Mathematical Investigations in Inquiry-Based Courses for Pre-Service Teachers

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Abstract

This study describes the efforts of a mathematics partnership in promoting inquiry-based mathematics instruction in university mathematics courses and the resulting impact on mathematical knowledge and attitudes toward mathematics. The subjects for the study are pre-service elementary and middle grades teachers taking a series of inquiry-based mathematics courses. A variety of measures are used in determining participants' knowledge of mathematics including objective tests, performance assessments, and portfolios. Additional measures such as classroom observations, focus groups, and surveys by external evaluators are used to measure changes in students' attitudes toward learning mathematics in such an environment. Implications for changes in other university mathematics courses will be discussed.

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People are usually more convinced by reasons they discovered themselves than by those found by others. **Blaise Pascal**

The above quote attributed to Pascal sums up the philosophy behind two mathematics courses for preservice teachers that are the basis for this study. This study describes the efforts of a mathematics partnership in promoting inquiry-based mathematics instruction in university mathematics courses and the resulting impact on mathematical knowledge and attitudes toward mathematics. Through our efforts, we hope to improve the confidence and competence of preservice K-8 teachers.

Numerous studies over the past 20 years have documented the need for changes in mathematics education at all levels, K-University. At the K-12 level, the National Council of Teachers of Mathematics (NCTM) produced the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). This document sought to promote reforms in mathematics education and included a strong emphasis on inquiry as a mode of instruction. Likewise, there have been some at the university level who have sought to foster an inquiry approach to mathematics instruction as opposed to the direct instructor lecture approach. For example, there have been attempts to make calculus instruction more inquiry-based (Hastings, 1997). More recently, Rasmussen and colleagues have initiated an inquiry-oriented course in differential equations (Rasmussen, et.al., 2006). Such reforms have not been without debate (Silverberg, 1999). While there have been reported gains in student performances in some studies, there is still much work to be done if we are to meet our national goals for achievement in mathematics.

Mathematics achievement data for both K-12 and the university level continue to point to the need for improvements in mathematics instruction. The Third International Mathematics and

Science Study (Mullis, et.al., 2003) concluded that many elementary and middle grades mathematics teachers do not have deep enough knowledge of mathematics to teach it in a conceptual way. Despite some recent improvements, many K-16 students still do not exhibit the mathematical competencies needed to function effectively in today's economy. Universities continue to offer an alarming number of remedial mathematics classes (McCray, et.al., 2003). For a good many students, the typical lecture style of mathematics instruction is not always effective. For university faculty, there needs to be a greater awareness of the specific mathematical content needed to prepare future teachers, as well as knowledge of methods of instruction aimed at reaching all college students.

To address some of these issues, a partnership involving two institutions of higher education (the University of Alabama at Birmingham (UAB) and Birmingham Southern College (BSC)), nine area school districts, and a non-profit mathematics organization (the Mathematics Education Collaborative (MEC)) was formed. This partnership sought and was awarded funding by the National Science Foundation for a Math and Science Partnership, the *Greater Birmingham Mathematics Partnership* (GBMP) (Cooperative Agreement #0632522). At UAB, faculty from mathematics, education, and engineering are participants. While at BSC, faculty from mathematics and education are participants. Participants from the school districts include K-12 teachers and administrators. Among its goals, this five year project included a goal of increasing the content knowledge of pre-service and in-service teachers of mathematics in grades K-12, with a special emphasis on serving grades 5-8 teachers. It has also sought to impact university mathematics courses and instruction.

One of the partners, MEC, based out of Bellingham, WA and Portland, OR, had already established a track record of implementing inquiry-based mathematics courses for K-12 teachers.

A series of nine-day courses, taught in a workshop fashion, had been developed to enhance teachers' knowledge of mathematics. The mathematics content in these courses consists of the "big mathematical ideas" of numerical reasoning, algebra, geometry, probability, and data analysis as identified in NCTM's *Principles and Standards for School Mathematics* (2000). The focus is on developing conceptual understanding of the mathematics as well as the ability to put mathematical ideas and skills to work in solving complex and relevant problems. All courses attend to the process strands of problem solving, reasoning, making connections, and communicating.

The courses model a learning environment that optimizes the learning of quality mathematics and will meet a broad range of learner needs. They allow access for those teachers who fear and/or dislike mathematics, yet challenge all participants. The courses offer teachers opportunities to struggle with complex, rich, and expandable mathematical tasks with the potential of arriving at the development of concepts that are foundational to the field of mathematics.

The summer courses were followed up by seeking to establish university mathematics courses that parallel the MEC/GBMP courses both in content and in methods of instruction and evaluation. This paper seeks to describe the implementation of two of these courses at the university level.

Method

To achieve one of our project goals, to promote inquiry-based mathematics instruction at the university level, we revised two existing courses in mathematics for elementary school teachers to fit the content, pedagogy, and assessment methods of two of the MEC courses. These two courses were re-titled, MA 313- Patterns, Functions, and Algebraic Reasoning, and MA 314-

Geometric and Proportional Reasoning. These courses were to be a part of a new middle grades mathematics certification program, as well as a part of the 12 hours of mathematics required of pre-service elementary school teachers. The details of the formulation of MA 313 are given below. The reformation of MA 314 followed a similar approach.

The evidence of the effectiveness of the summer GBMP courses in increasing content knowledge in in-service and pre-service teachers supported the development of the MA 313, *Patterns, Functions, and Algebraic Reasoning*, course to be taught at the university level. The audience was primarily undergraduate, early childhood and elementary education majors (ECE, ELE) taking the course as part of their 12 semester hours of mathematics coursework needed for certification. There were also a few middle school mathematics majors taking the course. Students had previously completed at least a course in intermediate algebra as a prerequisite to taking the course. One purpose of the UAB courses was to increase students' understanding of the basis of algebra and algebraic reasoning that they may not have gleaned from the traditionally taught algebra courses taken in college and high school. The emphasis was on depth of conceptual understanding and not so much on breadth of a lot of topics. We took our cue from George Polya who is attributed to have said, "*It is better to solve one problem five different ways, than to solve five problems one way*." Other purposes included increasing students' abilities to reason and communicate mathematically. Another purpose was to expose students to inquiry based methods of teaching and learning mathematics.

The students were predominately typical-aged undergraduates (in their 20's) with a handful of non-traditional aged students. More or less this was a required course for students to take and thus participation was not entirely voluntary. The UAB students had from 1 to 3 college mathematics courses (taught by lecture methods) in their backgrounds prior to taking

MA 313. Almost none of the students had taken a mathematics methods.

A major adjustment from the summer workshops to a university mathematics course was the necessity to assign grades for the MA 313 classes. While active participation was an expectation for both groups, the assessment of individuals' performances on tasks was even more crucial in MA 313. A grading system had to be devised that discriminated between various levels of performance in MA 313. Although summer workshop participants received a 'grade' on their portfolios, there was not the pressure of assigning grades for university credit. A problem solving rubric was used to give feedback to students. This rubric was used by MEC in the summer workshops and was adapted from the Oregon Department of Education's 1995-2003 Statewide Assessment. Students completed 2 menus of mathematics tasks. A selection of 2-3 problems from each menu was assessed in detail using the rubric. Similarly, a midterm and a final performance task were assessed using the rubric. Additional measures of students' abilities were gained through classroom observations, student presentations to the class, and one-on-one conversations with students. Also, students developed a mathematics portfolio and included a reflective summary of their experiences in the course. Project evaluators had designed a rubric for assessing summer participant portfolios and this was used within the course. Final grades were assigned based on students' demonstrated proficiencies of course objectives as reflected by the artifacts above.

The content and teaching approach for MA 313 were radically different from previously taught sections of the course. Although there was an emphasis on problem solving in the former courses, the problem solving was a lot more guided by the instructor as opposed to the student-centered problem solving of the revised course. The revised course placed the students in a more active role for solving problems and for sharing their work with others. The content of the revised course was much more focused around fewer topics than previous versions of the course. Whereas, former courses had focused on problem solving topics, patterns and functions, and topics in geometry, the revised course emphasized significant extended tasks that covered fewer concepts, but dealt with these in much greater depth. There was a unifying theme of algebraic reasoning and connections between algebra and geometry.

Another major shift in the revised course from the former methods was the emphasis on students' writing and explaining their thinking. While some writing had been included in former versions of the course, it was never with the same intensity as the revised course. Students were expected to explain, using written sentences, diagrams, equations, and graphs, their solutions to all problems. There was a shift from an emphasis on just getting a correct answer to the expectation that a correct solution must include a valid justification.

A final adjustment worth mentioning was the attempt to make students take more responsibility for their own learning. The practice of responding to students' questions without directly providing an answer was intended to help students become independent thinkers. By asking students questions and listening to their responses and having them explain their attempts at solutions, students were guided toward answering their own questions. Also, collaboration with peers on problems was another way of assisting them in developing their own understandings.

A similar approach was used in developing and implementing MA 314, *Geometric and Proportional Reasoning*. The content focus deals with number sense, geometry, measurement, congruence, similarity and proportional reasoning. Students develop inductive and deductive reasoning skills and make and explore conjectures about mathematical concepts. Justification of ideas leading toward formal proof is developed within the course.

In addition to reforming courses, the project also sought to gather data on views of higher education faculty on the influence of their participation in GBMP courses on their roles as university instructors. Focus groups of faculty from mathematics, education, and engineering were held and qualitative data was collected by project evaluators. Evaluators sought to identify factors which promote or impede the implementation of inquiry-based methods as well as positive aspects of implementing these methods.

Another part of our methodology was to observe university faculty and rate their instruction using the Reformed Teaching Observation Protocol (RTOP) (Sewada, 2002). A website describing the RTOP states, "The RTOP was developed as an observation instrument to provide a standardized means for detecting the degree to which K-20 classroom instruction in mathematics or science is reformed per the national science and mathematics standards." (http://physicsed.buffalostate.edu/AZTEC/RTOP/RTOP_full/) The RTOP consists of five subscales: Lesson Design and Implementation, Content: Propositional Pedagogic Knowledge, Content: Procedural Pedagogic Knowledge, Classroom Culture: Communicative Interactions, and Classroom Culture: Student/teacher Relationships. Project evaluators used the RTOP in their observation of six "traditionally" taught mathematics classes and four classes taught using inquiry-based instruction. Details of these observations can be found in the results section which follows.

Results

After approximately three years of attempting to implement inquiry-based instruction in university mathematics courses, project data has begun to show positive outcomes and challenges which remain to be addressed. Findings shared below will relate to the impact of course reforms on students and instructors. MA 313 has been taught more frequently than any of the other reform efforts and most of the results below are derived from this course. The result below were attained through focus groups conducted by an independent evaluator who worked as a part of the Greater Birmingham Mathematics Partnership evaluation team.

Three focus groups were conducted with preservice teachers in to monitor their beliefs about how students learn and their perceptions of the effectiveness of the reformed mathematics courses. All students in the focus group were enrolled in MA 314 or MA 313 at the time of the focus group. Some students had taken both courses.

Comments from the participants who had taken both MA 313 and 314 were much more positive than comments from students who had only taken one course. Those students saw the value in the methodology and planned to use the teaching strategies with their own students. One student mentioned, “Dr. Smith modeled what you would do with your students – I think this is very good … he didn’t specifically teach ‘teaching techniques’ but instead provided an example of what you should do in helping a student understand the problem.” Other students pointed to the importance of having to struggle to change their thinking about teaching and learning. But they all expressed a strong belief that their understanding of mathematics was much deeper than it would have been if they had taken a traditional mathematics course.

Some students who were taking their first reformed course expressed a preference for traditional instruction. One student commented, “I like the regular math classes better because you come in, get the syllabus, and you know what you have to do to make an A. Here, you’re never sure.” Another said, “I didn’t like the class at all – I was in the dark … I guess I’m just ‘old school’ in what I expect from courses. The uncertainty was very unsettling … I had no idea if I was right or wrong and that made me nervous.” Students who had taken only one course also expressed frustration that the instructor expected them to “teach themselves.”

Grading appeared to be a significant concern to preservice teachers who had only taken one course. They cited a general lack of “structure” in the course that caused them anxiety over their grades. One student said, “If it weren’t for a grade, it would be okay. But I have to keep my GPA up. I would rather take a test than worry about whether I wrote everything in my portfolio.”

Following is a summary of themes that emerged from each group of students:

<u>After one course</u>

Frustration of not knowing whether answers are correct

- Lack of structure and purpose
- Lack of clarity about grading
- Required to “teach yourself”
- Group work not always effective
- Okay sometimes, but shouldn’t be only method

<u>After two courses:</u>

- Learned much more about mathematics
- In some ways more difficult than traditional (have to think more, do more active work)
- In some ways easier than traditional (no memorizing or cramming, no longer mysterious)
- More rewarding to find answers yourself
- More confident about mathematics ability
- Can see how to use this in my own classroom

- Some lingering concerns about grading
- Group work still a problem for some

The evaluator also did class observations in which she rated the instructor and rated randomly selected students. The table below shows the results for an observation period during the fall semester of 2008. Three randomly selected students were observed at three time points during semester…beginning, middle, and end. The numbers in the cells indicate the number of students exhibiting the described behaviors during group problem-solving and processing:

<u>Student Behavioral Checklist - MA 313, Fall 2008</u>

N=3			
	Time 1	Time 2	Time 3
Understanding of Mathematical Ideas			
Uses variables to describe unknowns	0	2	3
Explains why equations make sense geometrically	0	1	2
Represents linear and quadratic equations in variety of ways	0	1	2
Productive Disposition			
Persists when answer is not known	1	2	3
Asks for guidance but not answers	0	1	2
Tries variety of strategies for approaching problem	1	2	3
Inquiry and Reflection			
Makes extensions and connections beyond immediate problem	0	0	2
Explores why it works and whether it will always work	0	1	2
Confusion and mistakes lead to further exploration	1	2	3
Communication			
Explains reasoning fluently	0	2	3
Asks probing questions	0	1	2
Shares ideas with class	1	2	3

The results of these observations show that the students did improve on the measured attributes over the course of the semester.

In interviews with faculty, one of the recurring themes that emerged is the idea that the summer courses helped IHE faculty to refine the way they were already teaching. For some faculty members, the course resulted in drastic changes in curriculum and pedagogy. But more common was the comment that the summer courses caused IHE faculty to be more reflective about their teaching and their students' thinking.

A focus group conducted with faculty members from UAB mathematics, engineering, and education departments centered on what elements of summer courses can and should be implemented at the higher education level. Faculty members recapped their experiences in summer courses by discussing the benefits of inquiry-based instruction in mathematics. The following themes emerged from their responses:

- The importance of struggle to the learning process
- That students learn by doing, not by watching

Although there was unanimous agreement about the value of inquiry-based instruction in general, two very different themes emerged from responses to questions about what elements of the inquiry-based instruction could be implemented at the university level. Some professors expressed strong beliefs in the need to reform university mathematics courses and make them more inquiry-based. One stated, "I never just lecture, I'm always turning them loose. Because I believe this is how they are going to learn – they are not going to learn while sitting there nodding their heads."

Other professors were more reticent to apply the principles of inquiry-based instruction in the context of their university classrooms and cited the following challenges:

- Class size—for group activities to be effective, smaller classes are needed
- Time—50 minutes is not long enough to engage in group problem-solving
- Fixed syllabus—there are certain objectives that must be covered
- Varying ability of students—can't reach every student doing these activities
- Grading—students must be held accountable for their understanding of the mathematics

IHE Classroom observations show a connection between attitudes expressed about inquiry-based instruction in the focus group and scores on the RTOP. Those professors who support inquiry-based instruction and expressed beliefs that is was the best way to teach students at the university level despite the challenges had higher scores on the RTOP than those who argued there were too many obstacles to implementing inquiry-based instruction at the university level.

Higher Education Mathematics Classroom Observation Results (RTOP)

	Traditonal UAB Courses Median (Range) (n=7)	Revised UAB Courses Median (Range) (n=3)
Lesson Design/Implementation	1 (0-3)	14 (11-15)
Propositional Knowledge	3 (3-6)	11 (10-12)
Procedural Knowledge	2 (0-6)	14 (14)
Communicative Interaction	1 (0-3)	13 (10-15)
Student/Teacher Relationships	2 (0-7)	14 (12-14)

The above data for the "revised" courses represents RTOP scores from an observation during the first semester the courses were taught. Revised courses MA 313 and MA 314 were observed for a second time during subsequent semesters. The table below shows changes within a single instructor over time.

Higher Education Reformed Courses Repeated Observations

RTOP Categories (maximum score on each subscale is 20)	**MA 313** Time 1	**MA 313** Time 2	**MA 314** Time 1	**MA 314** Time 2
Lesson Design/Implementation	14	18	15	18
Propositional Knowledge	10	17	12	17
Procedural Knowledge	14	18	14	18
Communicative Interaction	13	16	15	17
Student/Teacher Relationships	14	17	14	17

Discussion

The results for this study to date are encouraging, yet they also leave challenges to be met. From a student learning point of view, instructors point to a greater depth of understanding of mathematical concepts using inquiry methods. While many students also, state that they feel as though they are learning more deeply, some, particularly those taking only one course, are still unsure that the method produces maximum results. These students may not recognize their mathematics growth to the extent that the instructor does. They may still believe that if they don't find the correct answer in a short time then they have not been successful. The evidence is clear that almost all students are much better at communicating their understandings in writing and orally than they were in traditionally taught courses.

Decreasing mathematics anxiety is a desired outcome of the summer courses as well as the university courses. While no formal measures of math anxiety have been used, anecdotal evidence from students' reflective writings indicate that for many teachers in the summer courses math anxiety is reduce. While they struggle with problems and experience frustration, in the nine-day format they are able to see the positive things that have happened and these out weigh the negative emotions. It is not as clear that this transformation is as dramatic in the full semester courses. Because the courses are spread over 14 weeks, there does not appear to be the intensity and the "ah-ha" that teachers experience during the short summer courses. The participants in these two groups are also quite different in their levels of maturity with the summer group mostly being in-service teachers who are typically older and more experienced with different methods of teaching. For students who take two full semester reformed courses, their anxiety does appear to lessen. Their comfort with the method appears to grow with experience.

From the perspective of university professors, the benefits of the approach surpass the drawbacks. The depth of student learning appears greater in the inquiry format. The topics are narrower in focus and time can be spent exploring problems more intensely than in courses where covering many topics is an expectation. This also points to an area of concern for those courses that are expected to deliver a lot of content that will be needed in future courses. Thus, the age old question of depth versus breadth reappears. Instructors are pleased with the level of thinking that students exhibit in the inquiry approach and find classroom exchanges energizing. Students improve in their ability to express themselves in written form. This does lead to another concern of some instructors, the issue of grading. While rubrics help to make the grading more objective, the time to assess open-ended problems is much greater than grading traditional closed-form problems. The issue of grading for process and product and how to weigh these two dimensions takes time and thought.

Finally, this study has led to some emerging ideas and questions that warrant further study. It does appear that inquiry-based instruction becomes easier over time for both instructors and students. Students who have experienced traditional instruction for twelve or more years in which they expect a skilled instructor to reveal all truths to them have to adjust to new classroom expectations. This takes more time for some than others. Another clear point is that, in order for this transition to an inquiry based approach to be successful with university students, a supportive classroom environment must be established. Instructor must invest time in initial courses using such methods to explain what they are doing in terms of their teaching method and why they are using this method. This initial investment takes away class time, but it pays off by improved efficiency and increased student morale.

Questions that remain include, can all students make the transition to a new way of learning after experiencing, and possibly being quite successful at, a traditional approach to instruction for twelve or more years? How do we deal with students who resist this method of instruction and reflect this in evaluations of instructors? How can university faculty be encouraged to and rewarded for investing time and energy into implementing such methods in courses they teach? For which courses do the benefits of depth of learning outweigh the need for breadth of coverage? With the demands of promotion and tenure for university faculty, these questions deserve consideration.

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