Student Mathematical Discourse and Team Teaching

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Introduction

This project began with a comparison of the level of student mathematical discourse in two different classroom settings: a summer institute for K-12 teachers and undergraduate mathematics courses. The same instructors led the classes in both settings. In preparation for the summer institute, the instructors attended workshops focused on the importance of and methods for eliciting student mathematical discourse. After participating in the summer institute, the instructors returned to teaching their typical undergraduate courses. The initial research question focused on whether the classrooms in the two settings exhibit similar levels of mathematical discourse.

Background on Mathematical Discourse

The focus on discourse in the teaching of mathematics, which has been the subject of considerable research in K-12 teaching, is extending to undergraduate mathematics education. The research on discourse in K-12 mathematics teaching has identified the characteristics of quality discourse and its impact on student learning (NCTM, 1989, 1991, 2000, 2003). Research is now examining undergraduate mathematics teaching in order to determine the relevance of the K-12 findings in this setting.

While it is important for students to be communicating, not all types of student mathematical discourse have the same impact on development of students' conceptual understanding. When student discourse is oriented toward what one *does* rather than what one *thinks*, students may continue to believe that the reason they are given mathematical problems is to find answers that are numbers or calculations, or that working problems means searching for operations to perform. They also may not be able to assess their own understanding of mathematical concepts (Clement, 1997). A classroom environment where students are expected to go beyond explaining how they solve a problem to sharing their mathematical understanding can be established by the use of socio-mathematical norms. Yackel (2001) cites the importance of socio-mathematical norms of explanation and justification. These norms include "that students explain and justify their thinking, that they listen to and attempt to make sense of the explanation of others, and that explanations describe actions on objects that are experientially real for them" (Yackel, 2001, p.1 - 17). Further, Yackel emphasizes that the instructor and the students develop the norms interactively. The instructor sets the expectations and influences the ways in which students (especially shy and reticent students) participate in the discussion. The students contribute to the norms by increasingly acting in accordance with these expectations.

The Pilot Study

The Settings

The motivation for this study grew out of our experience working in a National Science Foundation Math Science Partnership (NSF-MSP) teacher institute project. The project represented a partnership between two state universities, 10 public school districts, and a non-profit professional development organization. The project staff included higher-education faculty from mathematics and education departments at public and private colleges, universities and community colleges; K-12 master teachers; and experienced professional development staff. The teachers participating in the project spanned all grade levels, from kindergarten through high school. One of the project's central activities was a series of three-week institutes over three consecutive summers. Each participating teacher attended two mathematics content courses each summer (for a total of six content courses throughout the project.) Each of these content courses was team taught (by four instructors) with a class size of approximately 30 and usually split into two sections of about 15 participants.

Goals of the project were to:

- Increase mathematics achievement of all students in participating schools
- Close achievement gaps for underrepresented groups of students
- Increase enrollment and success in challenging mathematics course work that supports state and national standards through coherent, evidence-based programs.

In order to achieve these goals, one focus of the project was to build the strong content knowledge "necessary to enable teachers to transform their classes into mathematical learning communities where students engage in high level discourse around important mathematical ideas. Teachers' content knowledge must be built in ways that connect important ideas clearly to school mathematics and using approaches that model effective instruction" (Dick, T. 2004, p.1). Specifically, the project's logic model proposed that student achievement in mathematics could be significantly improved by increasing the quantity and quality of meaningful mathematics discourse in the mathematics classes in the schools of the teacher participants. Hence, attention to the teacher participants' mathematical discourse was also stressed in the summer institute mathematics content classes.

<u>Pilot Investigation: Impact of summer institute experience on higher education faculty</u>

A key evaluation question for the project involved documenting the quantity and quality of student mathematical discourse in a sample of the teacher participants' classrooms to determine the project's impact on those classrooms. We hypothesized that the project might have a similar impact on the regular undergraduate classrooms of the higher education faculty teaching the institute mathematics content courses in which they paid particular attention to the participant mathematical discourse. With that in mind, we undertook a pilot case study of the student mathematical discourse in the regular classes of three members of the grant project instructional staff.

The primary subjects for this study were three higher-education grant project staff teaching in the same content course during the summer institute, one of whom is an author of this study. All subjects hold doctorates in mathematics. One of the subjects is a full time faculty member at a community college and has been teaching at the college level for over five years. The other two subjects are professors at private colleges, one a full professor with over 30 years college teaching experience and the other an assistant professor with over five years experience. All three subjects collaborated with a master teacher on the development of the institute content course and team-taught the course in pairs.

Methodology

For our case studies, observations of all three subjects were conducted during one summer institute of the grant projects and in their regular undergraduate classes the following fall term. During the summer institute, observations of two sessions led by each instructor were conducted by student research associates. During the fall term, two or three observations of each class being taught by each instructor were conducted by one of the authors. Each observation was videotaped and data on the quality and quantity of discourse was recorded using the discourse observation protocol developed by the grant project (Weaver and Dick, 2006). All observers were trained in the use of the discourse observation protocol.

The discourse observation protocol was developed specifically to record and measure the quantity and quality of student mathematical discourse in the classroom. The aspect of the observation protocol analyzed in this study involves the discourse taxonomy (see Table 1) that classifies each incident of student discourse by types that are grouped in levels. The first level of discourse including answering, stating, or sharing is the lowest cognitive demand level. Discourse in Level Five including justification and generalizing is the highest cognitive demand level. The observation protocol also records size of group. Only data from whole class episodes is presented in this paper. The mode of discourse and tools used are also recorded but are not examined in this study.

Level	Type of Discourse	Description
1	**Answering, stating, or sharing**	A student gives a short right or wrong answer to a direct question, or a student makes a simple statement or shares his or her results in a way that does not involve an explanation of how or why.
2	**Explaining**	A student explains a mathematical idea or procedure by describing how or what he or she did but does not explain why.
3	**Questioning or challenging**	A student asks a question to clarify his or her understanding of a mathematical idea or procedure, or a student makes a statement or asks a question in a way that challenges the validity of an idea or procedure.
4	**Relating, predicting, or conjecturing**	A student makes a statement indicating that he or she has made a connection or sees a relationship to some prior knowledge or experience, or a student makes a prediction or a conjecture based on his or her understanding of the mathematics behind the problem.
5	**Justifying or generalizing**	A student provides a justification for the validity of a mathematical idea or procedure, or makes a statement that is evidence of a shift from a specific example to the general case.

Table 1: Discourse taxonomy used classifying levels of discourse[1].

Data Analysis

Level of discourse was analyzed by computing the proportion of discourse at each level for each instructor. Comparisons were made between the proportions of discourse at each level for all three instructors. The trends in the distributions of proportions of

[1] From Oregon Mathematics Leadership Institute Spring 2007 Evaluation Report by D. Weaver, 2007, Portland, OR, RMC Research Corporation, p. 26.

discourse were similar for all three instructors. To facilitate the analysis of the comparison between the two settings, the data from all instructors were combined for each setting. Comparisons between proportions of levels of discourse in the two settings are justified by similar rates of discourse (1.72 instances per minute in the regular classes and 1.21 instances per minute in the summer institute). In regular classes, Level One discourse is the mode while in the summer institute course, Level Two discourse is the mode (see Figure 1). Further, in regular classes the proportion of discourse decreases as the cognitive level of discourse increases. Clearly, since Level Two is the mode for the summer institute course the decreasing trend is not present. Moreover, the proportion of discourse at the higher cognitive levels (four and five) tends to be greater in the summer institute as compared to the regular classes of all instructors.

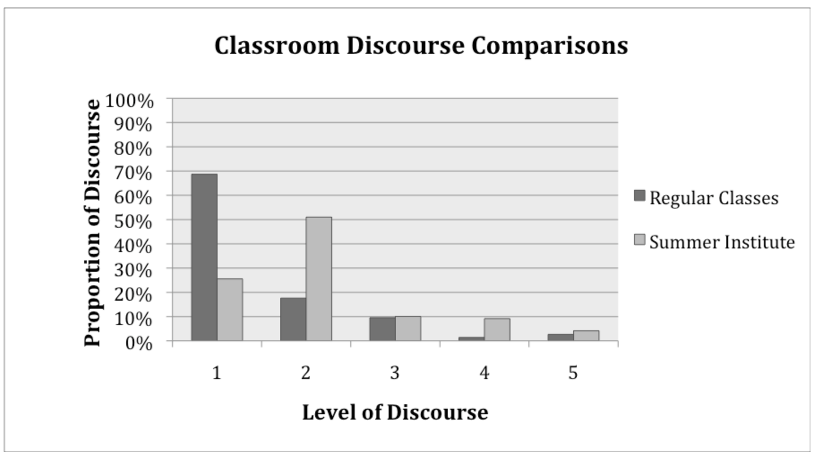

Figure 1: Level of discourse comparisons between regular and summer institute classes.

<u>Discussion and a new hypothesis on the impact of team teaching on student discourse</u>

The contrast between what was seen in the summer institute classrooms and the regular classrooms of the project instructors reveal differences in the proportions of discourse in these two settings. Discussions regarding factors that might have influence the levels of discourse during the summer institute that were not present in the regular classrooms returned again and again to the structure of the summer institutes classes. A team of two instructors facilitated each of these classes. For each lesson, one instructor was the lead and was primarily responsible for facilitating the activities and discussion. The second instructor acted in a supporting role assisting with the monitoring of small groups, time management, selection and sequencing of presentations and facilitation of discussion. Since team teaching was such an integral part of the summer institute classrooms and not present in the regular classrooms of the instructors, it was conjectured that the structure of the summer institute, particularly the employment of team teaching, may have had an impact on the level of student mathematical discourse present in these classrooms.

Background on Team Teaching

In their research on team teaching Cook and Friend (1996) identified four key components of co-teaching: (1) two educators, (2) delivery of meaningful instruction, (3) diverse groups of students, and (4) common settings. Utilizing these components, they went on to describe five forms of variation in co-teaching: (1) one teaching/one assisting: one instructor takes an instructional lead while the other assists, (2) station teaching: each instructor working on a specified part of the curriculum in the classroom, (3) parallel teaching: instructors plan together, but divide the class for instruction, (4) alternative teaching: divide class into one large group for main instruction and one small group for alternative instruction, and (5) team teaching: instructors take turns leading discussion and in other roles throughout the class.

Grassl and Mingus (2007) concluded that team teaching can allow for dynamic interaction between the instructors and between instructors and students, allowing students to experience different viewpoints of the instructors. Some advantages of team teaching cited in their study include: students hearing alternative ways of explaining the same concept; the availability of immediate feedback on how the class is progressing; the assisting instructor asking leading questions to clarify student thinking, make extensions, and/or connections; and the assisting instructor highlighting opportunities for student speaking. In addition, Grassl and Mingus found evidence that team teaching supported efforts to sustain reform teaching beyond the team teaching setting. Both instructors involved in their study taught the subsequently taught the course independently and noted that one of them taught the course with the "same spirit, organization and results" while the other instructor has "changed the nature of her exams to included more challenging problems, with higher expectations" (Grassl & Mingus, 2007, p. 596).

The summer institute course embodied the four components of team teaching stated above and primarily used the one teaching/one assisting form described. Within this framework, we formulated a new research question: does team teaching foster the increased level of student mathematical discourse observed in the summer institute classroom by affording an advantage similar to those suggested by Grassl and Mingus.

Team Teaching in the Summer Institute Classrooms

Re-examination of the data

The videotapes of the summer institute classes were reviewed with a focus on the actions of the second instructor. This review identified four categories of second instructor actions: monitoring small groups, time management, selecting and sequencing, and interjections into whole class discussion. While all of these second instructor actions added to the effectiveness of the instructional team, the direct effect on student discourse can only be isolated in the interjections to whole class discussion. The second instructor actions during the monitoring of small groups likely affected the level of discourse within the small groups, but cannot be shown to have affected discourse in whole group discussions. The skills developed by both instructors through working together on the selecting and sequencing of presentations appear to be readily transferable to the solo instructor classroom. Therefore, the episodes in which the second instructor made interjections into whole class discussions were further examined. These episodes were transcribed to facilitate detailed analysis of the student mathematical discourse and the role of the second instructor.

To determine whether the actions of the second instructor during the whole class discussion raised the level of student mathematical discourse, the student discourse following interjections by the second instructor during the whole class discussion were recoded. This recoding was necessary because the discourse in these episodes could not be isolated in the original observation discourse protocol records. Analysis of this recoding of student discourse revealed an entirely different distribution of levels of discourse following an interjection by the second instructor than was found in general whole class discussion during the summer institute class. The primary difference in these distributions was higher proportions of student discourse in Levels Three, Four and Five than were seen in other situations (see Figure 2). These higher proportions of higher-level student discourse provided initial evidence of the positive impact of the second instructor on student mathematical discourse.

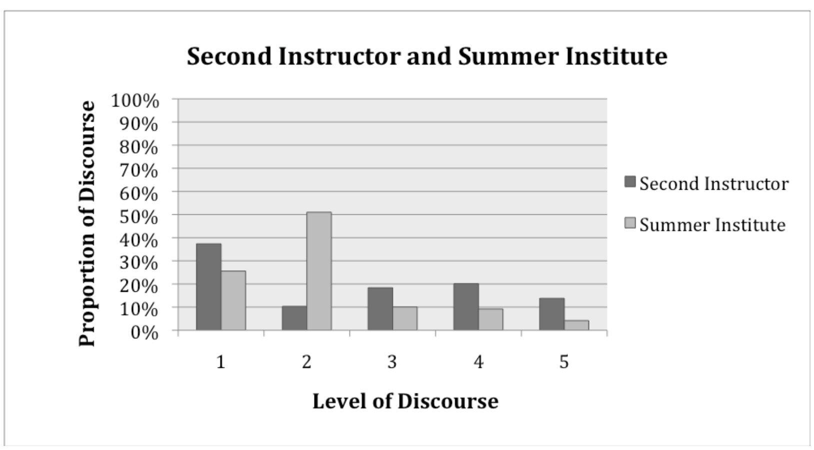

Figure 2:Level of discourse comparisons between summer institute class and second instructor episodes.

Motivated by the increased high level discourse, the second instructor's interjections into whole class discussions observed in these sessions were further analyzed and grouped according to their purpose and effect. The following classifications were developed to identify these four types of second instructor interjections.

1) *Side-trips* are instances when the second instructor pursued an opportunity to discuss an important mathematical idea related to but not necessarily the focus of the lesson.
2) *Pressing for clarification* occurs when the second instructor detected disequilibrium or confusion and encouraged more thorough explanation of the ideas being discussed.
3) An *extension* occurs when the second instructor prompted additional exploration and discussion in order to deepen the mathematical understanding of the primary lesson focus.
4) *Highlighting* serves to bring attention to a student's contribution that might otherwise have gone unnoticed.

Examples of Second Instructor Interjections

While all four types of interjection by the second instructor served to promote meaningful student discourse, each type plays a unique role in the classroom. The following examples are meant to illustrate these roles.

Side-Trip

The following example takes place during the whole class discussion of an activity in which participants were instructed to find models for geometric patterns.

> Presenting participant: This is how I did it exactly like Student H except I just put it into two t-charts so you can see so I could compare that the stage and the vertices or that the n column and the edges column were the same numbers and then I could compare the stage and the vertices and saw that there was a difference of one so I plugged in the n-1 wherever I had n.
>
> Second Instructor: And when you did that what did you get

Presenting participant: Um the um this n-1, n-1 + 1 over 2 and then simplified to n -1 times n divided by 2.

Second Instructor: So my question is that formula n-1 times n over 2 equivalent to Student H's formula which was n times n-1 over 2 plus n.

In this episode, the second instructor's interjection instigates discourse on determining the equivalence of two algebraic expressions. While this was an important discussion in light of the diversity of algebraic skill among the K-12 teacher participants, it was not the focus of the lesson. The second instructor made the decision, for the benefit of the class, to take this opportunity for a side-trip providing the opportunity for meaningful mathematical discourse that would not otherwise have occurred, but was not directly related to the main focus of the lesson.

Highlighting

In this episode the lead instructor is beginning the transition to a new task when a student makes a comment that prompts the second instructor to call attention to her comment.

Lead instructor: Hang on to that thought because there actually is some graph theory models there, correct?

Participant: Does the, are the students and the classes kind of like the vertices?

Lead Instructor: Well we are going to figure that out.

Second Instructor: Did you hear what Student E's concern was?

This example illustrates the second instructor calling attention to the question of a student that might have gone unnoticed by the class. As occurred in this episode, the highlighting may not lead immediately to student mathematical discourse. The value of the highlighting may be its role in the establishment and reinforcement of the socio-mathematical norms of the classroom that value the contributions of all participants and encourage meaningful mathematical discourse.

Pressing for Clarification

In this episode the participants are exploring the question of how to count the number of distinct ways that five dashes and two lines (seven items total) can be arranged. The participant begins his presentation with the seven factorial needed for counting the arrangements of all seven items.

> Presenting participant: (A)nytime you have a number of elements in a pattern such as repeating lines (in this problem) you have …seven elements so you have a factorial 7…
>
> Second Instructor: Does anybody have a question about where he got 7!? What is it representing?

The second instructor notices that some participants still seem unclear about the concept and presses the presenting participant for clarification. The ensuing discussion, although it takes a few prompts to move from the level of responding to the second instructor's queries, moves on to a higher level of discourse. Other participants gain access to the problem and help the presenter clarify by relating to previously explored problems.

> Second Instructor: So how do you see that? How do you see that was 7!? Can you be specific about where you saw that in the 7 things?
>
> Presenting participant: Ok, if you label . . .(the) dash(es) would be one through five, line one and line two. You can arrange each one of those seven different ways. There is a factorial of 7. Does that make sense?
>
> Respondent A: I am wondering does that work only if you are given the exact things that must be in those seven like you must have five dashes and you must have two lines? Would it work for the alphabet, say take the letters of the alphabet and stick any 7 letters in?
>
> Respondent B: Are you asking if you only put them in seven spots like there are only 7 spots but you are using the whole alphabet?
>
> …
>
> Respondent C: That would be more like the fifteen books and the three slots.

Respondent D: Or (when) there were nine toppings but we only chose three.

Respondent A: I am still not clear on the 7! … where it has been coming from before was the idea that for the first slot you have 7 choices, for the second slot you have 6 choices … this problem (with the 5 dashes and two lines) is different because when I go to choose what I am going to put in the first slot I have two choices.

Presenting participant: That is exactly the problem I had. I couldn't get around that. But you actually have 7 different choices. You can put dash one there or dash two there, they may look the same, but

Respondent A: Thanks, now I see it!

The second instructor's attention to the disequilibrium in the classroom prompts her to press the participant for a more thorough explanation. When participants begin relating (Level Four discourse) the discussion to previously explored problems, the presenting participant is able to make the connection and justify (Level Four discourse) his solution. Because of the second instructor's interjection, higher level discourse occurs, but more importantly participants' confusion is resolved. Resolving this confusion allows the discussion to progress to a complete solution of the problem.

Extension

In this episode the second instructor initiates an extension by asking a group to think about how to solve the pizza problem with the multiplication principle (they had a solution with the addition principle). During their group presentation they mentioned this task. The second instructor clarifies what she had asked them to do and instigates an investigation in table groups by the whole class.

Second Instructor: So the question I asked them, remember yesterday Kathy talked about strategy for when to add and when to multiply depending on how you wrote the problem down. So everyone that I saw, and correct me if I am wrong, kind of thought of this as 0 toppings, 1 topping, 2 toppings. So, you figured those out independently and added them together. Right? So, you kind of separated them into these disjoint mutually exclusive groups, right, and added them together. So, multiplication was invoked when you kind of built a pizza by making choices, a string of choices. So, what I asked them is could you change your perspective could you look at this problem differently through a multiplication lens instead of an addition and build that 2^6.

Following this second instructor interjection, the groups work at their tables. After the table group work, the lead instructor facilitates the ensuing group discussion. Without the earlier actions of the second instructor this episode would not have occurred.

Participant A: The fact that there are six toppings and we ended up with two to the sixth makes me think that when we don't have the base piece, like the ice cream or the 39 types of ice cream, that thing that you are loading it on is one. That's what I was trying to see. Is … if I had one pizza … the base pizza is one and I had four toppings I am thinking that the answer will be two to the fourth … on our Pascal's triangle … (the) 5th row down and so I was going to see would that be right if I had four toppings. And it would be.

Lead Instructor: What are you guys think(ing)?

Participant B: I was kind of thinking the same way as Participant A was. I was thinking about the base as one … I was trying to work from the top down (of Pascal's Triangle). And so if I had the one topping not the one at the top but the two.

Lead Instructor: Come up. Do you want to point (to Pascal's Triangle) while you are talking?

Participant B: … we were discussing prior to this that this was the 2 to the 1. Right? And this one up here was 2 to the 0. And so, it was one. So, if this was a one topping pizza and there was two total ways of doing it … it can either be on the pizza or it can be, it can be plain nothing on the pizza. … So then I have two toppings- so I in my mind I was trying to get to it the same way you are. …There is something with that two times two… So it can either be this is a topping on my pizza or not on my pizza. So that is two ways … And then this (second topping) can be on there, or these (both toppings) can be on there, and so (participant is motioning for each topping).

Participant A: I was playing with that piece, too.

Participant B: Yeah, there is suppose to be four ways. If this is my pizza and here is my topping on there (puts pen on document camera …) That is one way. This is another way (takes off pen and puts on a different pen). This is the two topping way (puts both pens on display). And then, the no topping way.

Right? … It is almost there. There is something with that two, that it is or it isn't.   I don't know if that helps or not.

…

Participant C: So there is the first topping, you either get it or you don't. Here is the second topping, you either get it or you don't. You either get it or you don't. Make sense? We are going to keep going. The third topping –

Participant A: There are your two's!

Participant C: Yeah those are my twos. I was so jazzed to see them. You either get it or you don't, you get it or you don't, you get it or you don't. So you make this tree diagram out six layers. It always goes by two's. So you've got every single one.

Participant A: So it's looking at it differently. Like it is either affirmative or it is not. I either want this and do I want to have the next one.

Participant C: So you have got yes or no at every step for every little piece so it is 2 to the 6^{th}. Yep.

The extension introduced by the second instructor allows the participants to justify (Level Four discourse) their approach, using the multiplication principle rather than the addition principle. As they share the solution and the justification, they are able to extend to a generalization (Level Five discourse).   The interjection of the second instructor leads the participants to a deepened understanding of the multiplication principle.

In these two last two episodes the second instructor attends to the disequilibrium in the classroom and introduces an extension. In both cases the student mathematical discourse moves to Level Four including relating and justifying. In fact, discourse reaches the highest cognitive level in the extension episode when participants generalize their results. While there are other instances of high level discourse in these classes, the actions of the second instructor clearly play a role in moving the discourse to a higher cognitive level.

Conclusion

The examination of the student mathematical discourse in regular classrooms and the summer institute classrooms revealed distinctly different proportions of cognitive levels of discourse. The regular classes exhibited a decreasing proportion of discourse as the cognitive level increased. In contrast, the summer institute classes displayed a higher proportion of discourse at the higher cognitive levels. We attributed at least some of these differences to the involvement of a second instructor in the summer institute classrooms.

The actions of the second instructor were identified as providing additional monitoring of small groups, time management, selecting and sequencing, and interjections into whole class discussion. Four classifications were developed for interjections into whole class discussion: side trips, pressing for clarification, extension, and highlighting. Higher proportions of Level Three, Four and Five discourse than seen in other situations occurred following interjections by the second instructor. Pressing for clarification and extensions by the second instructor provided the greatest opportunity for high level discourse.

One of the values of team teaching in this setting was an increased proportion of high level student mathematical discourse. Episodes when the second instructor acted to press for clarification or extend a discussion displayed the greatest increase. Opportunities for these types of moves may be more likely to be observed by the second instructor than by the lead instructor in part because the lead instructor is focused on the structure and flow of the entire lesson, while the second instructor is able to give full attention to the students and their questions, understanding and disequilibrium. Often the second instructor observed something that the lead instructor did not notice.

The opportunity to participate in this team teaching structure can provide valuable experience in acting in these second instructor roles. Practice in detecting opportunities for high level discourse, especially pressing for clarification and extensions, can develop an instructor's ability to notice and act on these opportunities when instructing "in solo." Further study is required to determine whether the skills gained through team teaching can be carried over to solo instruction. Additional observations of the subjects in the future could be undertaken to determine whether these instructors who experienced the benefits of team teaching are able to improve their abilities to detect similar opportunities for high level discourse in their solo classrooms.

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