Preparing to Teach Mathematics with Technology:

Lesson Planning Decisions for Implementing New Curriculum

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Introduction

Whenever a new curriculum is introduced, teachers must make decisions on exactly how to implement the curricula materials in their instruction. Teachers make decisions about 1) how to organize class activities for whole class or small group work, 2) the tasks to pose and key questions to ask, and 3) how resources will be used. Whether resources such as technology will enhance or hinder students' learning depends on teachers' decisions of how and when to use technology tools as an aid in teaching and learning mathematics (Lee & Hollebrands, 2008a).

Through a NSF-funded project, Preparing to Teach Mathematics with Technology[1] (PTMT), teacher education materials have been created to address the need to prepare teachers who are able to use technology to teach mathematics in ways aligned by recommendations from national organizations (Association of Mathematics Teacher Educators, 2006, National Council of Teachers of Mathematics, 2000). The curricula materials are written in the form of modules for about 18-20 hours of instruction. The first module completed focuses on teaching and learning data analysis and probability with technological tools for middle and high school topics (Lee, Hollebrands, & Wilson, under review). This module could be used with prospective teachers in collegiate-level mathematics education methods courses, mathematics or statistics courses, or in professional development workshops to prepare practicing teachers to teach statistics with technology. The module is currently under review for possible commercial publication and has been field tested at several universities. The research discussed in this paper focuses on mathematics teacher educators' planning and debriefing when using the Data Analysis and Probability module within a "Teaching Mathematics with Technology" methods course for middle and secondary prospective teachers.

Framework

Shulman's (1986) notion of teachers' pedagogical content knowledge (PCK) has greatly influenced teacher education and research on teachers in the past two decades. More recently, several authors have described technological, pedagogical, and content knowledge (referred to as TPCK and more recently TPACK) as a type of knowledge needed for teachers to effectively use technology for teaching specific subject matter (Koehler & Mishra, 2005; Mishra & Koehler, 2008; Niess, 2005, 2006). Niess (2005) describes four different aspects that comprise teachers' TPACK: 1) an overarching conception of what it means to teach a particular subject integrating technology in the learning process; 2) knowledge of instructional strategies and representations for teaching particular topics with technology; 3) knowledge of students' understandings, thinking, and learning with technology; and 4) knowledge of curriculum and curriculum materials that integrate technology with learning. Considering the components of TPACK, such a model for mathematics teachers should integrate mathematical/statistical content, technology, and pedagogy, with a focus on student thinking.

In considering a framework for the design and development of the first module on teaching data analysis and probability with technology, Lee and Hollebrands (2008b) specialized the notion of TPACK for teaching the specific content of data analysis and probability.. Lee and Hollebrands consider the development of teachers' technological pedagogical statistical knowledge [TPSK] as layered circles with a foundation focused on teachers' statistical thinking (Figure 1). Thus, the inner most layer, consisting of elements of TPSK, is founded on and developed with teachers' knowledge in the outer two circles.

Developing knowledge in the outer two layers of statistical thinking and technological statistical knowledge is essential to, but not sufficient for teachers having the specialized TPSK. The elements noted in each layer in Figure 1 are descriptors of the major foci of the knowledge, thinking, skills and dispositions the curriculum authors (Lee, Hollebands, & Wilson, under review) aim to develop as teachers' TPSK in the teacher education materials.

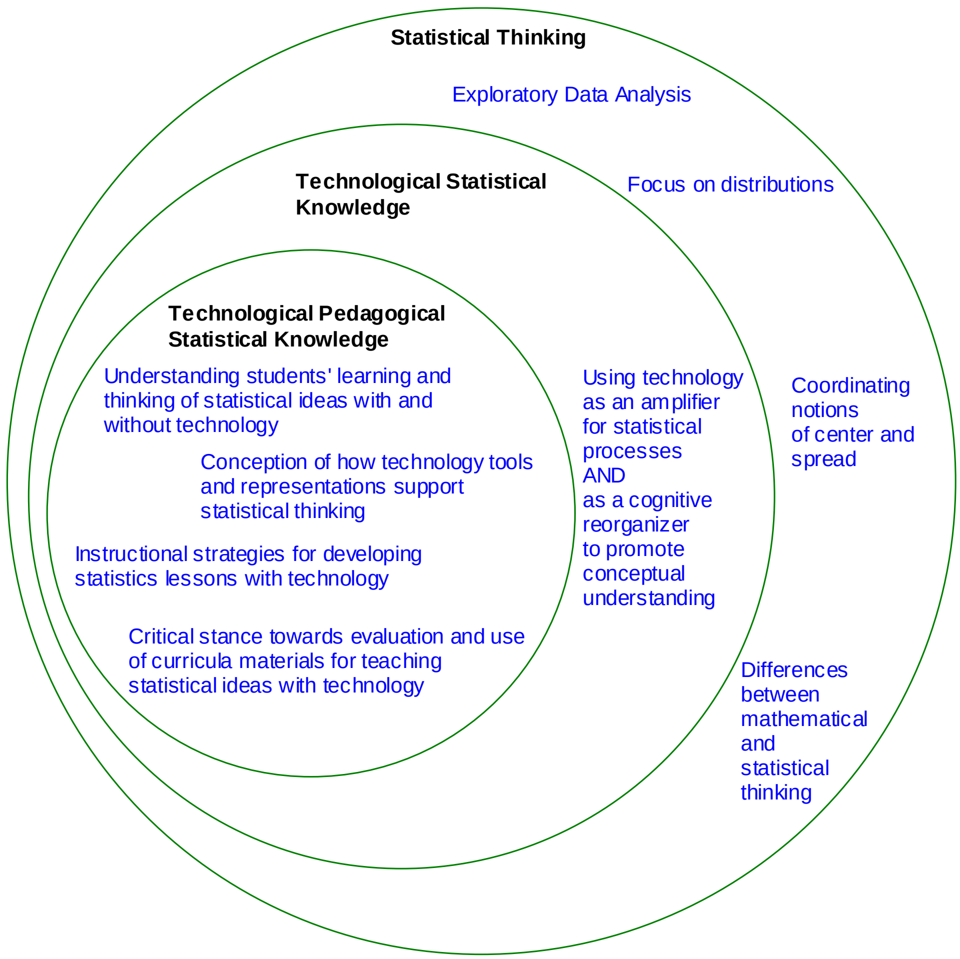

Figure 1. Framework for developing teachers' TPSK

For many teachers, both prospective and practicing, engaging in statistical thinking is a different process than that which they have been engaged in teaching and learning mathematics (delMas, 2004). Thus, it is important to engage teachers as active learners and doers of statistical practices. In the Data Analysis and Probability module, teachers are engaged in Exploratory Data Analysis (EDA). The practice of data analysis is seldom a linear process (Konold & Higgins, 2003) and should be informed by the personal experiences one has with the context of the data. The curriculum includes data that is likely of interest to teachers (e.g., national school data, college retention data, vehicle fuel economy, birth data). Using such contexts promotes the practice of asking questions from data, and the technology tools facilitate using a variety of representations to analyze data in novel ways that can motivate other questions to explore with the data. The PTMT materials also engage teachers in examining distributions graphically and characterizing the data with such constructs as bins (Rubin & Hammerman, 2006) and a modal clump (Konold & Higgins, 2003) before computing statistical measures. Questions promote the comparison of distributions as means to transition to thinking about data as aggregate. The materials are also designed to help teachers conceptually coordinate center and spread by understanding measures of center like mean, median, and midrange with respect to individual deviations from those measures. This notion is used in a univariate context to help students consider measures of variation (e.g., residuals, sum of squares) in a bivariate context when modeling with a least squares line. Throughout the materials, there is an aim to develop teachers' understanding that the way one thinks in statistical contexts is often different than thinking used in pure mathematical contexts. The materials provide experiences for teachers to consider how statistics is a tool for answering questions and that answers to questions are highly connected to the context of the data and rarely are strong enough to make definitive or conclusive statements.

Although statistical thinking is foundational in the TPSK framework, technology tools are used to engage teachers in tasks that simultaneously develop their understanding of statistical ideas with technology skills so they may experience firsthand how technology tools can be useful in fostering statistical thinking. Ben-Zvi (2000) has provided a useful way of organizing how a technology tool can support statistical thinking. Building from the work of Pea (1987), Ben-Zvi provides a useful lens on statistical technology use as ways to amplify or reorganize one's statistical work. Technology tools are typically used in two different ways in statistics. They can amplify our abilities to solve problems. The idea of an *amplifier* is that the tool expedites a process that could be completed without its use. For example, technology tools can be used to quickly compute and order numbers, generate large lists of pseudorandom numbers, and to generate graphical representations efficiently. Technology tools can also be seen as a *reorganizer*. Through dynamic features of dragging, the linking of multiple representations, and overlaying measures on graphs, technology tools can help students reorganize their statistical conceptions.

Throughout the module, findings from research on students' understandings of statistical ideas are used to make points, raise issues, and pose questions for teachers. After teachers have engaged in examining a statistical question with a technology tool, they are asked pedagogical questions that are aimed at developing their understanding of how technology and various representations can support students' statistical thinking. In addition, teachers are encouraged to consider the pedagogical implications of the differences between mathematical and statistical thinking. For example, an entire chapter is dedicated to teachers examining a video and written work of students' engaging in a comparison of two distributions. The videocase provides an in-depth opportunity to analyze how technology can support or hinder students and to push teachers to consider how their students will typically engage in data analysis tasks differently than the ways the teachers did as learners.

Research Questions and Methods

If PTMT teacher education materials are going to be used effectively in the education of prospective teachers, it is important to consider factors that may affect the use of the curriculum. In addition, since many teacher educators are not expected to be experts in the teaching and learning of every topic in mathematics, it is reasonable to believe that some teacher educators may not immediately be comfortable with the concepts and technologies used in the data analysis and probability materials. Thus, as part of the dissemination efforts, the project team is interested in using results from studying local implementation efforts to help inform the design of a facilitator's guide and needed faculty education efforts to help assist other mathematics teacher educators in optimally using the PTMT materials. Thus, we are interested in the following research questions:

- How do mathematics teacher educators make sense of the intended curriculum and make decisions for implementation?
- In what ways do mathematics teacher educators use components of technological pedagogical statistical knowledge in their planning and implementation of the curriculum?

To answer our questions, we are employing qualitative methods of inquiry to examine the local phenomena of planning and curricula implementation through the use of document collection and analysis, and non-participant observer techniques (Huberman & Miles, 2002).

Context of Study

The study took place during the five weeks devoted to the teaching of a unit on Data Analysis and Probability that was part of a course on "Teaching Mathematics with Technology." This course serves middle and secondary prospective teachers and a few beginning graduate students that have little experience using technology tools in teaching mathematics. During the Fall 2008 semester, the course met twice a week for 75 minutes each class meeting. During this five week unit, the module from the PTMT project (Lee, Hollebrands & Wilson, under review) was being used as the primary textbook. All students had purchased the text materials and were given a CD with the necessary technology files. For this preliminary research report, we chose to analyze the data from only two of the six chapters in the module, Chapters 3 and 4. This decision was made to limit the technology used by the mathematics teacher educators to a single technology (*Fathom*), and the fact that these two chapters are significant in length and Chapter 4 conceptually builds upon ideas developed in Chapter 3. See Figure 2 for details on the topics for each section in these two chapters.

Chapter 3: Analyzing Data with *Fathom*

Teachers analyze 2006 automobile data using *Fathom* to describe center and spread using dot plots and box-and-whisker plots. They will consider pedagogical issues related to the use of various graphical representations, measures of center and spread, and dynamic statistical software.

Section 1: Asking Questions from Data
Section 2: Examining Univariate Distributions
Section 3: Comparing Distributions Using Center and Spread
Section 4: Understanding Spread of a Distribution

Chapter 4: Analyzing Bivariate Data with *Fathom*

Building from ideas introduced in Chapter 3, teachers continue to analyze automobile data using *Fathom* to look for relationships between two quantitative attributes. They use the concept of variation and deviations from the mean in each univariate attribute to help conceptualize correlation and least squares regression as ways of describing the relationship in the bivariate data and developing a linear model for making predictions. The teachers will consider pedagogical issues concerning the difficulties students may have in analyzing bivariate data and the benefits and drawbacks for using conceptual underpinnings from univariate analysis to develop bivariate analysis techniques.

Section 1: Examining Relationships Between Two Quantitative Variables
Section 2: Conceptualizing Correlation
Section 3: Using a Line to Describe a Relationship Between Two Quantitative Variables
Section 4: Visualizing the Residuals
Section 5: The Least Squares Regression Line
Section 6: Exploring Additional Attributes on a Scatterplot
Section 7: Exploring the Effects of Outliers on Correlation and the Least Squares Line

Figure 2. Outline for Chapters 3 and 4 of Module Being Implemented

Participants

During the Fall 2008 semester, the course was being taught by a mathematics teacher educator who has taught the course during five prior semesters in the 2005-2006 and 2006-2007 school years and in Fall 2007. During those semesters, this instructor had implemented prior versions of the PTMT Data Analysis and Probablity module. Thus, she has seen many early versions of the materials, some of which have been changed dramatically over the years, including Chapters 3 and 4. This mathematics teacher educator has been part of the undergraduate teaching faculty since Fall 2005 and also has seven years of mathematics teaching experience at the middle and high school level. There was also a Teaching Assistant (third author) assigned to this course that would assist in planning, delivering instruction, and evaluation of students' work, and would need to be educated on the course materials so that she could teach the course on her own in future semesters. This TA was in her first year as a mathematics education PhD student and had seven years teaching experience at the high school level and two years experience teaching mathematics courses at a community college. To facilitate the apprenticeship model, the two instructors decided to meet weekly to engage in planning and debriefing sessions. These sessions were two hours in length. Since the focus of our study was not the instructors as individuals, we will collectively refer to them as mathematics teacher educators for our unit of analysis.

Data Collection

The sources of data for this study included audio recordings of the planning and debriefing sessions between the two mathematics teacher educators as well as a copy of their textbooks in which they each had made notes to themselves to use during instruction. The first author attended each of the planning sessions throughout a five week period when the instructors were teaching the material from the module. The first author was mostly a non-participant observer during these sessions, but did occasionally ask a clarification question or interject a comment about a point raised by the instructors that could lead to further discussion. A total of five sessions were attended and transcribed. Through observing how the teacher educators were planning for and implementing lessons from the module to foster their prospective teachers' TPSK, we wanted to gain insight into the ways the teacher educators needed to draw upon their TPSK for effective implementation of the materials. The use of planning and debriefing sessions as a source of data has been used in the collegiate setting by Bartlo, Larsen, and

Lockwood (2008) to study a mathematician's instructional decisions when planning and implementing an innovative research-based undergraduate abstract algebra curriculum that focused on conceptual rather than procedural understanding. Thus, we felt this method of data collection, rather than direct observation of the class, would provide information about the challenges the mathematics teacher educators were facing and a record of the instructional decisions that they made.

Analysis Methods

Our study uses the framework of TPSK that was used to guide the intentions of the curriculum designer to examine how the teacher educators planned and enacted material from two chapters in the module. The transcripts of the planning and debriefing meetings for Chapters 3 and 4 (two planning sessions, 4 hours total) were first analyzed for key themes about instructional decisions and their use of resources. We then coded the transcripts for instances where it appeared the mathematics teacher educators were drawing upon elements of technological, pedagogical, and/or statistical knowledge. The codes were then discussed among the authors until consensus was reached. Once the transcripts were coded we then looked for examples that indicated where the teacher educators drew upon their statistical thinking, technological statistical knowledge, and technological pedagogical statistical knowledge, as indicated in Figure 1.

Results

The coding process using the TPSK framework, as well as the open coding process generated several common themes concerning how the mathematics teacher educators intended and used the curriculum with the prospective teachers. In this section, we give an overview of the results organized by each of the posed research questions.

Question 1: How do mathematics teacher educators make sense of the intended curriculum and make *decisions for implementation?*

The planning and debriefing sessions provided evidence that the mathematics teacher educators had used the main student textbook materials and the technology files that are associated with Chapters 3 and 4. During their planning sessions, the mathematics teacher educators had discussions about specific language (e.g., predictor/response), definitions (e.g., coefficient of determination), and conceptual descriptions (e.g., standard deviation) that were present in Chapters 3 and 4 to use in their class discussions. They indicated some of these ideas may have been unfamiliar or difficult to them and that they anticipated would be unfamiliar to the prospective teachers. In addition, there were several references that the mathematics teacher educators made to the technical directions provided in the textbook for how to perform an action in *Fathom*. It was also evident that the mathematics teacher educators had opened and familiarized themselves with the associated technology files for each section in the two chapters. They knew the content of the files and even made a suggestion for additional files they felt would be helpful for one of the sections. All of these examples illustrate that the mathematics teacher educators utilized the student textbook and the technology files as key faculty resources for making sense of the curriculum as it was intended by the authors.

There is an additional faculty resource, the detailed answer key, which could have been referenced during their planning. There was little evidence of direct use of the answer key in their planning discussions for Chapters 3 and 4 about answers to mathematical or pedagogical questions. One of the instructors had made brief notes in their copy of the student textbook concerning answers to selected questions. While most of these answers were aligned with those that appear in the answer key, we were not able to determine the extent of how that instructor used the answer key. More evidence is needed to ascertain whether, and how well, the answer key is providing appropriate support for the instructors.

The majority of the time spent in the planning and debriefing sessions was actually spent discussing the details of how to implement the materials in Chapters 3 and 4 in the time allotted for the class sessions. The mathematics teacher educators discussed: 1) how much time they should spend on each section, 2) which sections they wanted to "cover" during class and which should be assigned for homework, 3) when they would need to access certain technology files during the lesson, and 4) and when they should have students reference the textbook during a lesson. These sessions included discussions of which parts of a section should be done in a whole class format or done in small groups with the teacher educators facilitating the small group work. They tended to assign work on the mathematics questions with the technology to be done in small groups where prospective teachers were to use the textbook as a guide, and reserved work on the pedagogy questions for whole group discussions.

Question 2: In what ways do mathematics teacher educators use components of technological pedagogical statistical knowledge in their planning and implementation of the curriculum?

In coding the discussions between the two mathematics teacher educators, we were not trying to characterize the elements of TPSK that might represent the knowledge of them as individuals. Instead, we were attempting to identify instances in their conversation where we believed at least one of them was drawing on understandings that were representative of one of the elements in the TPSK framework. As such, we are not describing the knowledge of an individual, but using the framework to characterize ways they may have been using elements of TPSK in their planning or implementation of the material in Chapters 3 and 4.

Statistical thinking. In their discussions about Sections 1 and 2 in Chapter 3, the mathematics teacher educators exhibited their understanding of the importance of having the prospective teachers be able to ask questions from the vehicle data set that would be supportive of conducting exploratory data analysis. They also exhibited attention to center and spread in distributions as they discussed the importance of the prospective teachers understanding correlation coefficient and R^2 as a measure of spread for bivariate data. They further attended to distributions and how data may be clumped in their discussion for how they could use the univariate distributions displayed in boxplots to ask the prospective teachers to conjecture how the two univariate data sets could be displayed for a bivariate analysis using scatterplots. (This particular example will be discussed further in the section below on TPSK.)

Technological statistical knowledge. The mathematics teacher educators displayed their understanding of how to use technology as both an amplifier as well as a reorganizer. In these two chapters, they often referred to the ease of using technology to quickly create data displays such as histograms, bar graphs, and box plots that would allow them more time to discuss comparisons across the different representations. Thus, they clearly knew how to capitalize on technology as an amplifier that could increase their ability to have deeper discussions regarding what the representations highlight or mask in the data with regards to center, spread, and viewing the data as individual points or in an aggregate (also illustrating their statistical thinking).

The mathematics teacher educators also illustrated their understanding of how technology could be used as a reorganizer for statistical concepts. They referenced the benefits of the ability to use sliders in *Fathom* to explore the connections between the value of a correlation coefficient and the spread of data in a scatterplot. They also frequently discussed how they wanted to highlight the ability in *Fathom* to change something in one representation and to observe effects in a linked representation. They indicated a goal for the prospective teachers to learn how to take advantage of linked representations for their own conceptual learning and in their future teaching.

Technological pedagogical statistical knowledge. There were several instances when the mathematics teacher educators discussed the importance of teaching statistics conceptually, and how this may be difficult for the prospective teachers since many of them had learned statistics in a formula-driven way. In these discussions they referenced the use of diagrams in the textbook (e.g., diagrams for understanding deviations from a mean and residuals from line of best fit) and features in *Fathom*, like the ability to move data points and visualize the effect of the mean and deviations as well as the line of best fit and associated squared residuals. They seemed to believe that the use of such tools could facilitate a conceptual approach.

There were also several examples where the mathematics teacher educators discussed ways their decisions to present material in a way that was different than what was explicitly suggested in the textbook materials. We believe these instances illustrate their critical stance towards the curricula (fourth element of TPSK) and their ability to draw upon their own technological statistical knowledge and statistical thinking abilities to make planned or impromptu changes to the PTMT curricula.

Example 1. One example of this was when one of the instructors wanted to alter the suggested sequence of activities in Section 1 of Chapter 4 to take advantage of how the technology could facilitate the prospective teachers' statistical thinking about how bivariate distributions can be displayed in a scatterplot. The class was to investigate if there is a relationship between city mpg and highway mpg for a set of cars. Figure 3 is a screen capture of the boxplots for city and highway mpg that prospective teachers were asked to graph and that appeared in the textbook. The goal of the activity was for prospective teacher to build from the two univariate distributions to consider how the two attributes co-vary and can be represented in a scatterplot.

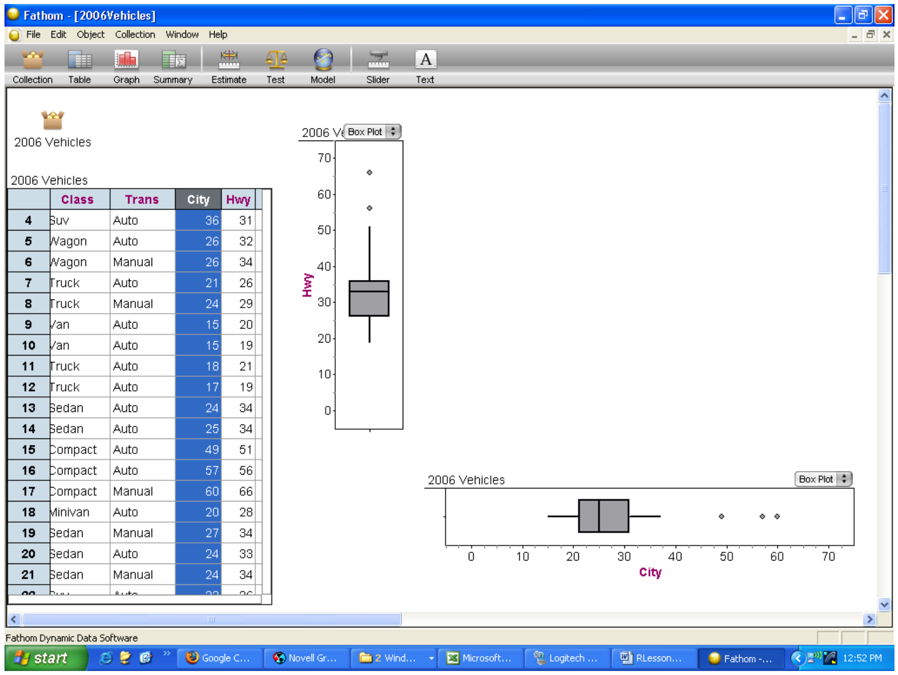

Figure 3. *Fathom* graph of two boxplots (Figure 4.5 in PTMT module)

During the planning for this lesson, one of the mathematics teacher educators pointed out that it would be good for the students to anticipate what the scatterplot will look like and draw it in before they actually create it in *Fathom*. "There's no question that asks them to explicitly, it might be nice to … say 'ok, based on your boxplots, what kind of scatterplot do you anticipate?'" The other teacher educator agreed, adding, "I think they viewed it more like an exercise, 'we'll do this because that's what the book says' as opposed to making any connections as to why we were doing it. But yeah I think that would be a good question, 'what would you anticipate the scatterplot to look like based on those two boxplots?'" Additionally, one of the mathematics teacher educators commented on how she "pointed out some things about the cluster being where you would expect the inter-quartile ranges to intersect." In the text there is no mention of the inter-quartile range, thus this is a piece of knowledge that she drew upon from her previous experiences with boxplots. She knew what students should know about the inter-quartile range and where that cluster should be in the scatterplot.

This example shows how these teacher educators drew upon their TPSK to notice that anticipating a scatterplot rather than simply following along and being shown a scatterplot could be a more effective way for prospective teachers to better understand the connections between the representations. Anticipating what the scatterplot will look like also could promote the prospective teachers to analyze the relationships between the two attributes – city and highway mpg. The teacher educators had to know the statistical concepts of the two plots, the pedagogical knowledge of anticipating the scatterplot, and technological knowledge of how *Fathom* can be used to display a scatterplot in relation to two boxplots. Within the framework, this example shows that the teacher educators had 'conception of how technology tools and representations support statistical thinking,' 'instructional strategies for developing statistics lessons with technology,' and 'critical stance towards evaluation and use of curricula materials for teaching statistical ideas with technology.'

Example 2. Another example of a mathematics teacher educators use of TPSK was when one instructor made an impromptu decision during class (which she discussed during a debriefing session) to use one of the technology files for Chapter 4 to help the prospective teachers make sense of residuals in least squared regression. The prospective teachers in this course had little experience with residuals and their concept image of them was somewhat limited. They were able to define a residual and to identify the graphical representation of the residuals associated with the squares shown with a moveable line in *Fathom*, that is, the vertical line segments connecting the actual and theoretical data points. Despite this knowledge, however, the homework presented a challenge for the students.

For homework (Section 4), students were asked to: 1) create a residual plot using *Fathom*, 2) adjust the moveable line, 3) consider the residual plot when determining the usefulness of the linear model, 4) sketch the location of the moveable line given a residual plot, and 5) describe some of the conceptual difficulties students may have and to list ways to help them understand. Most, if not all, of the prospective teachers in the course had not constructed a residual plot before this assignment. However, all of them were successful in doing so by following the instructions in Section 4. They also showed competence in adjusting the moveable line, but several admittedly had difficulty answering the other three questions. Time was allowed at the beginning of the next class meeting for the students to discuss their answers in small groups. The mathematics teacher educators noticed that the prospective teachers still had many unresolved questions regarding residuals. So, before moving ahead to Section 5, the decision was made to spend extra time on this topic.

One of the mathematics teacher educators decided to use a *Fathom* technology file for Chapter 4 to provide an additional demonstration of the residual plot and how it relates to the moveable line. The teacher educator used vertical translations of the moveable line to illustrate, once again, how the residual plot would respond. Figure 4 shows a screen capture of how the *Fathom* technology file was used, with the moveable line entirely above all of the actual data points, to help students understand why the corresponding residuals would have a negative numeric value. A similar translation was performed to exemplify all positive-valued residuals. This drastic manipulation of the moveable line further augmented the relationship between the moveable line and the residual plot, thereby enhancing students' understanding of residuals. Upon reflection of this demonstration, students were able to successfully respond to the remaining questions from Section 4 – questions that were left incomplete prior to the extended discussion.

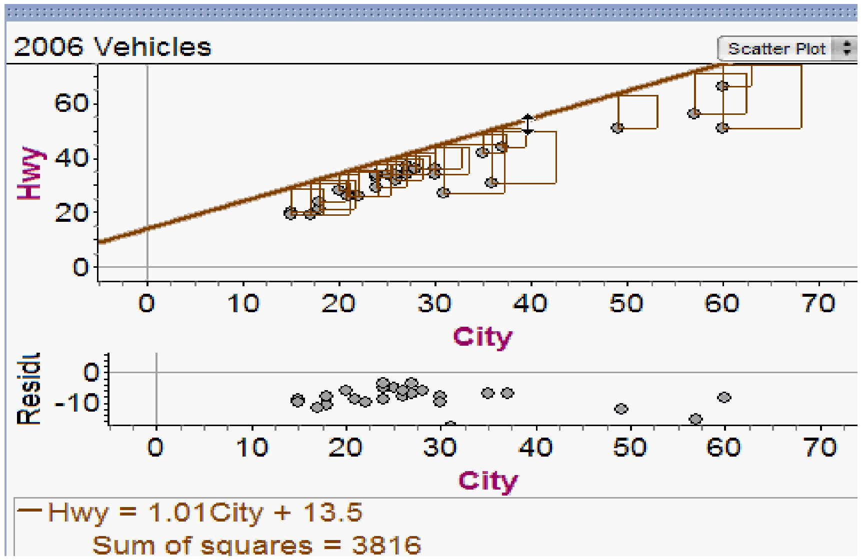

Figure 4. Screen capture of a technology file for Chapter 4.

In this example, the technology file and the dynamic environment of *Fathom* were used as an reorganizer to reinforce the connections between the scatter plot, moveable line, and residual plot - illustrating an instructional decision that was made for the development of statistical understanding through the use of technology. The mathematics teacher educator was able to conceptualize how the technology tool and representations within the dynamic environment would support students' understanding and statistical thinking.

Discussion

Our analysis of the planning and debriefing done by the mathematics teacher educators for the implementation of two chapters have helped us better understand the ways that instructors may make sense of the PTMT materials and the major decisions they need to make when implementing the module. It is clear that instructor resource materials need to include suggestions for how to allocate time for each section in the module, and ways to utilize small group work and whole group discussions. By learning what decisions teacher educators make when implementing new curriculum, we will be better able to tailor the instructor resource materials to address issues and difficulties mathematics teacher educators may encounter when first implementing the materials. For example, since residual plots are likely a new representation for many prospective teachers, perhaps Chapter 4 Section 4 (Visualizing Residuals) is best done as a whole class discussion. In addition, we will consider revisions to this section to include more description of a residual plot and to make better explicit connections to materials presented in Chapter 3 Section 4 on deviations from a mean, which lays a conceptual foundation for residuals from a line of best fit. Furthermore, our results can lead to planning professional development for faculty of these materials, which are planned as part of the dissemination efforts of the PTMT project. One idea for professional development that we will consider is the use of examples of how other mathematics teacher educators have implemented a section or chapter that can lead to discussions about pros or cons that the faculty can consider for their own local implementations.

Our results also indicate that the mathematics teacher educators did indeed need to draw upon the types of knowledge that Lee and Hollebrands (2008b) had identified in their TPSK framework (Figure 1) when implementing the PTMT materials. Not only does this help to validate that the framework may be useful for characterizing the elements of TPSK, but can help us in writing the instructor's guide in a way that makes this framework explicit to the mathematics teacher educators wishing to implement the PTMT material. One interesting finding to us was that it appeared that the examples we found in our analysis was that instances of TPSK seemed to occur when a mathematics teacher educator was extending the intended curriculum in a way that made sense to them for improving the understanding of their prospective teachers. In addition, it appears that an instructor needed to draw upon more than one of the four elements of TPSK in instances where they were creating learning opportunities for the prospective teachers.

1. understanding students' learning and thinking of statistical ideas with and without technology,
2. conception of how technology tools and representations support statistical thinking,

3. instructional strategies for developing statistics lessons with technology,
4. critical stance towards evaluation and use of curricula materials for teaching statistical ideas with technology.

In the example of building a scatterplot based on two boxplots, the mathematics teacher educator seemed to use elements 2, 3, and 4. The example of an instructor helping prospective teachers understand the residual plots seemed to include elements 1, 2, and 4. In our future analysis of the planning and debriefing sessions for the other chapters in the module, we would like to see if this notion generalizes to how and when the instructors seemed to use the four elements of TPSK.

The framework of TPSK was developed to describe the intended ways the curriculum materials were designed to develop TPSK for prospective teachers. Ultimately we want prospective teachers who learn from our materials to develop their own TPSK that will be useful for how they teach data analysis and probability to their future students in grades 6-12. However, our analysis certainly suggests that we must first attend to ways we can develop TPSK for technology-using mathematics teacher educators who may utilize our materials with their own prospective teachers.

Author Notes:

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References

Association of Mathematics Teacher Educators. (2006). *Preparing teachers to use technology to enhance the learning of mathematics: A position of the Association of Mathematics Teacher Educators*. Available at www.amte.net/Approved%20AMTE%20Technology%20Position%20Paper.pdf.

Bartlo, J., Larsen, S., & Lockwood, E. (2008). *Scaling up instructional activities: Lessons learned from a collaboration between a mathematician and a mathematics education researcher.* Paper presented at the Conference on Research in Undergraduate Mathematics Education. San Diego, CA.

Ben-Zvi, D. (2000). Toward understanding the role of technological tools in statistical learning. *Mathematical Thinking and Learning 2(1 & 2)*, 127-155.

delMas, R. C. (2004). A comparison of mathematical and statistical reasoning. In D. Ben-Zvi & J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning, and thinking* (pp. 79-95). Netherlands: Kluwer Academic Publishers.

Huberman, A. M., & Miles, M. B. (2002). *The qualitative researcher's companion.* Thousand Oaks, CA: Sage.

Koehler, M. J., & Mishra, P. (2005). What happens when teachers design educational technology? The development of technological pedagogical content knowledge. *Journal of Educational Computing Research*, 32(2), 131-152.

Konold, C., & Higgins, T. (2003). Reasoning about data. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to Principles and Standards for School Mathematics* (pp. 196-215). Reston: VA, National Council of Teachers of Mathematics.

Lee, H. S., & Hollebrands, K. (2008a). Preparing to teach mathematics with technology: An integrated approach to developing technological pedagogical content knowledge. *Contemporary Issues in*

Technology and Teacher Education [Online serial], *8*(4). Retrieved from: http://www.citejournal.org/vol8/iss4/mathematics/article1.cfm

Lee, H. S., & Hollebrands, K. F. (2008b). Preparing to teach data analysis and probability with technology. *Proceedings of the Joint Study of the ICMI/IASE*. Monterrey, MX, June. Available online http://www.ugr.es/~icmi/iase_study/Files/Topic3/T3P4_Lee.pdf.

Lee, H. S., Hollebrands, K. F., & Wilson, P. H. (under review). *Preparing to Teach Mathematics with Technology: An integrated approach to Data Analysis and Probability*.

Mishra, P., & Koehler, M. (2008). Introducing technological pedagogical content knowledge. Paper presented at the Annual Meeting of the American Educational Research Association (New York, New York).

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Niess, M.L. (2005). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education, 21*, 509-523.

Niess, M. L. (2006). Guest Editorial: Preparing teachers to teach mathematics with technology. *Contemporary Issues in Technology and Teacher Education* [Online serial], *6*(2). Available: http://www.citejournal.org/vol6/iss2/mathematics/article1.cfm

Pea, R. D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89-122). Hillsdale, NJ: Erlbaum.

Rubin, A., & Hammerman, J.K. (2006). Understanding data through new software representations. In G. Burrill (Ed.), *Thinking and reasoning with data and chance: Sixty-eighth yearbook* (pp. 241-256). Reston, VA: National Council of Teachers of Mathematics.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, *15*(2), 4-14.