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Incorporating Inquiry-Based Class Meetings with Computer-Assisted Instruction

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Incorporating Inquiry-Based Class Meetings with Computer-Assisted Instruction

What is the effect of incorporating inquiry-based group work sessions in a Finite Mathematics course in which the primary pedagogy is computer-assisted instruction? Our research at a major state university investigates in a randomized quasi-experimental study the relative effects of combining computer-assisted instruction with inquiry-based group work sessions, traditional summary lectures of material to be covered in the computer-based part, and the latter combined with regular in-class quizzing on lecture material. Results suggest that a group work session with individually written reports and regular feedback significantly improves students' ability in problem identification, showing evidence of problem-solving, and quality of explanation of reasoning leading to the solution over the other two treatments. This is accomplished without any significant difference in students' grades or in growth of accuracy in pre- and post-testing.

Student success as measured by grades, and greater efficiency in terms of cost effectiveness, have been a driving force in "course reform" over the past 15 years, particularly at large state universities (NCAT, 2008). One prevalent direction of course reform has been the development of, and widespread use of, sophisticated computer-assisted instruction. This approach has been often applied to large-enrollment service courses in mathematics. One such course is the Finite Mathematics course taken by non-technical students to fulfill a "university mathematics requirement."

At the University of Alabama at Birmingham, the site of this study, Finite Mathematics (MA 110) is taken by most pre-service elementary and middle school teachers. Pre-service elementary teachers are required to take four 3-credit-hour mathematics courses, two of which must be courses that satisfy the university Core Curriculum requirement in mathematics. Finite Mathematics, a freshman-level course, is the lowest level such course and most pre-service elementary teachers take it. Pre-service middle school teachers, in accordance with the mathematics curriculum of the Mathematical Reasoning track in the mathematics major, are required to take the Finite Mathematics course.

One goal of such a course might be to foster an appreciation of how mathematics, even simple mathematics, can be employed to solve approachable problems. Thus, the goal may well be more developing quantitative reasoning than training to acquire a specific compendium of skills (Wiggins, 1989). We take the position that incorporating an inquiry-based component into a computer-assisted instructional environment enhances student learning (compare Marrongelle and Rasmussen, 2008). Specifically, we will examine inquiry-based plus computer-assisted instruction against lecture plus computer-assisted instruction for effectiveness. However, what we investigate, and our methodology of simultaneously comparing different pedagogies within one term with random assignment of students to treatments, has few direct comparisons in the literature that we have found (but see Doorn and O'Brien, 2007). A number of studies (for example, Gautreau and Novemsky, 1997 and Hoellwarth, Moelter, and Knight, 2005) have compared different pedagogies over a longer time frame.

Theoretical Perspective

Our theoretical perspective is that of constructivism, somewhat informally described by D. Blais (1988). Invention and active construction is essential for the development of knowledge. The students in the course that we have first investigated, Finite Mathematics, are generally speaking mathematics-avoiders. Construction of mathematical concepts, as opposed to being told algorithms then being asked to implement them, is far from their experience. Nevertheless, we contend that the opportunity to construct will positively affect their self-efficacy and confidence, as well as their ability to explain and defend their conclusions. We contend that this will occur in conjunction with the algorithmic learning emphasized in the computer-assisted instruction.

Hypotheses

The research in this proposal investigates the relative effect of combining computer-assisted instruction with, respectively, inquiry-based group work sessions, summary lectures of material to be covered in the computer-based part, and the latter combined with regular in-class quizzing on lecture material. The hypothesis is that inquiry-based group work sessions differentially benefit students in terms of self-efficacy, content knowledge, and communications.

We hypothesize that the following results will be observed:

- H1. All sections will have similar grades regardless of treatment.
- H2. Group work sections will have differentially improved problem-solving ability.
- H3. Group work sections will have differentially improved mathematics communication skills.
- H4. Group work sections will have differentially improved mathematics self-efficacy.
- H5. Group work sections will have differentially improved long-term retention.

Research Design and Methodology

Since our goal is to compare three pedagogical treatments within an over-arching context of computer-assisted instruction, our methodology seeks to remove from consideration as many confounding factors as possible. Variability introduced by differing grading schemes, instructor differences, and university scheduling factors is to be avoided.

All students involved in the courses will have identical computer-assisted instruction and online assessment for grades. A student's grade is determined by total number of points earned out of 1000 with the following thresholds: A-880, B-750, C-620, D-500. About 79% of the grade in the course is determined by evaluation in the computer-assisted context: lab attendance (70 points), online homework (70 points), supervised online quizzes (70 points), and supervised tests (580 points). The remaining 21% of their grade, but reflecting more like 30% of their time on task, is determined by one of three pedagogical treatments, described below.

Students registered for one of three time periods (sections) in the Fall 2008 semester schedule, an early morning (8:00 AM), mid-morning (11:00 AM), or mid-afternoon (2:00 PM) time slot, two days a week (Monday/Wednesday), for their 50 minute class meeting and 50 minute required lab meeting in Finite Mathematics. (There was also an evening section of MA110 which was not part of the study.) Students in each time slot were randomly assigned to one of the three treatments. Three instructor/teaching assistant pairs agreed to participate in the experiment. Each instructor/TA pair teaches in each time slot, employing exactly one of the treatments, as pre-assigned to that subsection, thus administering one of each treatment over the day.

Pedagogies. The three pedagogies to be compared are:

- *Group*: inquiry-based group work without prior instruction, on problems intended to motivate the topics to be covered in computer-assisted instruction later;
- *Lecture*: a summary lecture with graduated examples on the topics to be covered in computer-assisted instruction later; and
- *Quiz/Lecture*: a briefer summary lecture with a 10 minute quiz each weekly class meeting on the material covered in the previous week's lecture.

Group. In the group work treatment, students are divided randomly at the beginning of each class into groups of four. All groups are given the same problem situation to investigate as a group, and strive to arrive at an understanding and solution. Discussion within each group takes place independently with the instructor and teaching assistant each playing the role of a Socratic facilitator, answering questions with questions. The problem is posed without prior instruction in the topic being introduced. An example of a problem is below.

Each student turns in each class meeting a written report on his/her investigation and solution of the problem(s) posed in that class period. The report is evaluated based upon the same rubric as the Pre- and Post-Test (described below). Response to the Challenge portion can only help, not hurt, a student's score. Students are aware of the rubric and receive written feedback consistent with the rubric. Time is allowed in each period for one or two of the groups of four to report voluntarily on their findings to the whole class.

Lecture and Quiz/Lecture. In the lecture treatment, the instructor gives a traditional lecture introducing the upcoming material. For instance, the concept of apportionment, distribution of indivisible objects in proportion to some entitlement, would be defined and examples, isomorphic to the above problem, would be worked through by the instructor. In the lecture/quiz treatment, the lecture is necessarily briefer, and the quiz is on basic material and examples from the previous lecture. The quiz is graded traditionally (correct answer with work shown) and returned.

The 21% of the final grade determined by the class meeting differs among the treatments as follows, each of 14 class meetings:

- for the group work, 10 points are earned for attendance and up to 5 more for evaluation of the solution and explanation turned in;
- for the lecture, 15 points are earned for attendance;
- for the quiz/lecture, 10 points are earned for attendance and up to 5 more for evaluation of the quiz.

Examples of Treatments

To provide context for the reader, we describe in what follows a specific typical class under each treatment. The example chosen is the class session on the topic of *apportionment*.

Group Work Example

This following problem is intended to bring out some of the mathematical and modeling issues involved in apportionment through inquiry and discovery. It is posed to the students without prior instruction in apportionment methods or fair division. In the group work process, both apportionment and more general fair division issues typically arise.

> Problem. Andy, Bert, and Connie are farmers. Their neighbor who is also a farmer is retiring next month and wishes to sell her 12 pigs for $480. Andy, Bert, and Connie can only afford to purchase the pigs if they pool their money. Andy can contribute $97, Bert can contribute $210, and Connie can contribute $173. How many pigs each should Andy, Bert, and Connie get?
>
> Challenge. After all of the money contributed to the purchase is tabulated but before the pigs are distributed, an extra pig is discovered hiding in the pen (13th pig). The neighbor decides to just include the extra pig in the $480 purchase. How many pigs each should Andy, Bert, and Connie get now?

Students have several different ways of approaching this problem, and some of those actually point out fundamental mathematical ideas involving modeling fairness. At this point in the course the students have not yet learned relevant terminology such as apportionment, fair division, and standard divisor. As students first approach the problem they often see it as a fair division problem. They generally see that each pig is worth $40 and give out the first 11 pigs accordingly as dictated by whole amounts of $40 contributed by each farmer. The interesting thing that arises with this problem is that students often decide to BBQ a pig or split the duties of raising the pig until it can be sold and the money distributed. This shows that students are thinking about a fair way to divide the value of the last pig. Though the ideas raised are important, since the topic is apportionment, the groups of students are then asked to divide the pigs under the assumption that the pigs are indivisible, thus making it a more traditional exercise in apportionment but without prior knowledge of any apportionment techniques.

Many students' efforts at a solution to the "Pig Problem" involve little or no knowledge of apportionment of indivisible items. Most students are not familiar with standard divisors, standard quotas, or any of the typical apportionment methods. However, despite this lack of formal knowledge of apportionment, many students working on the pig problem manage to "construct" Hamilton's Method for apportionment. In Hamilton's Method as applied to this problem, once each farmer's standard quota is calculated and rounded down to the nearest whole number (the lower quota) and each farmer given that number of pigs, the surplus items are given, one at a time, to the farmer with the largest decimal part of the standard quota until all surplus pigs have been distributed. Many students working on this problem, see the whole number part of the standard quota as the number of whole pigs each farmer can have based on his/her contribution to the purchase. The decimal (or percentage, or fractional part, depending upon how they express it) of the standard quota to many students clearly represents the portion of a pig that can be purchased with the money brought to the deal.

> *A typical solution.* The total cost of the pigs is $480; $480 divided by 12 pigs equals $40 per pig. Since Andy contributes $97 of the $480, he can use $80 of the $97 to purchase 2 whole pigs. He will then have $17 left over towards the purchase of an additional pig. The others have $10 and $13 toward the 12th pig. These students reasoned that since Andy had the most money left over after purchasing the maximum whole number of pigs, that he should have the surplus pig that must be distributed.

These students have used Hamilton's Method without knowing that they have used this method. When presenting Hamilton's Method in the lecture format, students are simply given instructions to give the surplus item to the group with the largest decimal part. Most of these students have very little, if any, insight into why this is done. The student mentioned above understands why she is awarding the surplus pig to Andy. This is a level of understanding of Hamilton's Method that is more difficult to attain when just presented with the steps to apportionment using Hamilton's Method.

> *A second typical solution.* As above, since the total cost of the pigs is $480, $480 divided by 12 pigs equals $40 per pig. Since Bert contributes the largest amount, $210, to the deal, he is given the 12th pig. The reasoning is that he had the largest impact on there being a deal at all. The idea of a "volume discount" was also mentioned by some groups.
>
> *An atypical solution.* In the same way as above the students arrive at an agreement that each pig is worth $40. Thus Andy gets 2 pigs with $17 left over, Bert gets 5 pigs with $10 left over, and Connie gets 4 pigs with $13 left over. For the final pig the students decided to hold a weighted lottery that allows Andy to put his name on seventeen pieces of paper in a hat, Bert on 10 pieces of paper in the hat, and Connie on 13 pieces of a paper in the hat. A single name is selected from the hat, and whomever's name is selected is the person that receives the 12th pig.

The latter is a well thought out solution addressing the issue of coming up with a fair way to divide the pigs. This solution was actually very valuable to the class discussion as well when many students who had thought of it in a different way objected to the fact that while Andy was the most likely individual to receive the pig he was in fact unlikely to receive the final pig when compared against the combined effort of the other two candidates even though he paid the most for the pig. A lot of the gain of this type of class work can be seen when situations like this arise and students have to explain or defend their answer to their peers, as well as the gain that students receive from critically examining their classmates' and groupmates' ideas in the first place.

The challenge for the pig problem also allows students a chance of discovering mathematical ideas. Solutions to the pig problem often depend on how groups did the initial problem. In the case of the students above who managed to construct Hamilton's method they often simply redo the method with a different standard divisor, 480/13 instead of 480/12. The interesting thing that these students in particular talk about is then that Andy actually ends up with only 2 pigs now even though an extra pig was found. They discuss this problem and how their solution can be fair if finding an extra pig allows someone to lose possession of a pig.

Some students look at the challenge completely differently. They decide that since they have agreed on a fair way to give out the first 12 pigs they will stick with that apportionment of these 12 and now must only decide who to give the 13th pig to. A common solution for students who have done Hamilton's method initially is to simply give the 13th pig to Bert because he was the one that contributed the most money in the first place. Another common answer is to give the 13th pig to the person who had the second highest decimal part under the distribution of the first 12 pigs.

Lecture Example

The lecture notes on apportionment that all instructors used for the lecture treatment are reproduced in Appendix 1. All three instructors worked from the same notes, though without any attempt to adopt the same lecture style. Lectures consisted of a 50-minute traditional lecture on the topic. Terms and procedures were introduced and exemplified. An effort was made to engage students actively in the lecture. For instance, students were given copies of tables, incomplete, to follow along and work on during the lecture. These are reproduced in the appendix.

Quiz/Lecture Example

In the Quiz/Lecture treatment, the same lecture notes were used as for the lecture treatment, and the instructor made whatever modifications were necessary to allow time (10 minutes, usually) for the quiz at the end of the period. In Appendix 2, the quiz given the class meeting following the apportionment lecture is reproduced. Such quizzes were graded traditionally and returned to students the next class period.

Computer-Assisted Instruction

All three treatments had access to the online textbook, online homework, quizzes, and tests, and multiple types of assistance during homework sessions. In the sections on apportionment, the homework assignment for the week included problems of all the major types covered in the lecture. An example of a homework problem is shown by screen shot in Appendix 3. Notice on the right of the problem statement the types of online assistance available to the student. If a student were to click on "Help Me Solve This," she would get detailed help on solving the problem (see second screen shot), but would subsequently have to go back and work a different version of the problem.

Measures

The classes took place in Fall Semester, 2008. Data gathered includes

- Course assessments, including Final Score and Test Sum (sum of four test scores).
- Pre-test and post-test content knowledge evaluation according to a rubric* that weighs problem identification (0 or 1), evidence of problem-solving (0, 1, or 2), and adequacy of explanation (0, 1, or 2) to extended responses on three problems typical of the material in the course; accuracy is separately evaluated (0 or 1).
- Pre- and post- responses to a Survey of Mathematical Self-Efficacy (Betz and Hackett (1983), Hall and Ponton, 2002).
- Focus groups selected from each of the three treatments.
- Student course evaluations using the online IDEA system (IDEA Center, 2009).
- RTOP observations of the instructors in each of the nine subsections (Reformed Teaching Observation Protocol, 2008).

(*All students were given copies of the grading rubric prior to both the pre-test and post-test.)

Pre-Test and Post-Test

The pre-test and post-test were given to students to evaluate their knowledge of the course content according to a five-point rubric. The pre-test and post-test included 3 problems each that would typically be presented in the Finite Mathematics Course (a percentage problem, a counting problem, and a probability problem). The students took the pre-test during the first lab meeting of the semester and took the post-test during the last class meeting of the semester.

Rubric. The students' solutions to the problems on the pre-test and post-test were evaluated based on a five-point rubric. The students were given copies of the rubric before taking the pre-test and post-test. The rubric contained the following specific guidelines for students:

Identify your problem (0 points or 1 point awarded)

- Take time to identify and define the problem that you are trying to solve.
- Return to your problem definition often (and perhaps redefine your goal, though this leaks into the next item).

Show Evidence of Problem Solving (0, 1, or 2 points awarded)

- Show your work and your thinking along the way.

- Don't erase! If you find an approach isn't getting you anywhere, draw a line through it, and go down a new path. (Also, you might later find you need the information that got erased.) This will show evidence of your persistence and flexibility.
- As you find something out about the problem, or about your approach, make a written note to yourself (and to the reader) on your paper. Give the reader insight into what you are thinking.
- Are you solving the problem that you initially identified? How do you know?

Explain Your Thinking (0, 1, or 2 points awarded)

- Take a moment to reflect on your results. Then reflect on how you can communicate your results.
- What did you find out? Present your findings on the problem clearly and concisely. (Some might call this "the answer," but that is only a part of complete work.)
- Give an explanation of your work appropriate to the audience (not so much your instructor, but your colleagues).
- Have you thought of any conjectures or new problems as a result of working on this problem?
- Reflect on how this problem might be connected to other problems that you have solved, or that you have been working on. Why is this problem important mathematically?

Student Rubric Training. Students in the group work sections of the course participated in training using the rubric. Students in these sections were asked to score a sample problem. During one of the class meetings in each group work section of the course, instructors provided three different sample solutions to a problem. Students worked in groups to score the three different solutions and to discuss, compare, and contrast the quality of the three different sample solutions. Each group shared their rubric-based scores with the class.

Instructor Rubric Training. Instructors and graduate assistants participated in several meetings throughout the Fall Semester designed as rubric training in order to improve the inter-rater reliability for the instructors using the rubric when scoring the pre-test, post-test, and weekly group meeting assignments. Prior to a training session, instructors and assistants were given blinded samples of student solutions to a problem and asked to score the student work. During the training session, instructors and assistants compared their ratings of solutions of student work to the rest of the training group. Differences in scoring were examined and discussed.

Inter-Rater Reliability.

Inter-rater reliability statistics are used to assess the level of agreement among multiple raters. The purpose of including this analysis was to estimate how consistently instructors were able to rate students' performance on each of the dimensions of the performance assessment rubric. A group of 6 raters (three MA 110 instructors and three graduate assistants) scored students' performance on each question of 120 randomly selected pre- and post-tests. In an effort to minimize rater bias, pre-tests were blinded and randomly assigned to each of the six raters. Raters received an equal number of tests. At the conclusion of the course, the process was repeated with post-tests. The Cohen's Kappa statistic was calculated to assess agreement. A minimum Kappa score of >0.40 (SPSS, 1997) was hypothesized *a priori* to indicate an acceptable level of reliability.

The analysis showed there was an acceptable level of agreement among raters for all rubric dimensions on all questions with the exception of “Evidence of Problem-Solving” for Item 1. Additional investigation into training procedures and item development will be necessary to understand the reasons for the lower agreement for problem solving in the first item. Further rubric training is scheduled to improve inter-rater reliability on all dimensions for all items.

Table 1. Inter-Rater Reliability on Pre- and Post-Tests

Item and Constructs	Kappa
Item 1	
Identify the Problem	0.794
Evidence of Problem Solving	0.369
Explain Thinking	0.425
Item 2	
Identify the Problem	0.627
Evidence of Problem Solving	0.588
Explain Thinking	0.562
Item 3	
Identify the Problem	0.694
Evidence of Problem Solving	0.621
Explain Thinking	0.461

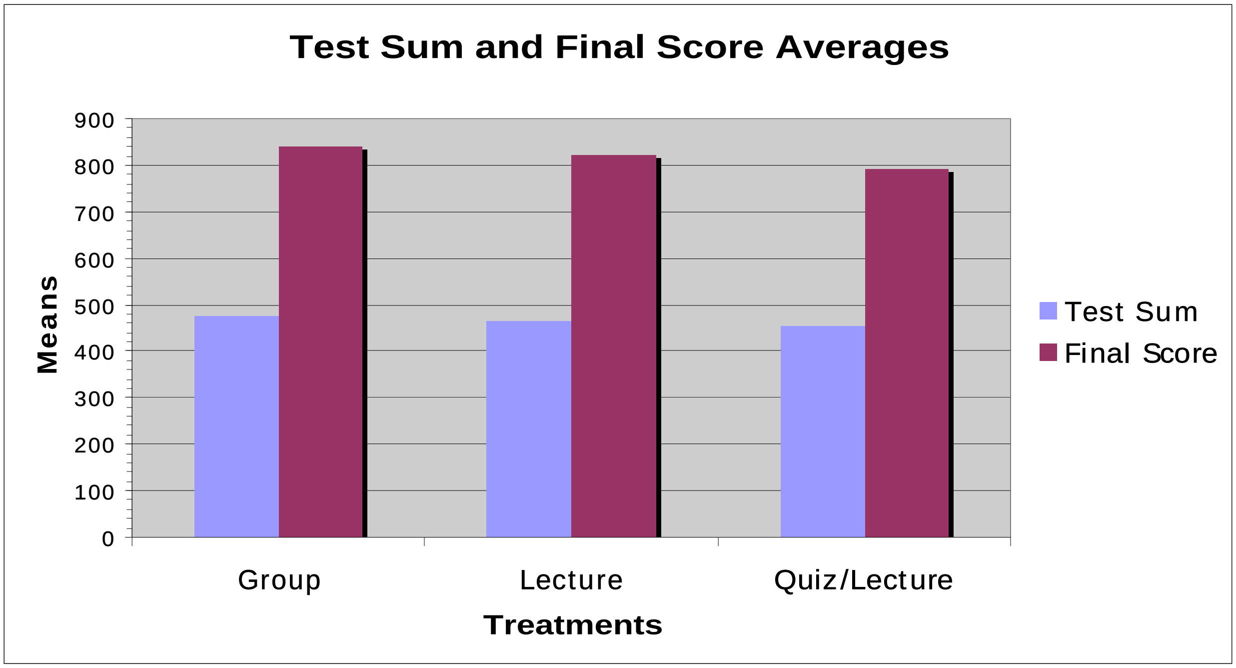
Results

The following results are available. Analysis of data gathered continues. In particular, the one-year-delayed post-test data has yet to be gathered.

Result 1: Hypothesis H1 Supported

As hypothesized (H1), there were no significant differences between treatments in terms of Final Scores (determining grades). See chart below. In particular, there were no significant differences between treatments in terms of the Test Sum (sum of the four test scores), the most rigorously determined part of the Final Score. Analysis of between-subjects effects for Group, Instructor, and Group*Instructor were not significant at $p<0.05$ (after Scheffe). (N=245: Group=80; Lecture=77; Quiz\Lecture=88. An additional 39 students did not take all tests, either dropping after the first few days, officially withdrawing, or quit attending.)

Figure 1. Test Sums and Final Scores

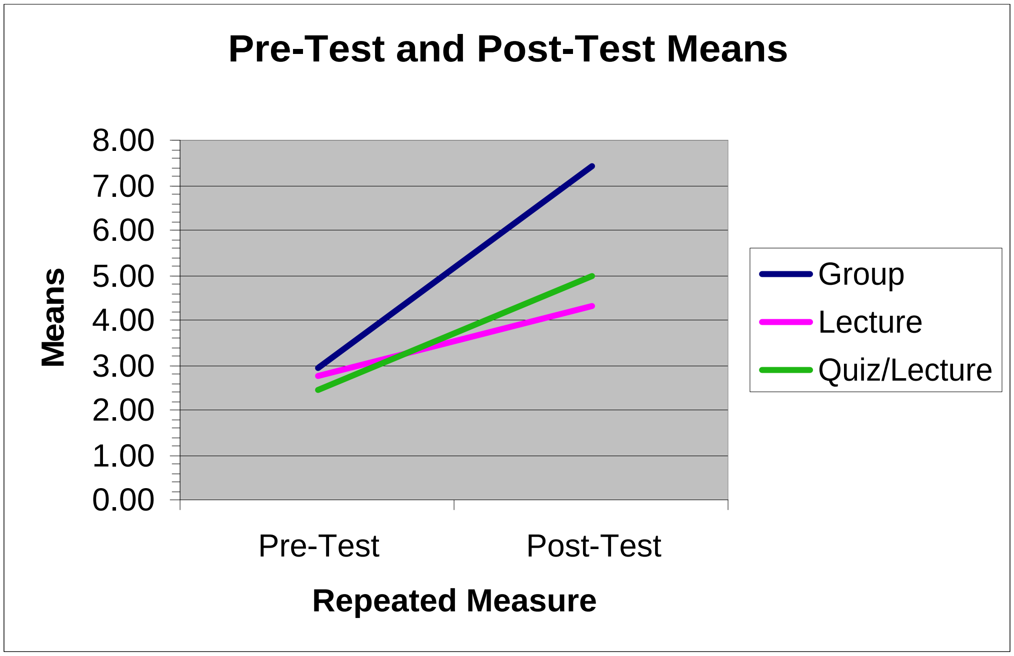


Result 2: Hypotheses H2 and H3 Supported

In support of hypotheses H2 and H3, there were significant differences between the Group treatment and each of the other two treatments in terms of Pre-Test and Post-Test Scores. The Pre- and Post-Tests consisted of three problems, each scored on a rubric which awarded on each problem 0-1 point for Problem Identification, 0-1-2 points for Evidence of Problem-Solving, and 0-1-2 points for Explanation, for a total of 15 points possible on the test. Raters participated in three practice sessions of rating the same responses to problems, and subsequent discussions resolving differences, prior to grading the Pre- and Post-Tests (note one exception). Repeated Measures ANOVA (Wilks' Lambda), and univariate analysis of difference scores, each indicated significant differences at $p<0.05$. In particular, the Time main effect (Pre- to Post-) was significant ($\lambda=0.50$), as was the Time*Group interaction effect ($\lambda=0.86$). Moreover, the between-subjects effects for Group versus each of the other treatments was significant, while the between-subjects effect for Lecture versus Quiz\Lecture was not significant, at $p<0.05$ (after Bonferroni). (N=214: Group=71; Lecture=68; Quiz\Lecture=75.) See chart below for comparison.

Confirmatory non-parametric analysis was also employed. A series of Wilcoxon tests were run on each rubric dimension to assess differences using nonparametric statistics. The Wilcoxon scores yielded similar results as the repeated measures analysis of variance.

Figure 2. Pre-Test and Post-Test Comparison.



Result 3: Hypothesis H4 Not Supported

Hypothesis H4 was not supported. The responses to the Survey of Mathematical Efficacy, given pre- and post-course were subjected to factor analysis. Five factors were evident; 7 (out of 34) questions were eliminated from the analysis as not meeting the threshold of significant contribution. All treatments showed significant improvement in mathematics self-efficacy, with no significant between-treatments effects overall, nor in any single factor. Apparently, succeeding at a mathematics course in and of itself improves self-efficacy and washes out any other effects. Further study is required.

Further Analysis. More detailed analysis is forthcoming on other interaction effects including Instructor and Section (time of day). We will also do further analysis on the Pre- and Post-Test subscores in order to tease apart Hypotheses H2 and H3.

Focus Group Summaries

Focus groups were held for each of the three treatments. The main ideas emerging from each treatment group are summarized below. No unexpected ideas emerged from the focus groups.

Lecture

- too much material to cover in one class period per week
- not enough specific feedback about mistakes from computer
- not enough time to get a deep understanding of material
- need fewer topics explored in more depth
- computer instruction could be good with more time for lecture

Quiz/Lecture

- need fewer topics explored in more depth
- computer instruction could be good with more time for lecture

- quizzes were motivating
- use of peer helpers (TAs) was important
- not challenging enough until the end
- easy to "beat the system"

Group

- lack of structure was a problem
- no guidance from instructors was frustrating
- forced to "teach yourself "
- group work was good, but would be better with a brief lecture introduction (5-10 minutes)
- need some "teaching" - need balance between lecture and group work

Limitations

There are several limitations to the study. We discuss each of the major limitations of which we are aware below.

Unit of Significance. It can be argued that the unit of significance for this study should be the class section rather than the student. The reason is that all the students in one class had the same experience, and it is to be expected that their variance is correlated. One the other hand, such correlation of variance with different class means would have the tendency to suppress differences between treatments. Since the differences observed were so large, we do not think this has any tendency to weaken the conclusions.

Rater training on rubric. The inter-rater reliability in scoring pre- and post-tests is within the acceptable range, but at the low end. We plan continued rater training. We will also consider sharpening the rubric to bring out more clearly differences among scores of 0, 1, and 2, where appropriate.

Accuracy on pre- and post-test. The pre- and post-tests were independently scored for accuracy of answers apart from the rubric scoring. On each of the three problems, 1 point was awarded for a correct answer and 0 points for an incorrect or incomplete answer. Over all treatments, the expected value for accuracy was about 1 out of 3 correct. Moreover, there was no significant difference ($p<0.05$) among the three treatment groups with respect to accuracy. This merits further study.

Differences among instructors. The study was designed to minimize differences in treatments arising from different instructors. For the most part, this effort seems to have been successful. However, one significant instructor difference did appear: two of the instructors differed significantly ($p<0.05$) on the pre- to post-test analysis of score differences. This significant difference appeared when the full 3-question test was analyzed, but disappeared when the analysis was based only upon questions 1 and 3. Moreover, other differences remained significant when analysis was based only upon questions 1 and 3. Each question is a substantial exercise for the student, requiring construction of an answer. We conclude that the study results are not weakened by this limitation. An examination of question 2 revealed that it was arguable that the pre- and post forms of the question were not equivalent. More attention will be paid in the future to pre- post-test design.

Conclusions and Implications

We draw the following conclusions and implications from the study.

1. The inclusion of a group work class meeting in lieu of a weekly lecture does not appear to affect adversely student success as measured by grades. (But it does not appear to improve accuracy.)
2. Group work does have a positive effect on problem-solving and communications abilities (as measured together by our rubric-based score. Further analysis to separate hypotheses H2 and H3 is needed.)
3. Success in a mathematics course increases mathematics self-efficacy among a population taking one of the lowest entry-level courses that carry college credit. (Further study is needed on other mathematics courses.)
4. The addition of a weekly paper and pencil quiz to lecture treatment, over and above the regular quizzing done within the computer-assisted instruction, does not affect student performance in terms of grades or problem-solving/communication. (We will eliminate the Quiz/Lecture treatment from further studies.)

This research will inform our teaching of Finite Mathematics. In Spring Semester, 2009, we will teach all sections of Finite Mathematics using the group work treatment. We will continue to gather data to corroborate the results of the research reported above.

We expect to extend this study to Basic Algebra, Intermediate Algebra, Pre-Calculus Algebra (Oerhtman, Carlson, and Thompson, 2008), and Pre-Calculus Trigonometry, using essentially the same experimental design. Our projected study of Basic Algebra in Fall Semester, 2009, will have two treatments: Group and Lecture, as described above. Many pre-service elementary school teachers start in the non-credit course, Basic Algebra, and take Intermediate Algebra, and Pre-Calculus Algebra, in addition to Finite Mathematics.

As part of our NSF-supported Mathematics and Science Partnership, we have designed courses that emphasize mathematical reasoning and are entirely inquiry-based. These include two recommended for pre-service elementary teachers: *Patterns: the Foundation of Algebraic Reasoning*, and *Geometry and Proportional Reasoning*. The same courses are required for pre-service middle school teachers in the Mathematical Reasoning track in the Mathematics Major. Studies are underway in these two courses. As yet few pre-service elementary teachers take both of the recommended newly-designed courses. One long-term goal of our research program is to provide evidence that the recommended courses are substantially better in terms of student learning for pre-service teachers.

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Appendix 1: Notes for Apportionment Lecture

Section 14.3: Apportionment Methods

According to the Constitution of the United States, representatives are to be apportioned among the states according to each state's population. The Constitution does not specify how this is to be done. This is an example of an apportionment problem.

Example 1: The Republic of Margaritaville is composed of four states A, B, C, and D. According to the country's constitution, the congress will have 30 seats, divided among the four states according to their respective populations. The table below shows each state's population.

State	A	B	C	D	Total
Population (in thousands)	275	383	465	767	1890

Standard Divisor: found by dividing the total population under consideration by the number of items to be allocated.

$$\textbf{Standard Divisor} = \frac{\textbf{Total Population}}{\textbf{Number of allocated items}}.$$

Find the standard divisor for example 1.

Standard Quota: found by dividing that group's population by the standard divisor.

$$\textbf{Standard Quota} = \frac{\textbf{Population of a particular group}}{\textbf{Standard divisor}}$$

Find the standard quotas for each state in example 1. What is the sum of these standard quotas?

The Apportionment Problem: The apportionment problem is to determine a method for rounding standard quotas into whole numbers so that the sum of the numbers is the total number of allocated items.

Lower Quota: the standard quota rounded **down** to the nearest whole number.

Upper Quota: the standard quota rounded **up** to the nearest whole number.

Example 2: A rapid transit service operates 130 buses along six routes, A, B, C, D, E, and F. The number of buses assigned to each route is based on the average number of daily passengers per route, given in the table below.

Route	A	B	C	D	E	F	Total
Average Number of Passengers	4360	5130	7080	10,245	15,535	22,650	65,000

The following are four different apportionment methods:

1. **Hamilton's Method**:
   a. Calculate each group's standard quota;
   b. Round each standard quota down to the nearest whole number (find lower quota);
   c. Give the surplus items, one at a time, to the groups with the largest decimal parts until there are no more surplus items.

* Use Hamilton's Method to apportion the buses in example 2.

The Quota Rule: A group's apportionment should be either its upper quota or its lower quota. An apportionment method that guarantees that this will always occur is said to **satisfy the quota rule.**

2. Jefferson's Method:

a. Find a modified divisor, *d*, such that when each group's modified quota (group's population divided by *d*) is rounded down to the nearest whole number, the sum of the whole numbers for all the groups is the number of items to be apportioned. The modified quotients that are rounded down are called **modified lower quotas**.

b. Apportion to each group its modified lower quota.

c. Note that the modified divisor, d, will be *less than* the standard divisor, since we want the quotients (the modified quotients) to increase before rounding down.

* Use Jefferson's Method to apportion the buses in example 2.

3. Adam's Method:

a. Find a modified divisor, *d*, such that when each group's modified quota (group's population divided by *d*) is rounded up to the nearest whole number, the sum of the whole numbers for all the groups is the number of items to be apportioned. The modified quotas that are rounded up are called **modified upper quotas.**

b. Apportion to each group its modified upper quota.

c. Note that the modified divisor, d, will be *more than* the standard divisor, since we want the quotients (the modified quotients) to decrease before rounding up.

* Use Adam's Method to apportion the buses in example 2.

4. Webster's Method:

a. Find a modified divisor, *d*, such that when each group's modified quota (group's population divided by *d*) is rounded to the nearest whole number, the sum of the whole numbers for all the groups is the number of items to be apportioned. The modified quotas that are rounded are called **modified rounded quotas.**

b. Apportion to each group its modified rounded quota.

c. When using Webster's method, the modified divisor, *d*, can be less than, greater than, or equal to the standard divisor. Thus it may take a bit longer to find a modified divisor that satisfies Webster's method.

* Use Webster's Method to apportion the buses in example 2.

14.4 Flaws of Apportionment Methods

Alabama Paradox: An increase in the total number of items to be apportioned results in the loss of an item for a group. In 1881, the chief clerk at the U.S. Census Office used Hamilton's method to compute apportionments for both House sizes. He found that adding one more seat to the House in order to have 300 seats would actually decrease the number of seats for Alabama from 8 seats to 7 seats.

Example of the Alabama Paradox: A small country with a population of 10,000 is composed of three seats. According to the country's constitution, the congress will have 200 seats, divided among the three states according to their respective populations. The table below shows each state's population. Use Hamilton's method to show the the Alabama Paradox occurs if the number of seats increased to 21.

State	A	B	C	Total
Population	5015	4515	470	10,000

Population Paradox: Group A loses items to group B, even though the population of group A grew at a faster rate than that of group B. The population paradox illustrates another flaw of Hamilton's Method.

Example of the population paradox: A small country with a population of 10,000 is composed of three states. There are 11 seats in the congress, divided among the three states according to their respective populations. Using Hamilton's method, the table below shows the apportionment of the congressional seats for each state.

State	Population	Standard Quota	Lower Quota	Hamilton's Apt.
A	540	0.59	0	0
B	2430	2.67	2	3
C	7030	7.73	7	8
Total	10,000	10.99	9	11

Now suppose the population of the country increases as shown below.

State	A	B	C	Total
Original Population	540	2430	7030	10,000
New Population	560	2550	7890	11,000

a) Find the percent increase in the population of each state.
b) Use Hamilton's method to show that the population paradox occurs.

The New-States Paradox: The addition of a new group changes the apportionments of the other groups. If a new state is added, the apportionment of other states may be changed.

Balinski and Young's Impossibility Theorem: There is no perfect apportionment method. Any apportionment method that does not violate the quota rule must produce paradoxes, and any apportionment method that does not produce paradoxes must violate the quota rule.

Worksheet for Examples for Apportionment Lecture

Table for Example 1:

State	A	B	C	D	Total
Population	275	383	465	767	1890
Standard Quota					

Tables for Example 2:

Hamilton's Method:

Route	Passengers	Standard Quota	Lower Quota	Decimal Part	Surplus Buses	Final Apportionment
A	4360	8.72				
B	5130	10.26				
C	7080	14.16				
D	10,245	20.49				
E	15,535					
F	22,650					
Total	65,000					130

Jefferson's Method: (Using d = 486)

Route	Passengers	Modified Quota	Modified Lower Quota	Final Apportionment
A	4360	8.97		
B	5130	10.56		
C	7080	14.57		
D	10,245	21.08		
E	15,535			
F	22,650			
Total	65,000			130

Adam's Method: (Using d = 512)

Route	Passengers	Modified Quota	Modified Upper Quota
A	4360	8.52	
B	5130	10.02	
C	7080	13.83	
D	10,245	20.01	
E	15,535		
F	22,650		
Total	65,000		131 *

* We want this sum to be 130, not 131.

If you try d = 513, your sum will be 129, so we need to use a modified divisor between 512 and 513.

Adam's Method: (Using d = 512.7)

Route	Passengers	Modified Quota	Modified Upper Quota	Final Apportionment
A	4360	8.50		
B	5130	10.01		
C	7080	13.81		
D	10,245	19.98		
E	15,535			
F	22,650			
Total	65,000			130

Webster's Method: (Using d = 498)

Route	Passengers	Modified Quota	Modified Rounded Quota	Final Apportionment
A	4360	8.76		
B	5130	10.30		
C	7080	14.22		
D	10,245	20.57		
E	15,535			
F	22,650			
Total	65,000			130

Tables for the example of the Alabama Paradox:

Apportionment with 200 seats Using Hamilton's Method

State	Population	Standard Quota	Lower Quota	Decimal Part	Surplus Seats	Final Apportionment
A	5015	100.3	100			
B	4515	90.3	90			
C	470	9.4	9			
Total	10,000	200	199			200

Apportionment with 201 seats using Hamilton's Method

State	Population	Standard Quota	Lower Quota	Decimal Part	Surplus Seats	Final Apportionment
A	5015	100.8	100			
B	4515	90.75	90			
C	470	9.45	9			
Total	10,000	201	199			201

Table for example of the Population Paradox:

State	Population	Standard Quota	Lower Quota	Decimal Part	Surplus Seats	Final Apportionment
A	560	0.56				
B	2550	2.55				
C	7890	7.89				
Total	11,000	11				11

Appendix 2: In-Class Quiz on Apportionment

Directions: Provide a written account of your understanding and solution of the following problem. You must show all of your work AND the correct answer to receive full credit.

Problem: The table below shows the number of students enrolled in three different biology courses, A, B, and C. The biology department has 25 teaching assistants to be divided among these three courses, according to their respective enrollments.

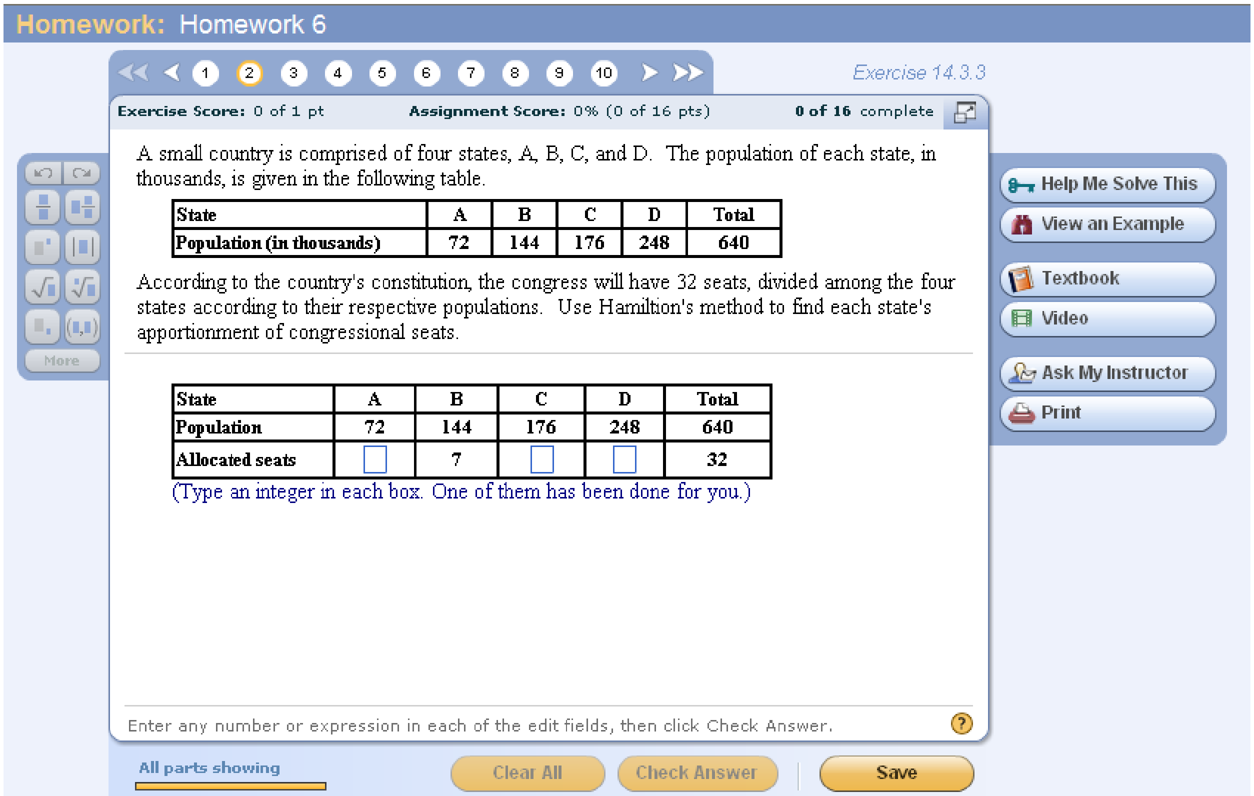
Course	A	B	C	Total
Population	825	521	154	1500

a) Apportion the teaching assistants using Hamilton's Method. [3 points]

b) Apportion the teaching assistants using Jefferson's Method. Be sure to clearly indicate the modified divisor that you are using. [2 points]

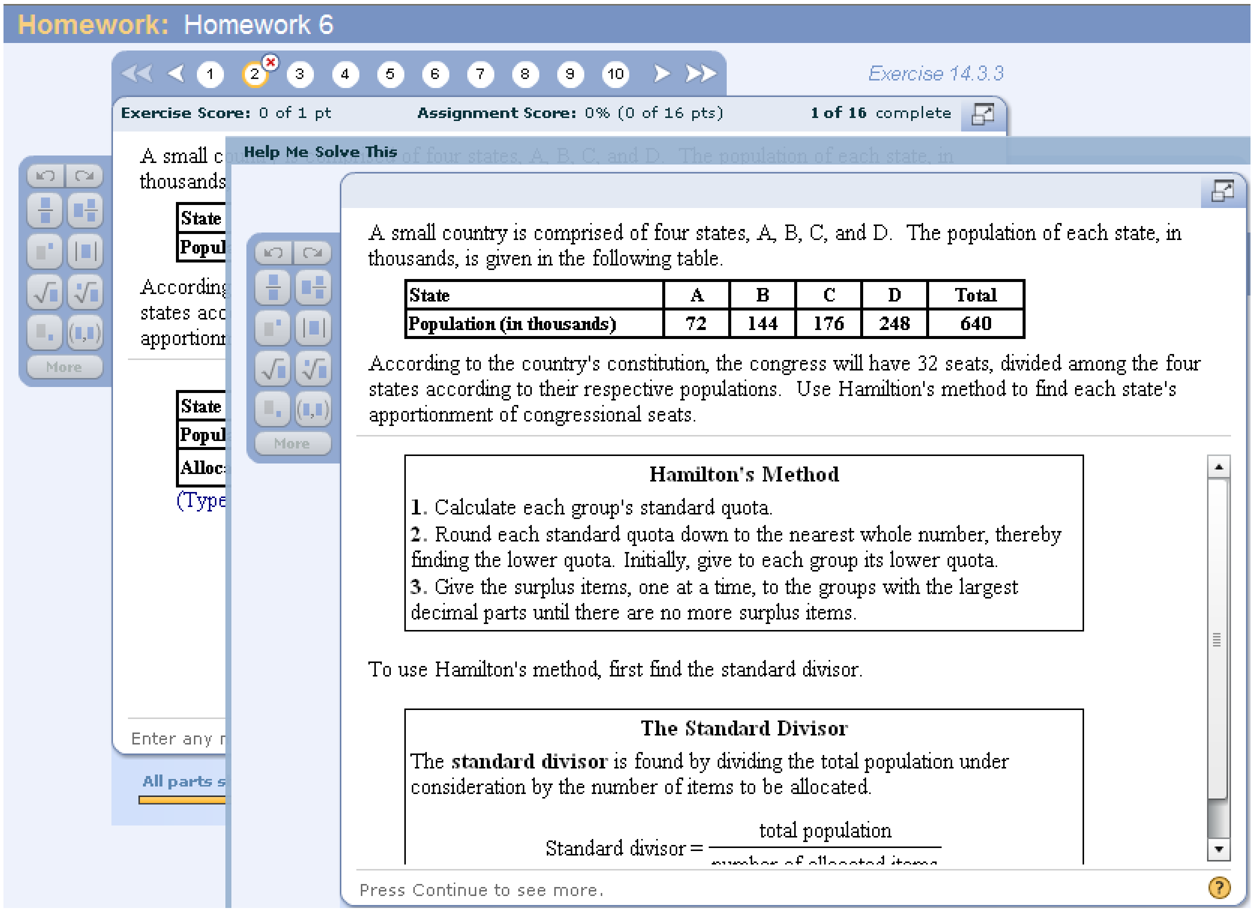
Appendix 3: Screen Shots of Computer-Assisted Instruction

Homework 6 is on Apportionment. An example of a Hamilton's apportionment method problem is below.



A puzzled student might click on "Help Me Solve This."

“Help Me Solve This” brings up the following screen:



Note that Problem 2 is marked “wrong.” The student will have to go back and work a different version of the problem after receiving this help.