An Analysis of College Mathematics Departments' Credit Granting Policies for Students with High School Calculus Experience

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Longitudinal studies conducted by the United States Department of Education's National Center for Educational Statistics (NCES) show the percent of students taking calculus in high school has been increasing steadily since 1982. While only 5.9% of high school graduates completed a calculus course in 1982, 14.1% completed a calculus course in 2004 (NCES, 2007). It is estimated that over 500,000 students enrolled in a calculus course in 2007-2008 (Bressoud, 2008). The largest percent of these students enroll in AP Calculus courses, but others participate in dual credit programs, the International Baccalaureate program, or enroll in other courses with calculus in the title.

The high school experiences of these students may vary depending on the type and quality of the calculus program in which they participate. Since standard placement tests used by college mathematics departments do not include calculus topics, departments must use other available information to determine appropriate first courses for these students and to determine whether credit should be granted for their high school experience. It is unclear if common credit granting policies for students with high school calculus experience are accurately placing students.

This research investigated differences in calculus achievement, as students entered college, between different subgroups of students with prior high school calculus experience. Using common credit granting policies, students were

categorized into three groups: students who earned college credit for Calculus I by successful completion of an AP Calculus Exam; students who earned college credit by successful completion of a dual credit (or dual enrollment) calculus course in high school; and students who completed calculus in high school but did not earn college credit for Calculus I.

Common Credit Granting Policies

The two most common methods for mathematics departments to grant credit for Calculus I are by successful completion of College Board's AP Calculus AB or BC exam or by successful completion of a dual enrollment calculus course (Laurent, 2009). Ninety-five percent of colleges and universities grant credit based on AP exam scores, with scores of 3 and 4 being the most common requirements (Laurent, 2009). Other students may earn credit for Calculus I by successful participation in dual credit programs through partnering colleges or universities. For this research, dual credit courses are defined as college-level courses taught on a high school campus by a high school teacher where students receive both high school and college credit after successful completion of the course. Fifty-nine percent of colleges and universities grant credit for Calculus I based on successful dual credit course participation, with a grade of at least C being the most common requirement (Laurent, 2009).

College Board's Advanced Placement (AP) Program

The College Board's Advanced Placement (AP) program has seen steady increases in numbers of students taking the AP Calculus Exams. In 2007, over 276,000 students took an AP Calculus Exam; a 30% increase over five years (College

Board, 2008). These numbers, however, do not include a large population of students who take AP Calculus courses but choose not to take an AP Exam.

The AP Calculus program has specific requirements and expectations for their courses as well as an audit process in place to insure program consistency (College Board, 2008). Additionally, students with successful AP Exams scores have been shown to earn higher grade point averages in advanced courses compared to students who did not place out of Calculus I, controlling for SAT and high school class rank (College Board, 2008).

Dual Credit Calculus Courses

Dual credit courses are offered in 71% of all public high schools, more commonly in rural communities than urban (Waits, Setzer & Lewis, 2005). The National Alliance of Concurrent Enrollment Partnerships (NACEP), established in 1999, provides an accreditation process for dual enrollment programs that includes curriculum and evaluation standards. However, very few higher education institutions are accredited through this agency (NACEP, 2008).

Inconsistencies and lack of oversight in dual enrollment programs has recently been questioned (Bressoud, 2007b; Cheifetz & Schmierer, 2007; Lutzer, Maxwell & Rodi, 2005). For example, 40% of 4-year colleges who participate in dual enrollment programs reported that they never have control over the final exam administered in the course. An additional 30% indicated they only sometimes have control over the final exam (Bressoud, 2007b). It is unclear from the literature whether students receiving dual credit for Calculus I and students receiving credit by AP Calculus Exams are equally qualified for Calculus II.

Responsibility for Evaluation

The Mathematical Association of America (MAA) has guidelines for undergraduate mathematics programs which include the departmental responsibility for establishing placement policies. In addition, the guidelines state that departments should periodically review the effectiveness of their placement procedures (MAA, 2003). However, a survey of mathematics department placement policies for students with high school calculus experience found that only 25% of colleges had evaluated their policies for this population (Laurent, 2009). Additionally, several schools reported that it had been a long time since they last evaluated their policies.

Assessment of Calculus Achievement

The Calculus Concepts Placement Test (CCPT) was administered to 143 first-year freshmen attending the St. Louis College of Pharmacy (STLCOP), to determine if there is a difference in calculus achievement between different subgroups of students with prior high school calculus experience as they enter college. This research question compared groups with predetermined, independent variable characteristics. Therefore, a causal-comparative research design was used. An ANOVA was performed to compare groups and the Dunnett C test was used for post hoc comparisons.

Students' high school transcripts, dual credit college transcripts, and College Board AP Exam score reports were used to categorize students for the analysis. Additionally, ACT math sub-scores (MACT) were obtained so they could be controlled in the analysis.

To answer the research question, students were grouped based on their earned credit status as determined by the following definitions: No Credit: Took Calculus in high school but did not earn credit for Calculus I; AP Calculus AB Exam: Earned credit for Calculus I with a satisfactory AP Calculus AB Exam score; and Dual Credit: Earned credit by successfully completing a dual credit course with a grade of C or higher.

It is possible that the mathematics ability, as measured by the MACT, of students enrolled in AP Calculus courses versus dual enrollment courses may differ. Therefore, an ANCOVA was performed to compare means while adjusting for MACT. Again, the Dunnett C test was used for post hoc comparisons.

Instrumentation

The Calculus Concepts Placement Test (CCPT) used in this study was a modified form of the Calculus Validation Exam (CVE) developed by the mathematics department faculty at the United States Military Academy (USMA). This test has been used for the past four years to determine placement into their advanced mathematics program (Retchless, Boucher & Outing, 2008). The content of the test covers single variable calculus topics typically taught in a first semester calculus course. The questions were designed so that a calculator is not necessary to solve the problems; therefore, no technology was available to students during the test.

Using two years of data provided by the USMA, student scores on the CVE had a small positive correlation to both Math SAT subscores and AP Calculus AB exam scores (Pearson correlation coefficient of .249 and .309, respectively). A factor analysis suggested only one clear factor being assessed with this instrument. An item

analysis of the CVE was performed based on the results of one year of USMA testing. The original 35 questions were reduced to 23 for this study (nine multiple choice and fourteen free response), improving the Cronbach alpha from .675 to .714.

The CCPT was administered two days prior to the beginning of fall semester, 2008, prior to any calculus instruction at the institution. Students were given fifty minutes to complete the exam. To encourage participation and a good effort on the exam, bookstore gift certificates were offered to the fifty students scoring the highest on the exam.

Sample

Based on St. Louis College of Pharmacy (STLCOP) application data, 152 students, who committed to attending the college in the fall, were identified as eligible to participate in this study. All potential participants in this study were traditional students, having entered college in the fall following their senior year of high school. Students who indicated in their college application that they had taken calculus or intended to take calculus in their senior year of high school were asked to participate. Of the 152 students identified, one student indicated that she was not eligible to participate because she did not take calculus in high school, two students chose not to participate, two students withdrew from the college and therefore did not participate, and four students did not attend orientation. The test, therefore, was administered to 143 STLCOP students.

Of the 143 students participating in the study, 138 submitted ACT scores in their college applications, 58 submitted AP Calculus Exam scores and 29 submitted college transcripts with dual credit grades for Calculus I.

The ACT Math sub-scores (MACT) of this sample ranged from 24 to 36 with a mean of 28.3. AP Calculus AB Exam scores for the sample ranged from 1 to 5 with a mean of 3.10. Dual credit grades reported on college transcripts ranged from an A to a C, with a mean of 3.7 on a 4 point scale. This mean includes 20 students who earned A's, eight who earned B's and only one student who earned a C. Dual credit transcripts represented 16 different higher education institutions in the Midwest.

The AB Exam average for the sample is similar to the national average of 2.94 reported by the College Board (College Board, 2008). The average AP Calculus AB score for STLCOP students includes eleven estimated scores based on Calculus BC Exam results. These estimations are reported by the College Board and are indicated on the score report sent to the college.

Item Analysis of the CCPT

After students completed the CCPT, an item analyses was conducted on the 23 items hypothesized to assess calculus achievement. Initially, each of the items was correlated with the total score for the test (with that item removed). All the correlations were greater than .30 except six items. Based on these results, these six items were eliminated from the scale. Item-total correlations for the revised 17-item assessment yielded only one correlation that was less than .30. This item was also eliminated resulting in a 16-item modified CCPT with a Cronbach's alpha of .833.

Scores on the modified CCPT ranged from 0 to 21 out of a possible 29 with a mean of 5.48 and standard deviation of 5.00. The low mean should not be surprising, since students did not prepare for this exam and most of them had no exposure or practice with calculus over the summer. Additionally, the majority of these students

registered to re-take Calculus I in their first semester of college. For those students who earned credit for Calculus I, however, these scores are disturbing.

Inter-rater Reliability

The scoring of the exam was as follows. For multiple choice questions, students earned one point for correct responses and zero points for incorrect responses. For the free-response questions, students earned two points for correct responses, one point for partially correct responses, and zero points for answers that show no understanding of the question.

Ten percent of the tests were randomly selected to check for inter-rater reliability. A mathematics graduate student and a college mathematics professor were given copies of the tests along with scoring guidelines and were asked to re-grade them. No markings from the original grading were on the tests. The graduate student's scores and the mathematics professors scores were correlated to the researcher's scores for the test. Results showed an inter-rater correlation of .970 and .987, respectively.

Analysis of Results

Because both scores of 3 and 4 are common requirements for colleges to grant credit for Calculus I, it was decided to conduct two different analyses (Laurent, 2009). The first analysis categorized the sample into the following groups based on a credit granting policy that requires a 4 on the AP Calculus AB exam. Students' earned credit status was determined by the following definitions: No Credit: Took Calculus in high school but did not earn credit for Calculus I; AP Calculus AB ≥ 4: Earned credit for Calculus I with an AP Calculus AB score of 4 or higher; and Dual

Credit: Earned credit by successfully completing a dual credit course with a grade of C or higher. The second analysis was similar except that the second group was defined as AP Calculus AB ≥3: Earned credit for Calculus I with an AP Calculus AB score of 3 or higher.

In this study, there were a few students who participated in dual enrollment programs who also took the AP Calculus AB Exam. If the students' scores on the AP Exam were high enough to earn credit, they were placed into the second group.

Analysis 1 – No Credit versus AP Calculus AB ≥ 4 versus Dual Credit

A one-way analysis of variance (ANOVA) was conducted to evaluate the relationship between the modified CCPT and students' earned credit status. The independent variable, earned credit status, included three levels: No Credit, AP Calculus AB ≥ 4, and Dual Credit. The dependent variable was the modified CCPT score. One student earned both dual credit and earned an AP Calculus AB score ≥ 4. This student was placed into the AP Calculus AB ≥ 4 group.

Results of the analysis are shown in Table 1. The overall ANOVA was significant, $F(2,140) = 33.80$, $p < .01$. Results indicated a strong relationship between students' earned credit status and the modified CCPT as assessed by a partial η^2, with earned credit status accounting for 33% of the variance of the dependent variable. The AP Calculus AB ≥ 4 group had the largest mean (M = 12.14), followed by the dual credit group (M = 4.61) and the no credit group (M = 4.17).

Because the overall *F* test was significant, follow-up tests were conducted to evaluate pair-wise differences among the means. Levene's test of equality-of-error variances was significant, $p < .01$, therefore unequal variances were assumed and the

Dunnett C test was used for post hoc comparisons. Results showed the AP Calculus AB ≥ 4 group's mean placement test score was significantly different from both the no credit group mean ($p < .01$) and the dual credit group mean ($p < .01$). Differences in the no credit group and dual credit group means were not statistically significant ($p = .88$).

Table 1
Means and Standard Deviations of the Modified Calculus Concepts Placement Test by Earned Credit Status (No Credit, AP Calculus AB ≥ 4, Dual Credit)

Earned Credit Status	n	M	SD
No Credit	93	4.17	3.70
AP Calculus AB ≥ 4	22	12.14*	5.57
Dual Credit	28	4.61	4.28
TOTAL	143	5.48	5.00

* $p \leq .01$.

The correlation between students' MACT scores and the modified CCPT was calculated. The Pearson correlation was .488, indicating a strong relationship between the two variables.

Because students were not randomly assigned to groups, it is possible that the groups' general mathematics ability were not equivalent prior to taking calculus in high school. Indeed, MACT means were different for the three groups (see Table 2). Therefore, it is necessary to control for math ability as measured by the MACT. Five of the 143 students in the sample did not report ACT scores and thus were not included in this analysis.

A one-way analysis of covariance (ANCOVA) was conducted. The independent variable, earned credit status, included three levels: No Credit, AP Calculus AB ≥ 4, and Dual Credit. The dependent variable was the modified CCPT score and the covariate was MACT. A preliminary analysis evaluating the homogeneity-of-slopes assumption indicated that the relationship between the covariate and the dependent variable did not differ significantly as a function of the independent variable $F(2,132)=1.366$, MSE = 15.20, $p = .26$, partial $\eta^2 = .02$. The ANCOVA was significant, $F(2, 134) = 17.78$, MSE = 15.28, $p < .01$. The strength of relationship between the earned credit status and the dependent variable was very strong as assessed by a partial η^2, with the earned credit status factor accounting for 21% of the variance of the dependent variable, holding MACT constant.

Table 2
Means and Standard Deviations of the Modified Calculus Concepts Placement Test by Earned Credit Status (No Credit, AP Calculus AB ≥ 4, Dual Credit) – Controlling for MACT

		MACT	Modified CCPT	
Earned Credit Status	n	M	M	Adjusted M
No Credit	90	27.74	4.17	4.61
AP Calculus AB ≥ 4	20	30.85	12.14*	10.69*
Dual Credit	28	28.84	4.61	4.53
TOTAL	138	28.34	5.48	5.48

* $p \leq .01$.

The means of the modified CCPT adjusted for MACT changed only slightly (See Table 2). The AP Calculus AB ≥ 4 group had the highest adjusted mean (M =

10.69). Again, the no credit group and dual credit group had lower means (M = 4.61 and M = 4.53, respectively).

Follow-up tests were conducted to evaluate pair-wise differences among these adjusted means. The Holm's sequential Bonferroni procedure was used to control for Type I errors across the three pair-wise comparisons. Again, there were significant differences in the adjusted means between the AP Calculus AB group and the dual credit group ($p < .01$) and between the AP Calculus AB group and the no credit group ($p < .01$). However, there was not a significant difference between the dual credit group and the no credit group ($p = .93$).

Analysis 2 – No Credit versus AP Calculus AB ≥ 3 versus Dual Credit

The analysis described above was performed again with slightly different group definitions. For this second analysis, the independent variable, earned credit status, was defined as follows: No Credit, AP Calculus AB ≥ 3, and Dual Credit. The dependent variable was, again, the modified CCPT score. With these new definitions, 20 students who were previously in the no credit group were now in the AP Calculus AB ≥ 3 group. Two dual credit students were also moved into the AP Calculus AB ≥ 3 group.

Results of the analysis are shown in Table 3. The overall ANOVA was significant, $F(2,140) = 37.59$, $p < .01$. Results indicated a strong relationship between students' earned credit status and the modified CCPT as assessed by a partial η^2, with earned credit status accounting for 35% of the variance of the dependent variable. The AP Calculus AB ≥ 3 group had the largest mean (M = 9.84), followed by the dual credit group (M = 4.50) and the no credit group (M = 3.21).

Table 3
Means and Standard Deviations of the Modified Calculus Concepts Placement Test by Earned Credit Status (No Credit, AP Calculus AB ≥ 3, Dual Credit)

Earned Credit Status	n	M	SD
No Credit	73	3.21	2.93
AP Calculus AB ≥ 3	44	9.84*	5.35
Dual Credit	26	4.55	4.30
TOTAL	143	5.48	5.00

* $p \leq .01$.

The overall *F* test was significant; therefore follow-up tests were conducted to evaluate pair-wise differences among the means. Levene's test of equality-of-error variances was significant, $p < .01$, therefore unequal variances were assumed and the Dunnett C test was used for post hoc comparisons. Results showed the AP Calculus AB ≥ 3 group's mean placement test score was significantly different from both the no credit group mean ($p < .01$) and the dual credit group mean ($p < .01$). Differences in the no credit group and dual credit group means were not statistically significant ($p = .91$).

A one-way analysis of covariance (ANCOVA) was conducted to control for math ability as measured by the MACT. A preliminary analysis evaluating the homogeneity-of-slopes assumption indicated that the relationship between the covariate and the dependent variable did not differ significantly as a function of the independent variable $F(2,132)=1.501$, MSE = 14.19, $p = .23$, partial $\eta^2 = .02$. The ANCOVA was significant, $F(2, 134) = 23.61$, MSE = 14.30, $p < .01$. The strength of relationship between the earned credit status and the dependent variable was very

strong as assessed by a partial η^2, with the earned credit status factor accounting for 26% of the variance of the dependent variable, holding MACT constant.

The means of the modified CCPT adjusted for MACT are reported in Table 4. The AP Calculus AB ≥ 3 group had the highest adjusted mean (M = 9.11). Again, the no credit group and dual credit group had lower means (M = 3.77 and M = 4.39, respectively). Follow-up tests were conducted to evaluate pair-wise differences among these adjusted means. The Holm's sequential Bonferroni procedure was used to control for Type I errors across the three pair-wise comparisons. Again, there were significant differences in the adjusted means between the AP Calculus AB group and the dual credit group ($p < .01$) and between the AP Calculus AB group and the no credit group ($p < .01$). However, there was not a significant difference between the dual credit group and the no credit group ($p = .48$).

Table 4
Means and Standard Deviations of the Modified Calculus Concepts Placement Test by Earned Credit Status (No Credit, AP Calculus AB ≥ 3, Dual Credit) – Controlling for MACT

		MACT	Modified CCPT	
Earned Credit Status	n	M	M	Adjusted M
No Credit	71	27.41	3.24	3.77
AP Calculus AB ≥ 3	41	29.83	9.94*	9.11*
Dual Credit	26	28.54	4.50	4.39
TOTAL	138	28.34	5.48	5.48

* $p \leq .01$.

Comparison of Analysis 1 and 2

Changing the score requirement on the AP Calculus AB exam from 4 to 3 produced a lower mean score for the AP group (12.14 and 9.94, respectively). However, if either of these scores is used to grant credit for Calculus I, significant differences are found between students earning credit with this method versus the other two groups. Additionally, using either score, there are no differences between the dual credit group and the no credit group.

Limitations

The students participating in this research are not a random sample of the population and represent only a subset of students who take calculus in high school. Because of the heavy emphasis in science in the pharmacy curriculum, it is assumed that students enrolled at STLCOP have an interest in the life sciences. Other students who commonly take mainstream calculus in college (those interested in engineering, or physical sciences, for example) are not represented.

It was decided that students participating in this study would not have an opportunity to review prior to taking the calculus assessment since the purpose of the study is to examine what the students learned and retained from their high school calculus experience. Because this test was administered to students as they entered college in the fall, and most students had completed their high school calculus course the previous spring, it is not clear how the scores would change if students were given time to review. Additionally, students were not allowed to use calculators on the CCPT used in this study. If students' high school calculus courses emphasized calculator usage, some of these skills may not be measured on this assessment.

Conclusion

The results of the CCPT indicated that students who earned credit for Calculus I by AP Calculus AB Exam had significantly higher calculus achievement than their dual credit counterparts, even when controlling for ACT math sub-scores. In fact, dual credit students' calculus achievement was statistically equivalent to students took calculus in high school but did not earn college credit. It is not clear however, how the subgroups in this study compare to students who complete their first calculus course in college. Are calculus achievement levels of students who successfully complete college Calculus I courses comparable to those who successfully complete the AP Calculus Exam or are they comparable to those who successfully complete dual credit programs? The answer to this question will determine whether college mathematics departments should focus their attention on the oversight of dual credit programs or if they should look more closely at the AP Calculus program and exams to determine the reasons for its success.

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