Examining Mathematical Knowledge for Teaching in Secondary and Post-Secondary Contexts[1]

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Abstract

In this paper we highlight areas we believe need attention as the construct Mathematical Knowledge for Teaching (MKT), including Common Content Knowledge (CCK) and Specialized Content Knowledge (SCK), is generalized to secondary and post-secondary contexts. These constructs were developed in the context of research on elementary school teachers' knowledge. Elementary teachers, however, typically differ from teachers of higher grades in their content preparation. We present a set of theoretical questions that arose from our examination of definitions of CCK and SCK as we attempted to utilize those definitions to characterize the nature of MKT at secondary and undergraduate levels. We illustrate these issues with data from two post-secondary mathematics instructional settings.

Objectives of the Research

The construct of "mathematical knowledge for teaching" (MKT) was developed in the context of elementary school teachers. MKT is an umbrella term used to describe the multiple types or

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components of knowledge used in the work of teaching mathematics. MKT includes knowledge of mathematics content, pedagogical content knowledge (PCK), and the specialized content knowledge (SCK) that people who do the mathematical work of teaching develop that other users of mathematics might not have the opportunities to develop. Most of the research involving these constructs has also been conducted with elementary or lower secondary school teachers.

To date, considerably less research on the knowledge needed for teaching has been conducted in the context of secondary or post-secondary school. Typically, elementary teachers have less formal preparation in mathematics than teachers of higher grades. Given the interconnected nature of knowledge, it is reasonable to presume that there are interactions between a teacher's content knowledge and the other components of knowledge used in teaching. As researchers begin to examine the use of knowledge in teaching at higher levels, it seems prudent to examine these conceptualizations of knowledge (e.g., SCK, PCK) and consider how transferable they are from one context to another. In particular, we examine definitions of common content knowledge (CCK), SCK and PCK and consider instantiations of these definitions in the context of secondary pre-service teacher preparation and post-secondary mathematics instruction. This is a theoretical exploration that was inspired in part by the fact that distinctions among CCK, PCK, and SCK for elementary teacher have been recognized and accepted in mathematics education community but these distinctions appear less compelling and clear at higher levels.

Relevant Research literature and Theoretical Perspective

Many researchers have examined teachers' knowledge and the roles knowledge plays in shaping teaching practices (Borko & Putnam, 1996; Schoenfeld, 2000; Schoenfeld, Minstrell, &

van Zee, 1999; Sherin, 2002; Shulman, 1986). In such an approach, knowledge is seen as one of several factors influencing teachers' goals and their approaches to accomplishing those goals as they plan for and enact instruction. While it is undoubtedly the case that teachers need knowledge of mathematics content, researchers have found it challenging to establish relationships between measures of teachers' content knowledge and student achievement (Ball, Lubienski, & Mewborn, 2001; Wilson, Floden, & Ferrini-Mundy, 2002). Teachers' having more courses in content is not strongly correlated with higher achievement for their students (Begle, 1979; Monk, 1994). These and other findings about knowledge resources teachers use have directed attention to other kinds of knowledge. Of particular note are influences researchers have found of PCK and SCK.

PCK is the label used to describe what teachers know about (among other things) which topics typically cause students difficulty, how different ideas tie together and are organized in curricula, and how particular examples or explanations can be useful in teaching particular concepts. Since the identification of this type of knowledge (Grossman, Wilson, & Shulman, 1989; Shulman, 1986), researchers have found that PCK plays important roles in teachers' practices and the learning opportunities such practices create for students. For example, researchers have shown that teachers' knowledge of the different strategies that their students would use to approach problems is positively correlated with student achievement (Fennema et al., 1996).

In addition to having PCK at one's disposal, Ball and colleagues have identified different types of mathematical knowledge needed for teaching. CCK is the mathematical "knowledge of a kind used in a wide variety of settings – in other words not unique to teaching; these are not specialized understandings but are questions that typically would be answerable by others who know mathematics" (Ball, Hoover Thames, & Phelps, 2008, p. 399). On top of this knowledge common to others, teachers use SCK to engage in a type of mathematical work to follow and understand students' ideas and solution strategies. SCK is "the mathematical knowledge 'entailed by teaching' – in other words, mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (Ball et al, 2008, p. 399). These mathematical tasks include following students' mathematical thinking, evaluating the validity of student-generated strategies, and making sense of student-generated solution paths. Researchers have examined the knowledge needed to do this work and found connections between teachers' possession of this knowledge and elementary students' achievement (Ball & Bass, 2000; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Franke & Kazemi, 2001; Hill, Ball, & Schilling, 2008; Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004).

Methods of Inquiry

In this theoretical endeavor, we made use of research on knowledge needed for teaching and data from two research projects. We focused our analysis on the explicit definitions of CCK, SCK, and PCK as well as the operational definitions as found in the literature on elementary teachers' knowledge. We examined those definitions and their relationships to typical characteristics of elementary teachers (e.g., their level of content preparation, their experiences doing mathematics) implicit in those definitions. Next, we compared and contrasted those characteristics with characteristics typical of secondary/post-secondary teachers and examined potential implications for the definitions of CCK, SCK, and PCK. To illustrate these issues we use data from observations of classroom practice and from studies of post-secondary teachers' practices. These data examples are used to illustrate the complexities in distinguishing among these components/types of knowledge and provide specific illustrations that may help with the development of definitions of CCK, SCK, and PCK at upper grade levels.

Findings

The results of this research are a set of questions, illustrated by vignettes from our data sets that demonstrate central issues in generalizing the constructs of CCK and SCK to secondary and post-secondary mathematics education contexts. We discuss two major questions that have arisen for us in this analysis.

Defining "Common" and "Specialized"

The first question is "What is the relationship of CCK to SCK for those holding a bachelors degree or higher in mathematics?" The assumptions embedded in the elementary context are that CCK is knowledge held by an average mathematically literate citizen and that SCK is different. However, amongst those teaching in secondary and post-secondary contexts, what should be considered *common* content knowledge? Is conceptual understanding of the *common* content knowledge amongst those with bachelors degree or higher level mathematics the same as SCK? For example, recognizing the mathematical accuracy of a definition, considered part of SCK for elementary teachers, is CCK for those with more mathematical education. With this new population, is there a similar differentiation?

To illustrate, consider the following example from a secondary mathematics classroom. A teacher poses the following problem: Suppose that a staircase comprises ten steps and that you can climb the stairs one or two steps at a time. In how many different ways can you climb these ten steps? (Rubel & Zolkower, 2007/2008). The teacher has an image of one solution, using combinations and counting methods. One group of students comes up with the diagram in Figure 1 and solution:

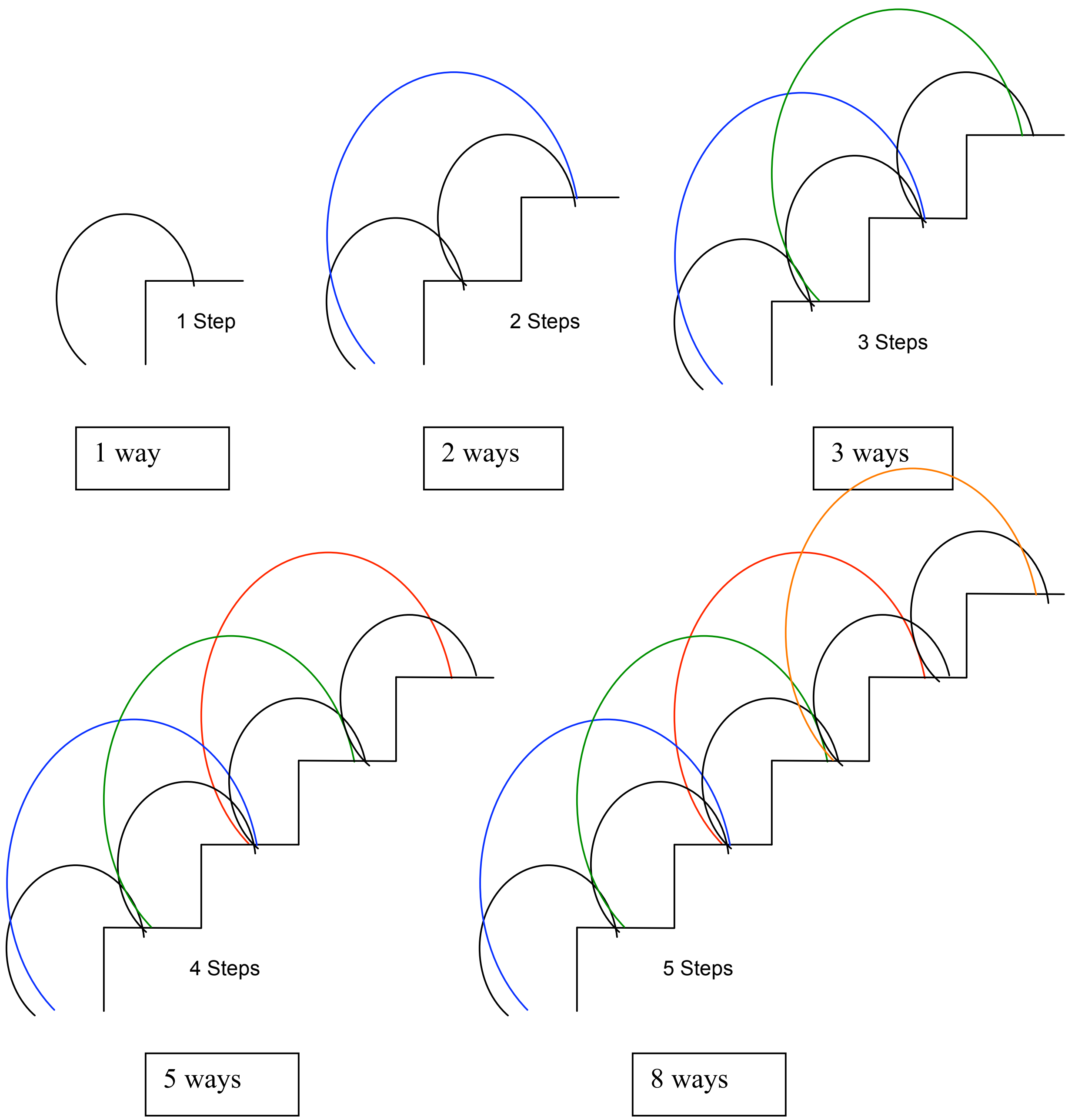

Figure 1
Student Work on Problem

The students conclude the pattern of growth of the number of ways as one increases the number of steps is the Fibonacci sequence. So for 10 steps there are 89 ways.

This example presents several questions about the mathematical knowledge teachers need to make the next pedagogical decision in this situation. Is this sequence really the Fibonacci sequence? Why? For a mathematics major, we contend the knowledge needed to determine if the sequence is really Fibonacci is common content knowledge for the population of similarly situated mathematical knowers – those possessing at least an undergraduate mathematics major. On the other hand, the ability to unpack the students' solution, interpreting their representation, may or may not be a mathematical expectation of others who have a major in mathematics. Is this knowledge SCK? In addition, to decide what to do with this solution, teachers need to be able to determine how well they can advance their mathematical goal of the lesson – combinatoric solutions – and still address this solution. So other mathematical questions the teacher might consider are: How does this solution connect to the combinatorial solution? Is this an important mathematical connection to make? Overall, in assessing the mathematical knowledge needed for teaching, do we consider the knowledge needed to deal with this solution common content knowledge, specialized content knowledge, pedagogical content knowledge, or something else? Why?

The Nature of Mathematicians' Work

Our second question is, "What is the relationship between the type of work mathematicians do in their research and while teaching mathematics?" Researchers have distinguished between CCK and SCK by saying that SCK is "*Specialized* because it is not needed or used in settings other than mathematics teaching" (Ball et al, 2008, p. 396). In other words, there is mathematical work that is required while teaching that is not required in the other contexts. This distinction is relatively clear in the context of elementary school teaching where the work required (e.g., considering the pedagogical entailments of particular examples) goes beyond what a mathematically literate person might do in their day-to-day lives.

When we consider the day-to-day lives of mathematicians, however, the distinction seems less clear. Consider the teaching task of examining, evaluating and formulating a response to a student-generated solution. This is a type of work that researchers of elementary teachers assert necessitates (and enables the development of) specialized content knowledge. Now consider the nature of the work of mathematicians. In the course of their typical work, mathematicians evaluate their peers' solutions and provide feedback about those solutions. This occurs informally as colleagues share ideas and possible solutions to problems. It also occurs more formally when mathematicians examine proofs and solutions while listening to their colleagues' presentations and while reviewing manuscripts for publication.

In both contexts, the mathematician needs to make sense of the mathematical ideas and reasoning presented by someone else and to determine whether the reasoning is correct or incorrect. In both contexts the mathematician must also formulate a response about the proposed solution, either to the student or (directly or indirectly via a journal editor) to their peer.

Several questions arise from this thought experiment. Are the two types of work described above the same? Do they draw on the same type of mathematical knowledge for teaching? Furthermore, elementary and secondary teachers generally do not examine the mathematical work of their peers. Does that mean that the knowledge used while checking the validity of student-generated solutions is common content knowledge for mathematicians but specialized content knowledge for others? Is the work the same when formulating a response to these different people and is the knowledge used to do that work the same in both contexts?

Conclusions and Implications for Further Research

Our examination of the literature on mathematical knowledge for teaching and data from secondary/post-secondary teaching contexts has raised many questions that we believe are important for the research community to consider as investigations of teachers' knowledge broaden beyond elementary school contexts. We anticipate that the questions posed in this report are just a small sample of the full set of issues that researchers will need to attend to as they use and refine the existing theoretical constructs and definitions of mathematical knowledge for teaching. Further research on teaching and teachers at secondary and post-secondary levels can help strengthen the literature base in this area by identifying aspects of current theory and definitions that are generalizable and others that are in need of refinement.

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