Preliminary Report

Pedagogical Content Moves in an Inquiry-Oriented Differential Equations Class: Purposeful Decisions to Further Mathematical Discourse

Hollylynne Stohl Lee
North Carolina State University
hollylynne@ncsu.edu

J. Todd Lee
Elon University
tlee@elon.edu

Karen Allen Keene
North Carolina State University
karen_keene@ncsu.edu

Krista Holstein
North Carolina State University
kaholste@ncsu.edu

Peter Eley
North Carolina State University
pmeley@ncsu.edu

Megan Early
North Carolina State University
meearly@ncsu.edu

Many universities have been encouraging professors to incorporate inquiry-oriented instructional strategies in their collegiate level classrooms. Several researchers have investigated a variety of research questions concerning the use of inquiry-oriented strategies in mathematics courses such as Differential Equations and Abstract Algebra (e.g., Bartlo, Larsen, & Lockwood, 2008; Wagner, Speer, & Rossa, 2007). In a mathematics classroom, inquiry includes a focus on fostering mathematical discourse among students and teachers. While general discourse practices such as telling or revoicing have been carefully analyzed for their effects on the mathematical discussion (e.g., Park et al, 2008,) most do not focus on the ways in which the instructor draws upon specific content knowledge when making a discursive move. There seems to be little research focusing on the *pedagogical content moves* made by mathematics professors in the university classroom. We define a pedagogical content move to be a discursive or inscriptive act by an instructor that is purposely used to promote or further the mathematical agenda in the classroom. Such moves require that the teacher draw upon his/her content knowledge specifically related to the current classroom discourse. Thus, as we study a particular differential equations class using inquiry-oriented curriculum materials (IODE),, we have been pondering the following:

What is the nature of a professor's pedagogical content moves used in an inquiry-oriented classroom?

Background Literature and Framework

For two decades now, researchers have built upon Shulman's (1986) work that suggested that the knowledge needed for teaching specific disciplines requires a blend of understanding about pedagogy and content understanding in the form of pedagogical content knowledge (PCK). Additionally, Hill, Ball and Schilling (2008) proposed a framework for the many types of knowledge that teachers use. In particular, they suggest that Mathematical Knowledge for Teaching (MKT) consists of three kinds of subject matter knowledge: *common content knowledge, specialized content knowledge,* and *knowledge at the mathematical horizon* and three elements of PCK: *knowledge of content and students, knowledge of content and teaching,* and *knowledge of curriculum*. Thus, inherent in this framework is that the three elements of PCK must draw upon the three types of content knowledge. We hypothesize that by focusing on an instructor's pedagogical content moves, we may be able to further describe how one uses content knowledge in specific ways that promote or further the mathematical agenda from the instructor's perspective.

Some prior research has begun to focus on pedagogical issues related to mathematics instruction at the undergraduate level. For example, in studying the implementation of the same differential equations curriculum materials, Wagner et al

(2007) analyzed the specific problems a professor encountered in facilitating mathematical discussions. The professor in that study had taught DEs for many years from a traditional perspective and was new to inquiry-oriented instruction. In particular they found that the professor struggled to respond to unexpected student responses during whole class discussions. This is similar to the work of Bartlo et al (2008) that shows that the mathematics knowledge that a professor brings to an abstract algebra classroom is broad in certain ways but that there are pedagogical situations when building content connections and understanding student thinking is a challenge. We are hoping that our research in a context of an experienced inquiry-oriented professor can identify categories of positive instances where he is able to draw upon his content knowledge to make discursive and inscriptive acts in promoting his mathematical agenda.

Rasmussen and Marrongelle (2006) identified two pedagogical content tools that they posit can extensively further the mathematical discourse and learning in a university level differential equations class. One of their pedagogical content tools is a transformational record in which an instructor introduces an inscription for representing a mathematical idea that is used by students in their reasoning and in their recordings of a mathematical idea. We are interested in moves along these same lines used by an instructor and the mathematical content understandings that drive the move.

Research Methodology

Data collection was conducted in Spring 2008 in a college level Differential Equations class in the southeastern United States (enrollment of 25) using a classroom teaching experiment methodology (Cobb, 2000). Most students in the class were mathematics, science, or engineering majors, had finished Calculus III, and about one third of the students had taken at least one prior course with this particular mathematics professor. The professor had been using inquiry-oriented strategies in his other courses (e.g., Abstract Algebra, Mathematical Reasoning) for several years, but had only taught Differential Equations once about 7 years prior and was implementing the specific inquiry-oriented differential equations materials (Rasmussen, 2003) for the first time that semester. Prior to many teaching sessions, the professor met with one of the researchers to discuss the material to be taught and make a planned trajectory. They also met immediately after class for debriefing sessions to reflect on the lesson and discuss any issues or questions that arose that may affect the content and teaching strategies used for the next class.

The class was designed to be inquiry-oriented with each class session involving cycles of learning: whole class discussion, followed by small group discussion, followed by whole class discussion. The learning environment of the classroom established by the professor required students to discuss the mathematics they were learning, express their own ideas, and make sense of, and agree or disagree with others' ideas.

The data used for analysis for this paper was drawn from the videotaped class episodes, field notes from a non-participant observer, and video/audio taped debriefing sessions held immediately after class. To begin our analysis, we reviewed videotapes and field notes of most class sessions throughout the semester. We then chose six class sessions that came from different points in the semester with different content foci to use in our analysis. For each class session we created descriptive timelines (chunked by 5-10 minutes) that provided an overview of the content, student work, and teacher moves during that class. We reviewed the descriptive timelines from each of the six class sessions, as well as the field notes, to identify episodes that appeared to contain several lengthy sections where the professor was facilitating mathematics discussions.

From the chosen class sessions, we identified five critical episodes where it was noted that the teacher was making discursive or inscriptive acts that appeared purposeful for furthering the mathematical agenda. Once these episodes where identified we used a coding scheme that was both top-down using methods of Miles and Huberman (1994), and generative in nature using grounded theory (Strauss & Corbin, 1990). It was top-down in the sense that we used our framework from prior literature to identify instances where the instructor was initiating a conversation, possibly using one of the typical discursive moves such as telling or questioning, or interjecting something in a conversation using revoicing. We also looked for instances of the use of tools from two categories of pedagogical content tools described by Rasmussen and Marrongelle (2006). The coding was generative in nature as we made notes of ways the instructor was drawing upon his content knowledge to make a discursive or inscriptive move that led to the emergence of additional categories. Each category was discussed by the project team. Some categories were thrown out if there was not enough confirmatory evidence that it was a consistent and useful move made by the instructor, and other categories were merged together and relabeled if they captured similar types of moves. These discussions included input from the instructor (second author) as to his intentions for making such moves. The final list of categories was determined by consensus of the project team.

Results

When we were initially using the discursive moves in our coding, we were often asking ourselves, "Why was he revoicing? Why was he questioning?" We wanted to know the purpose behinds these acts. Often, the purpose seemed to be:

- drawing upon knowledge students should have from previous courses or other disciplines such as biology (lateral curriculum knowledge, Shulman, 1986),
- considering future content material in the current DE course or subsequent courses offered in the department (vertical curriculum knowledge, Shulman, 1986),
- deepening the students' knowledge of the current concepts under consideration, or
- promoting the students' abilities to think like mathematicians and develop mathematical habits of mind (Cuoco, Goldenberg, & Mark, 1997).

We consider all of these purposes part of the professor's mathematical agenda. We then focused on characterizing *how* the instructor seemed to be promoting the mathematical agenda. From this process, we identified several major categories of pedagogical content moves that helped us describe how the instructor was promoting or furthering the mathematical agenda. The professor:

1) Uses everyday language or contexts
2) Fuses context & mathematics
3) Utilizes representations
4) Builds upon students' ideas
5) Focuses upon or orders appropriate groups/ideas

What follows is a brief description and an example of each category of a pedagogical content move that we have identified.

Uses Everyday Language or Contexts

The instructor often used common language or real world contexts not directly used in the DE curriculum materials to describe mathematical ideas that may help the students connect their understanding of an everyday idea to the mathematical idea. For example, when attempting to provoke the students to recognize that the use of a particular differential equation and its curve did not appropriately model the context of a problem, he asked the students to consider whether or not there was a mathematical "uh-oh" in their work that they found "disturbing". His purpose of using these informal words was to invoke a perturbation for the students about how the differential equation was modeling the real word context in the problem. However, rather than explicitly asking them to consider the differential equation as a model of the context, he used these informal words as way to develop their mathematical sense-making skills that should be a part of thinking like a mathematician about the way models are used. Thus, his purpose was to promote furthering one aspect of his mathematical agenda for the class—to develop the students' mathematical habits of mind, or as he calls it, their mathematical "gut." (Keene et al, 2009)

Fuses Context and Mathematics

We borrow the idea of fusion from Nemirovsky, Tierney, and Wright (1998) to mean situations where the professor appears to blend the mathematical concepts and the contexts of the problems under study. Attempts to fuse the mathematical objects under study with the contexts can be used to help students understand: 1) how the mathematics is connected to the context, and 2) how they can draw upon their understanding of the context to better understand the mathematics. For example, the professor was having the students develop a system of differential equations for a general predator-prey model. The students decided to discuss lions and antelopes. At this point in the class, the students determined that the differential equation for the antelopes would have a constant times the population, times some quantity that represented the carrying capacity: $\frac{dA}{dt} = kA(1 - \frac{C}{A})$ where A is the population of the antelope and C is the carrying capacity. During the discussion of their general model, the professor stated "If I start above the carrying capacity, *it* drops down" (gesturing the shape of a curve). The pronoun "it" and the gesture suggesting the shape of the curve both seem to be representing a fusion of the context of the problem (the population) as well as the graph of the curve. The use of fusion can help promote connections to other disciplines (e.g., biology) as well as considering the importance of mathematics as a modeling tool.

Utilizes Representations

The professor often used a variety of representations during class discussions that were not explicitly part of the DE curriculum materials. The introduction of these representations seemed to be a powerful pedagogical content move as he drew upon his understanding of mathematics. At times the representations used were familiar ones to the students, and other times new or novel ways to represent the ideas presented by the professor.

During one class session, the professor was developing the idea of a flow line as a representation of whether a solution to a differential equation is increasing or decreasing in a given interval. This representation is explicitly a part of the IODE curriculum materials. To help them build upon ideas they should have learned in Calculus, he brings in the use of a sign chart where one can represent whether a function is increasing or decreasing on all given intervals in the domain. For example, for a given function *f*, one can represent the local behavior of the function within the domain $-3 \leq x \leq 5$ by examining *f′* and noting whether the function is increasing or decreasing (see Figure 1). While the representation of a sign chart does not exactly map onto how a flow line represents an entire solution space for an autonomous differential equation, the professor intended to use it to help bridge from what the students already knew about increasing and decreasing intervals in a function to thinking about the issue of flow lines differential equations.

f'	$-$	$+$	$-$
f	dec	inc	dec

$-3 \quad 1 \quad 3 \quad 5$

Figure 1. Example sign chart

This type of pedagogical content move illustrates how the professor can draw upon his depth of mathematical knowledge and purposely help tie together ideas from previous mathematical concepts to the current content being learned. It can further a mathematical agenda of students viewing mathematics as connected conceptually, rather than just isolated skills of using sign charts or generating flow lines. In this class episode, many of the students expressed that the usefulness of sign charts finally made sense to them.

Builds Upon Students' Ideas

We often found that the professor capitalized on students' ideas to enrich the discussions or to focus the students on different aspects of the mathematics under consideration. The students' ideas surfaced in the classroom in the form of questions as well as statements. The professor would then take their question or statement and use it to pose additional questions for the class to consider. For example, during one class session, the professor had the students examining a situation where two students are memorizing a list according to the rate of change equation $\frac{dL}{dt} = 0.5(1 - L)$, where *L* represents the fraction of the list that is memorized at any time *t*. He asked the class to consider the following question in their small groups: “According to the rate of change equation, will two students, who started with a different amount of knowledge, ever have the same rate of learning?”

After some time for students to work in groups, he asked the students to stop group work and to come together for a whole group discussion. A student asks a question to the professor, but some students are still talking. The following dialogue then occurs:

Professor: Let’s listen to this question, because it is a good one. Do I mean what?

Student: Will they ever be learning at the same time at the same rate?

Professor: At the same time will they be learning at the same rate? That is a separate question than what I was asking.

Student: ok.

Professor: I was asking, at any time, is this person learning at the same rate as this person is learning right then, then being at time zero [pointing to graph on board].

Others: Oh, oh….

Professor: There is a separate question, that I like, so I will pose to you and give you 30 seconds to talk about. Is there any time, now notice if you pick a time you can draw a vertical line and see what is going on with all the solutions [gestures a vertical line on graph displayed on board]. At any time is it the case that these two people will be learning at the same rate? Talk to your group.

The student's clarifying question led the professor to consider how exploring the students' question, in addition to his originally posed question, may be helpful for students. He considered the mathematical value of examining a "same time, same rate" situation, make a connection to how same time could be considered through a vertical line on the graph, and how it may help students to better understand the problem, especially his originally posed question. The professor's in-the-moment pedagogical move seemed to be aimed at deepening the understanding of the current mathematics. This pedagogical content move illustrated how he was able to listen carefully to students' questions or statements that can gave him insight into their understanding of the task and how he had to draw upon his deeper understanding of the rate of change context to consider how to build off the student's ideas.

Focuses Upon or Orders Ideas from Students

In this type of pedagogical content move, the professor would draw upon his understanding of the content to purposely choose which student idea to focus on in a discussion or choose an order for whole class presentations of students' ideas that would allow the ideas presented to build to a specific point he was trying to make. For example, in a lesson focused on using the Euler method to approximate a solution, he assigned different groups to investigate the problem using different time steps (Δt). He then asked the groups to present in a specific order starting with the group that investigated the largest time step and then proceeding through the groups in decreasing Δt order. In this way, he was able to facilitate an ongoing group discussion comparing a current group's approximation to those presented previously. This pedagogical content move was initiated by the professor, and not a suggested part of the IODE curriculum materials. However, it demonstrated the professor's use of mathematical knowledge about the effects of Δt on the approximations. In addition, he wanted this point to be made through an investigation and discussion with the students, rather that merely stating what the effect would be of decreasing the time step. As such, engaging the students in this investigation took a lot of classroom time, but was promoting an inquiry approach to mathematics—an approach the professor wanted them to value and engage in as mathematicians.

Discussion

Certainly, all of the pedagogical content moves we have identified are not unique to this professor and could likely be observed in the classrooms of others. But they do require that the professor draw upon his mathematical knowledge in a way that is needed to make teaching decisions. Hill, et al (2008) (and other papers from these authors and their colleagues) would likely call this knowledge "specialized mathematical knowledge" that is needed especially by teachers. What became most interesting to us is not that the instructor had to have this knowledge, but *how* and *why* he was putting his mathematical knowledge into action in his classroom. Thus, the notion of a pedagogical content move as being one that can promote or facilitate a mathematical agenda in the class can allow us to capture the ways a mathematician needs to draw upon their mathematical knowledge in the act of teaching.

One aspect of the mathematical agenda that seems to keep coming to the fore in many of the instances we found in this classroom is the professor's desire to get the students to experience being mathematicians and to develop the habits of mind of a mathematician. Just as Cuoco et al (1997) have been promoting the use of these habits of mind as ways that can help prepare high school students for advanced study in mathematics, professors at the collegiate level will likely have developed mathematical habits of mind as part of their mathematical agenda for their students. Furthermore, the mathematics professors who are going to implement inquiry-oriented courses certainly have mathematical depth knowledge and understand what it means to be a mathematician. However, as has been shown in the research of others (e.g., Bartlo et al, 2008, Wagner et al, 2007), these mathematics professors may struggle with the pedagogical demands of teaching in an inquiry-oriented class. By identifying the pedagogical content moves of professors who use inquiry-oriented approaches, we may be able to help develop ways to assist other mathematics professors develop pedagogical strategies to explicitly draw upon their mathematical knowledge.

References

Bartlo, J., Larsen, S., & Lockwood, E., (2008). Scaling up instructional activities: Lessons learned from a collaboration between a mathematician and a mathematics education researcher. In *Proceedings for the Conference on Research inUndergraduate Mathematics Education*. San Diego, CA. Available online at http://www.rume.org/crume2008/Bartlo_LONG.pdf.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers. In E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education.* (pp. 307-334). Mahwah, NJ: Erlbaum

Cuoco, A., Goldenberg, E. P., & J. Mark. (1997). Habits of mind: an organizing principle for mathematics curriculum. *Journal of Mathematical Behavior*, 15(4), 375-402.

Hill, H., Ball, D., & Schilling, S. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39.* 372-400.

Keene, K., Lee, T., Lee, H., Eley, P., Early, M., & Holstein, K. (2009). An investigation of one instructor's ,mathematical knowledge for teaching: developing a preliminary framework. *Conference on RUME online proceedings*.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook.* Thousand Oaks, CA: SAGE Publications.

Nemirovsky, R., Tierney, C., & Wright, T. (1998). Body motion and graphing. *Cognition & Instruction*, *16*(2), 119.

Rasmussen, C. (2003*). Differential equations instructional materials*. Non-published materials.

Rasmussen, C. & Marrongelle, K. (2006). Pedagogical content tools: Integrating student reasoning and mathematics in instruction. *Journal for Research in Mathematics Education, 37*. 388-420.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15* (2), 4-14.

Strauss, A. & Corbin, J. (1990). *Basic of qualitative research: Grounded theory procedures and techniques.* Thousand Oaks, CA: SAGE Publications.

Wagner, J., Speer, N. & Rossa, B. (2007). Beyond mathematical content knowledge: A mathematician's knowledge needed for teaching an inquiry-oriented differential equations course. *Journal of Mathematical Behavior, 26*. 247-266.