and what factors do they believe affect the margin of error?
- 2) What are students' intuitions and informal understandings of confidence?
- 3) In what ways do students relate margin of error and confidence?

Our consideration of these questions led to the development of preliminary frameworks describing ways in which tertiary students may think about the concepts of margin of error and confidence.

Results

In this section we describe the margin of error and confidence frameworks that arose from our qualitative analysis of students' responses to the margin of error interview scenario. We begin by presenting our results related to margin of error, and follow with results related to confidence.

Margin of Error

The margin of error interview scenario asks students to describe their understanding of the term margin of error within the context of a political poll, and to

discuss the reasons why two reported margins of error differ. In their discussion, students focused on what we have termed *human error*, or errors that are potentially correctable. Based on our analysis of students' responses, we were able to identify four classifications of human error: Data Error, Research Design Error, Respondent Error, and Sample Size Error.

Data Error: Data errors include general collection and handling errors. One student's response that "any types of mathematician errors, any types of polling errors, are all going to be calculated into that margin of error" is typical of this classification.

Research Design Error: Research design errors include errors in the sample design. This classification includes most sample bias. Typical responses in this category would be that the two margins of error differ because "maybe they only asked people in certain parts of the country".

Respondent Error: Respondent errors are errors caused by the individuals being polled. The response that "margin of error is if they answered a question in a weird way" is typical of this classification.

Sample Size Error: Sample size error is attributed to the size of the sample. For instance, a representative response is that "if the Associated Press has a large margin of error maybe they didn't have a large sample size". While it is true that having a larger sample size will lower margin of error by reducing sample variability, we saw no evidence to suggest that the students saw margin of error as a measure of sample variability.

Confidence

It was clear from our analysis of students' responses to the margin of error interview scenario that most participants were not using the term confidence in the intended statistical sense. The interview data provided strong evidence that students' made two key assessments in relation to *confidence*: (1) *who* was confident, and (2) *what* is one confident in. From an iterative analysis of the interview data, four classifications of the term *confidence* arose: Confidence as Trustworthiness, Confidence as Usefulness, Confidence as Accuracy, and Confidence as a Statistical Measure.

Confidence as Trustworthiness: Confidence as trustworthiness describes the confidence that a consumer of polling results has in a polling company. For instance, one student stated that, "my confidence in them will be based on if they've been right in the past, if I trust them." In this statement we see that the student is talking about their own personal confidence in whether or not they can trust the polling company.

Confidence as Usefulness: Confidence as usefulness describes the confidence *a consumer* of polling results has in the predictive value of a polling result. For instance, one student stated that they are "[more confident in the second poll than the first] because it would be easy to see that Obama is leading the race in the second poll." This student assumes a level of *personal confidence* dependent on whether they feel the results are useful in making a decision or prediction.

Confidence as Accuracy: Confidence as accuracy describes how confident *a polling company* is that their reported statistic accurately represents the population parameter. Representative of this classification of meaning is the student response, "they made a

tighter range there which means they were pretty confident that they were gonna fall in there”. Here the student is talking about how confident *the polling company* is in *their* results.

Confidence as a Statistical Measure: Confidence as a statistical measure describes the confidence level assigned to a reported statistic by *the polling company* through a calculation or process, and not just a feeling. For example, the student response, “there is no data to look at here to determine what confidence level they had and I don’t think the numbers here give us enough information to determine confidence” indicates that the student sees confidence as being assigned by the polling company itself and that specific data can be used to determine a quantitative measure.

Discussion

It is clear from our results that students participating in our study thought about both margin of error and confidence in a colloquial sense rather than a statistical sense. This tendency may have several implications. For one, viewing margin of error as correctable human error obscures the role of random sample variability. Moreover, thinking of confidence in a colloquial sense may keep students from seeing confidence as a quantifiable measure tied to repeated sampling.

Students’ understandings of the relationship between margin of error and confidence are also likely to be hindered by a colloquial interpretation of margin of error and confidence. For instance, seven of the eight students interviewed in our study agreed with the hypothetical student response, “The second poll has a lower margin of error because they are more confident in their results”. Note that this interpretation is consistent with students’ colloquial usage of the terms error and confidence. These results

suggest that the statistically correct relationship between margin of error and confidence is not intuitive, and moreover, that attributing margin of error to correctable human error may support a statistically incorrect understanding of their relationship.

Conclusion

Our analysis of students' responses revealed emerging and informal views of margin of error and confidence. We believe our frameworks offer a valuable lens through which we may begin to understand ways in which tertiary students think about margin of error and confidence. There remain, however, tremendous opportunities for continued research on tertiary students' understanding of margin of error and confidence. Below we offer suggestions in the form of specific questions that arose during our investigation.

1. How would students respond if human error were eliminated?
2. How does increasing the sample size decrease the margin of error and increase confidence if not by decreasing variation?
3. What is the "it" captured by the interval? The population parameter, or the sample statistic?
4. Are the students expressing confidence in terms of the interval or the point estimate?

While the terms *margin of error* and *confidence* have very strong and significant statistical definitions, they also have strong and significant colloquial meanings. This study indicates tertiary students' statistical understandings may be difficult to develop because of strong colloquial associations with the terms *error* and *confidence*, as well as an absence of awareness of the role of repeated sampling.

References

- Ben-Zvi, D., & Garfield, J. (2004). Statistical literacy, reasoning, and thinking: Goals, definitions, and challenges. In J. Garfield & D. Ben-Zvi (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (pp. 3-15). Dordrecht, The Netherlands: Kluwer.
- Gal, I. (2004). Statistical literacy: Meanings, components, responsibilities. In J. Garfield & D. Ben-Zvi (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 47-78). Dordrecht, The Netherlands: Kluwer Academic.
- Luzter, D.J., Rodi, S.B., Kirkman, & Maxwell, J.W. (2005). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States: Fall 2000 CBMS Survey*. Retrieved on 5/17/05 at <http://www.ams.org/cbms/cbms2000.html>
- Liu, Y. (2005). A theoretical framework for understanding teachers' personal and pedagogical understanding of probability and statistical inference. Unpublished doctoral dissertation, Peabody College, Vanderbilt University.
- National Council of Teachers of Mathematics. (2000). *Principals and standards for school mathematics*. National Council of Teachers of Mathematics, Inc., Reston, VA.
- Noll, J.A. (2007). Graduate teaching assistants' statistical knowledge for teaching. Unpublished doctoral dissertation, Portland State University, Portland, OR.
- Saldanha, L.A. (2004). "Is this sample unusual?" An investigation of students exploring connections between sampling distributions and statistical inference. Unpublished doctoral dissertation, Vanderbilt University, Nashville, TN.
- Saldanha, L.A., & Thompson, P.W. (2002). Conceptions of sample and their relationship to statistical inference. *Educational Studies in Mathematics*, 51, 257-270.
- Saldanha, L.A., & Thompson, P.W. (2007). Exploring connections between sampling distributions and statistical inference: An analysis of students' engagement and thinking in the context of instruction involving repeated sampling. *International Electronic Journal of Mathematics Education*, 2(3).
- Thompson, P.W., & Liu, Y. (2005) Understandings of margin of error. In S. Wilson (Ed.), *Proceedings of the Twenty-seventh Annual Meeting of the International Group for the Psychology of Mathematics Education*, Roanoke, VA. Vicksburg, VA: Virginia Tech.