Examining Novice College Mathematics Instructors' Conceptions, Experiences, and Teaching Practices Related to College Algebra

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Abstract

This paper reports the preliminary findings from an ongoing study on mathematics graduate Teaching Assistants (TAs)' conceptions of mathematics in general, and algebra in particular, their prior learning experiences in mathematics, and how may such conceptions and experiences be related to the TAs' current practices in teaching pre-calculus algebra to college students. The TAs either hold a polarized view on the nature of algebra, or tend to characterize algebra as a mix of multiple topics and activities. Most TAs contributed successful algebra teaching and learning to practice and strengthening basic skills. Cultural backgrounds and past learning experiences explain partially the differences in the TAs' teaching styles. No obvious correspondences were found between the TAs' conceptions of algebra and teaching styles.

Background

Graduate Teaching Assistants (TAs) are essential members of the college mathematics instructors' community and play crucial roles in teaching developmental and general education

mathematics content. Yet they form a special group of mathematics educators since the majority of them are novice instructors and have had little formal or systematic training in teaching mathematics as mathematics educators at the elementary and secondary levels would be required to have. The past decade has seen increased efforts invested by mathematics education researchers in characterizing mathematics TAs' knowledge, beliefs, attitudes, teaching practices, and the interrelations among these aspects (Belnap, 2005; DeFranco and McGivney-Burelle, 2001; Gutmann, Speer & Murphy, 2002; Meel, 2000; Speer, 2001, 2004, 2008). Interested researchers even formed working groups to meet annually at conferences such as PME-North America to share research agenda, ideas, and findings (Gutmann, Hsu, Marrongelle, Murphy, Speer, Star, and Terrell, 2004). Nonetheless, the body of research on mathematics TAs is still relatively limited in terms of its theoretical foundations, scopes, depths, methods, and connections to practice (Speer, Gutmann & Murphy, 2005).

Research Objectives and Questions

In the summer of 2008 the researcher launched a project that attempts to examine mathematics graduate TAs' conceptions of mathematics and algebra, experiences in learning mathematics, and how these may be related to the TAs' current teaching practices. These TAs are teaching pre-calculus algebra courses and most of them intend to earn a master's degree rather than a doctorate.

The researcher's interests in this project originates from his observation that most of the mathematics TAs involved in existing studies have been Ph.D-bound and teaching or preparing to teach calculus-level courses (Gutmann, Speer & Murphy, 2005). To understand fully the diverse backgrounds, experiences, knowledge, conceptions, and practices of mathematics TAs across various types of programs and institutions, it is necessary to extend the research to include

those TAs who are currently aiming for a master's degree in the mathematical sciences and are instructing pre-calculus mathematics courses such as college algebra or intermediate algebra. Such extended research becomes even more imperative when we consider the fact that algebra is the main high-stake mathematics course in high schools and a gatekeeper subject for studying college mathematics, sciences, and engineering. The teaching and learning of algebra at college level do resemble those at the secondary school level in some aspects, while fundamental differences also exist between the two levels in some other aspects.

With the data and findings from the project, the researcher hopes to develop and disseminate a better understanding of how and why mathematics TAs choose to handle mathematical subjects and topics in certain ways, and what professional development opportunities should and could be offered to the TAs for the improvement in their teaching skills and in student learning outcomes.

Specifically, the project aims to answering the following two research questions:

1. What kinds of conceptions do mathematics graduate TAs hold with respect to the nature, teaching, and learning of mathematics in general, and of algebra in particular?

2. How may such conceptions, as well as the TAs' past experiences in learning mathematics and algebra, be related their teaching styles in college algebra classrooms?

Conceptual Framework

Existing research on mathematical beliefs has produced various forms of categorization schemes (Speer, 2005). One of the most fundamental scheme focuses on teachers' beliefs about mathematics (Ernest, 1989; Lerman, 1990). Another major scheme centers on teachers' beliefs about mathematics teaching and learning (Kuhs and Ball, 1996). Based on these two themes, as well his recent work on teachers' mathematical knowledge for teaching (Li, 2007a, 2008), the researcher developed a conceptual framework to map and characterizes a mathematics

instructor's conceptions of a particular mathematical subject. The framework encompasses three major dimensions:

Conception of the Mathematics Subject Matter

This dimension refers to a mathematics instructor's perception of the nature, objects, and main topics of this subject, whether and how this subject is related to some other disciplines, and whether and how it is important or useful to the students and to the society, etc.

Conception of the Teaching of the Subject

This dimension refers to a mathematics instructor's belief on the major objectives in teaching this subject and specific topics, the most reasonable ways to sequence the topics and the most effective ways of presenting each of these topics, as well as the main factors that influence the quality of teaching, etc.

Conception of the Learning of the Subject

This dimension refers to a mathematics instructor's understanding of whether and why this subject and particular topics may be difficult to a certain group of students, the major barriers and difficulties in student learning, what it means for a student to sufficiently understand the subject, how students learn most efficiently and conceptually, the major factors contributing to a student's success in learning this subject, etc.

In this framework the term *conception* could be used interchangeably with others such as *belief* or *view*. A teacher's professional conceptions often overlap with his or her knowledge, to a certain extent or in certain aspects. For the purpose of this study, the researcher defines a fundamental distinction between a mathematics teacher's beliefs and knowledge as follows: an teacher's conceptions of a mathematical subject are mostly subjective and hardly to be determined as right or wrong, whereas a teacher's mathematical knowledge can be assessed as

valid or invalid or of high or low quality, based some commonly accepted mathematical or professional criteria (Li, 2007b).

Research Context and Methods

The research study takes place at a public university in the western United States. It has a highly diverse student population of nearly 38,000. Each semester the mathematics department appoints over 20 graduate TAs to teach developmental and pre-calculus mathematics courses, and provides on-going professional development and mentoring opportunities which include (a) beginning-of-the-semester TA workshops that address teaching and classroom management strategies, students' learning characteristics, curriculum analysis, assessment design and grading; (b) TA peer teaching observations, evaluations and reflections; (c) TA group planning, rubric design, and grading; (d) TA mentors observing TAs' teaching and providing feedbacks afterward.

Since Summer 2008 data collection in this project has gone through three stages:

1. In the first stage, the researcher designed and administered a questionnaire with 20 mathematics graduate TAs in August 2008 at the TA professional development workshop. These TAs are in the first or second year of their graduate studies in mathematics. Most of them plan to earn a master degree then find a job as college mathematics instructor. They have either taught developmental or introductory mathematics courses in previous semesters, or had mathematics tutoring experiences with college or high school students. Seventeen of these TAs were about to start teaching college algebra courses in the fall semester.

The questionnaire collects information on the TAs' academic backgrounds and probes their conceptions of mathematics in terms of its importance, learning difficulties, and strategies for working with students. The researcher also took observation notes during the workshop, including the 20-minute micro-teaching presented by each individual TA on an assigned topic.

2. In the second stage, based on data from the questionnaire and micro-teaching observations, the researcher selected eight TAs with different academic backgrounds and teaching styles, and observed three consecutive sessions taught by each of them in September and October.

3. In the third stage, shortly after the completion of the observations with each of the eight TAs, the researcher conducted an hour-long, semi-structured interview with each TA to discuss (a) his or her teaching styles (why he or she prefers such styles, what prior mathematics teaching and learning experiences have influenced the styles, whether the styles are effective in the classrooms, etc.), (b) a few specific scenarios in the three observed lessons and the TA's thinking and reasoning behind the decision-making in each scenario, and (c) his or her conceptions of the algebra course, student characteristics, and issues in algebra teaching and learning. All the interviews were audio-recorded upon consents of the interviewees.

Data analysis has been ongoing and intertwined with the data collection stages. The TAs' responses to each question in the questionnaire were categorized and summarized for common themes or patterns. Videos from the mini-teaching activity and teaching observations (with the eight selected TAs) are being reviewed and cross-referenced with the observation notes. The current units of analysis are the episodes in which the TA is explaining an important algebraic concept or process (e.g., exponential growth and the formula for compound interests), or there is an interesting interaction between the TA and the students. Post-observation interviews with the TAs are being partially transcribed, which is focusing on several key interview questions. Existing transcripts are being coded and categorized according to the conceptual framework.

Preliminary Findings

Existing data demonstrates the diversity in the TAs' conceptions, past experiences, and current teaching practices. It also reveals the complexity in how these three aspects are linked.

Conceptions of Mathematics

The TAs provided a variety responses to the question "How would you convince your students that mathematics is very important?". The most popular answers include:

- It is a set of practical skills for solving real-life problems and interpreting daily information
- It teaches logical, analytical, and critical thinking skills
- It helps you find a job or impress your boss; Employers are putting more values on an employee's mathematical competence
- It is the language for understanding other subjects, foundation for all fields of science and technology
- It is a puzzle to discover, it is fun, it has clarity, and there is right or wrong answer

Conceptions of Algebra

Most TAs didn't seem to have previously given many thoughts the nature of algebra. They responded to the question "What is algebra about?" in two different ways. One was to list several algebraic activities and topics (e.g., "generating structures from a collection of things", "functions", "equations", "rules", "guidelines for manipulating numbers and variables", "logical thinking", "solving problems", "applications", "linear vs. exponential relationships"). The other way was to focus on one of the two competing aspects of algebra: basic skills versus modeling and application. Two of the TAs held a very traditional view that algebra is a set of symbolic skills, rather than about modeling or application. One of them stated that "I don't know much about the use of algebra. I'm not an applied person. I don't like the reality part". The other TA claimed that he didn't like the modeling approach to teaching college algebra because "algebra is missing from this course".

In phrasing their thoughts on a related question, "Why is algebra important?", the TAs put different emphases on four basic notions of algebra: (a) It is a set of rules or basic skills, (b) It is a set of tools for solving a variety of problems, (c) It is a logical way of thinking and reasoning, and (d) It is the foundation for more advanced studies. Some of these notions, especially (a) and (b), were directly translated into the TAs' beliefs about algebra teaching and learning. For instance, most of the TAs stressed "practice" and "improving basic skills" as the most important factors for being successful in studying algebra.

Conceptions and Experiences

Educational backgrounds and past learning and teaching experiences do have influences on the TAs' mathematical conceptions. For instance, a major difference between the beliefs of the American TAs and some of the international TAs lies in the real-world applications of algebra. One of the TAs who had Asian school education background stated, "In my home country I just learned algebra as basic skills, and never paid attention to the application part. Now I realize the importance of applications, and it is the biggest challenge to my teaching." Another TA from Asia said that "I used to view algebra as only about equation solving. Now [after teaching algebra here] I feel algebra is more about modeling real world situations and predicting future".

Conceptions and Practices

Some of the TAs believed that, in order to succeed in algebra, it is crucial for students to work collaboratively, ask many questions, and explain ideas to each other. Correspondingly these TAs managed to establish an interactive environment where the instructor and the students pose and respond to questions frequently. They intentionally arranged group activities and discussions during each class session. Beyond sharing a relatively outgoing personality, these TAs' preference of an interactive classroom environment often originated from contrasting

learning experience during their high school and college mathematics studies. For example, some teachers and professors just "talked to the board", which were "very boring" and "shut many students down", while others have been very engaging and enthusiastic.

Some other TAs chose to teach by lecturing. The reasons vary: it may be because they are shy, or they want to maintain a good control of the pacing due to time constraints, or because of their learning experiences in a foreign educational system where straight lecturing is the norm of mathematics teaching, students are only supposed to take notes and rarely ask questions.

No simple correspondences seem to exist between the TAs' conceptions of algebraic subject matter and the TAs' algebra teaching styles. A TA who views algebra as mainly about symbolic rules and skills might teach by straight lecturing or in very interactive approaches. Those TAs who did recognize modeling and application as essential component of algebra demonstrated varied levels of interaction with their students during algebra teaching.

Discussion

In this study, the TAs' notions of mathematics and its importance form a spectrum. This could be contributed to the fact that the TAs have gone through many years of mathematics studies and been exposed to various levels and types of mathematics subject. Specific to algebra, their conceptions are either polarized (a choice between skills and application) or become an unsystematic blend of algebra topics and activities. This phenomenon echoes the competing views or various conceptions of algebra that have coexisted in the contemporary history of mathematics education (Chazan, 2000; Kieran, 2007; Usiskin, 1988). As mathematics students and novice algebra instructors, these TAs have not had enough opportunities to think about the nature of algebra as a developed, multifaceted while also dynamic body of knowledge. That could be the main source of polarization or mixture in their descriptions of algebra. Professional

development activities for TAs should include an introduction and discussion on the nature and historical development of algebra, as well as the recent trends of algebra curriculum in the past few decades.

As summarized by Speer (2005), studies on mathematics teachers' beliefs have revealed both consistencies and inconsistencies between teachers' professed (stated) beliefs and their attributed beliefs (as reflected in their practices). The conceptual framework and preliminary findings in this study suggest a potentially new explanation for such complicated relationship: teachers' professed conceptions consist of multiple dimensions and there may not be even inter-dimensional consistency in the first place because of the complex nature of a mathematical subject like algebra. Depending on which aspects of the subject a teacher is focusing on at the moment of reflection, the teacher could provide quite distinct narrative of what he or she believes regarding the subject matter and its teaching and learning. When such inconsistency is translated into a teacher's instructional practices, it may be observed or interpreted as an inconsistency between the teacher's professed beliefs and attributed beliefs. If this analysis is true, future studies on teachers' conceptions of a mathematical subject have to consider the multifaceted characteristics of the subject, and probe teachers' views and perspectives from various directions.

It will be also worthwhile to contrast the data and findings from this ongoing study with those from existing studies on Ph.D.-bound graduate TAs who are teaching calculus or higher-level mathematics courses. A sample research question would be whether and how factors such as mathematical preparation, career goals, and the course being taught have significant impacts on the TAs' conceptions and teaching practices.

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