Riding the Double Ferris Wheel: Students' Interpretation and Understanding of Trigonometric Functions in Realistic Settings

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This paper examines how two students understood the notions of angular velocity and movement and how that understanding contributed to their construction of a trigonometric function that detailed the height of a rider on a double Ferris wheel over time. The analysis is carried out from a situated perspective with particular emphasis on when students reasoning took place and how that facilitated their understanding. This paper contributes by detailing how the students' conceptions of angular velocity and movement change as they construct graphs and functions. Furthermore, it illustrates how the students utilized those conceptions to contribute to their ability to reason about quantities co-varying in the problem and also what it meant to compose the angular movement with the function for the height verses change in radians.

This paper is divided into three main sections. The first section is a background section in which I give a brief synopsis of the literature on trigonometry, detail the theoretical perspective that will be used in the paper and discuss the task and methods utilized. The second section analyzes the students' engagement in the task and also how they constructed their mathematical understanding of the phenomenon. Finally, the last section will discuss how this examination can shed light on the role that function composition and co-variation play in modeling periodic phenomena using trigonometric functions.

Background

Literature Review

The literature analyzing trigonometry, particularly student understanding of trigonometry is sparse and reflects a disparate number of different perspectives. Weber (2005) has examined students relationships between the triangle, unit circle and the sine and cosine functions. He demonstrated how non-traditional classroom instruction made an impact in the students' ability to reason about trigonometric functions. Ozimek, Engelhardt, Bennett, & Rebello (2004) found that students in their study had significantly greater difficulty dealing with the unit circle than the function and triangle representations of sine functions. They also found that there was no significant transfer from trigonometry to physics. Shama (1998) suggested that students' understanding of periodicity was related to Gestalt structures, where periodic phenomena are "understood as a whole process with unified laws (273)". And Gerson (2008) analyzed how students understood different representations of periodic functions. Furthermore, she showed that students developed their own concept images of these functions despite the fact that the teacher focused only on procedural fluency.

Some authors have also examined how the teaching of angle measure, radian and arc length have possibly led to a lack of coherence in the trigonometry curriculum (Thompson, Carlson & Silverman, 2008; Thompson 2008). These authors contend that the trigonometry curriculum does not allow enough time or emphasis on students creating meaning about angle measure and arc length. Furthermore, they also note that teachers of trigonometry need to have professional development that encourages coherence between their understanding of angle measure, arc length and trigonometric functions.

Theoretical Perspective

The situated perspective takes as first principle that individuals' mathematical activity cannot be separated from the situation in which it arises (Lave, Murtaugh, & De La Rocha, 1984; Nunes, Schliemann, & Caraher, 1993; Säljö & Wyndham, 1993; Brown, Collins & Duguid, 1989). The use of this perspective requires that we re-examine how students come to know and the patterns of action that they use to undertake tasks. Greeno (1998) defines the *emergent problem space* as one in which the problems that are in dispute during any interaction are a product of the context in which any problem arises. Consequently, salient characteristics of any problem are not implied in the original problem, but rather arise in the interaction of the person or persons. This problematizes characterizing characteristic features of a problem as surface or relevant because the status of the feature depends on the person solving the problem and how they are interpreting the situation in a moment-to-moment fashion. Thus, when examining Andrew and Oscar's attempts at dealing with the tasks of the experiment, I consider what features become salient and what questions Oscar and Andrew ask at any given moment to understand the nature of the problem. As well, I examined how the nature of the task and the patterns of their activity contributed to their understandings of the graphs and function of Sandra's height versus time on the double Ferris wheel.

Methods

A small group teaching experiment (Confrey, 2000) was conducted over a period of two weeks. Three groups of two students attended two, 1 ½ -2 ½ hour working sessions in which they were asked to consider a series of tasks relating to the path travelled by a rider on a double-Ferris wheel. The students were given an applet, which mimicked the movement of the double-Ferris Wheel (figure 1) that they could control and move as they wished. Furthermore, later in the interview, the students were given numerical values for the size of the wheels and periods of the wheels movement. The teaching experiment centered around four basic tasks:

- Construct a representation of the Sandra's ride.
- Construct a qualitatively correct height versus time graph of Sandra's ride.
- Construct a mathematically accurate graph of the Sandra's height versus time on the Double-Ferris wheel. . (Eg. Add scale and important points to the graph and their corresponding values.)
- Create a function for Sandra's height versus time.

The students were allowed to use as much time as they wished to complete the tasks and they were frequently asked questions about the particular work they were doing or how they were thinking at any one moment. Furthermore, there was consideration that the students may not be able to move on to later parts of the task without some intervention. As my intention was to characterize the students' activity as they interacted with the wheel, I do not see this as being problematic. Any time my own actions serve to further student understanding of the task, I will account for it in the analysis.

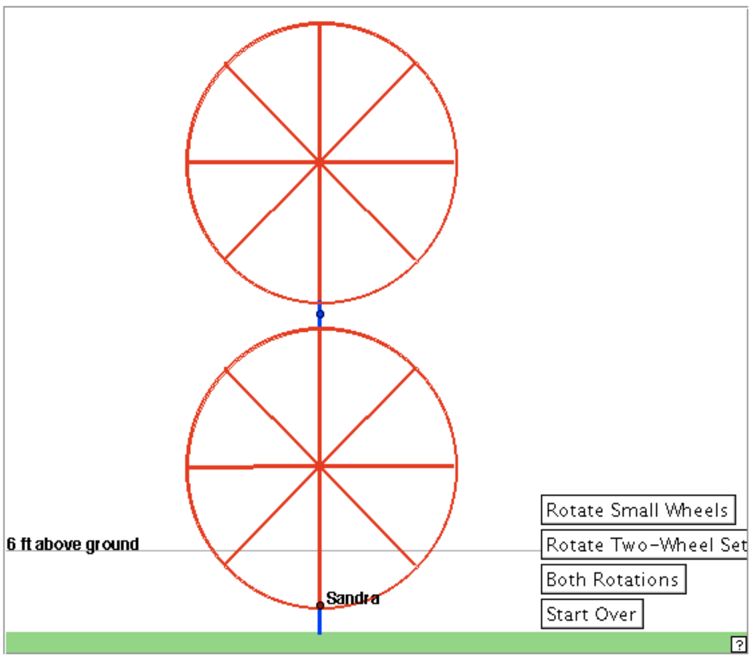

Figure 1[1]

The two students analyzed in the following sections of the paper were both enrolled in a class on thinking about high school mathematics from an advanced perspective. They were both engineering majors and were on their way to earning a minor in mathematics. They both considered teaching high school mathematics as an option to working in engineering. Furthermore, both students had extensive

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[1] Keymath. (2009). [Applet for the double Ferris wheel]. *Discovering Advanced Algebra Resources at Keymath.Com.* Retrieved from http://www.keymath.com/x3361.xml.

background in modeling, as evidenced by their references to modeling the phenomenon and talking about how they would model circular motion.

Analysis

Initial Impressions

Andrew and Oscar first drew a parametric graph of Sandra's ride on the double Ferris wheel. Both were convinced that such a graph was sinusoidal in nature. Andrew reasoned that this was because the graph, once it reached its lowest point would repeat. For Andrew, this repetition meant that the graph was periodic and hence would be sinusoidal. Oscar echoed these sentiments. When asked why they believed that, they stated that they had seen similar graphs in their modeling classes and they always ended up being sinusoidal. It is following this interaction that the first mention of angular velocity arises. In this case, for Andrew, angular velocity was necessary as the argument for a sinusoidal function. When queried as to why that was the case, he said that it was because they were wheels and the motion of wheels needs to be expressed in terms of angular velocity. He related the periodic nature of the movement (the wheel returns to the same point after a certain number of revolutions of the component wheels) to the functions roots in trigonometry. However, angular velocity is soon to be problematic as Andrew and Oscar engage further in the tasks.

Separating the two wheels

Angular velocity arose again as Andrew and Oscar attempted to add scale to the height versus time graph (Figure 2). The students had drawn a qualitatively correct graph of the height versus time of Sandra on the wheel. During that discussion, I asked them to characterize how many times the small wheel had turned verses how many times the big wheel had turned. There was some disagreement between the two students as to how far the small wheel had actually travelled. Oscar contended that the small wheel[2] moved three revolutions for every two revolutions of the big wheel. However, Andrew contended that the small wheel only moved one revolution for every two revolutions of the big wheel. He ascribed the appearance of the small wheel revolving three times to his count being "with reference to the position of the big wheel." Consequently, when he added the position of the big wheel plus the position of the small wheel with "reference to" the big wheel, he arrived at the position of the small wheel with reference to it's starting position. For example, if the big wheel has moved ¼ and the small wheel has moved 1/8 with "reference to the big wheel," then the small wheel 3/8 with regard to the starting point. Thus, when Andrew and Oscar looked at the velocity of the wheel, Andrew contended that the small wheel moved more slowly than the large wheel. When Andrew was made aware of the time that each wheel took to make a revolution, he had problems compromising how he calculated the speed of the wheel with the data that he was given. Consequently, he claimed that the only way

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[2] In this teaching experiment, Andrew, Oscar and I refer to the wheel that Sandra is directly connected to as the small wheel. The lever arm, in blue on the applet, is referred to as the big wheel. The other red wheel without Sandra is never referred to and never becomes a salient issue for the pair.

that that could occur would be for the big wheel's velocity to be added to the small wheel's velocity, a statement that Oscar clearly disagreed with. Consequently, at the end of the interaction both Andrew and Oscar agreed to disagree.

It's important to note that while Oscar and Andrew continued to work on this graph and to construct its scale, this particular point of contention was not addressed. In fact, while working on the height versus time graph, dealing with angular velocity is not an explicit issue. The relationship between how fast the wheels are turning and the height versus time graph is implicit. The speed at which each wheel moves or how they work in conjunction to create the motion need not be addressed because points can be considered in isolation. The only indication that angular velocity is a concern would be the relationship between the scale of the graph and the shape of the graph. However, because Andrew and Oscar only drew a single period of Sandra's height versus time that relationship is not explicitly addressed. Consequently, when constructing the height versus time graph and plotting the maximums and minimums, Andrew and Oscar rarely had to deal with angular velocity directly.

Rate of Change in Height Versus Time and the Angular Velocity

The role of angular velocity returned as an explicit contention when they were asked to give their intuitions about the function for height versus time that could be drawn from their completed graph (Figure 2). Andrew and Oscar both agreed that the function for this graph would most likely be sinusoidal. They mentioned that they had seen graphs like this in physics and they had sinusoidal functions. Furthermore, fluctuating of the height versus time graph up and down also indicated that this might be sinusoidal. Andrew returned to the applet and he once again said that he was trying to figure out the angular velocity. Oscar vocally expressed not wanting to discuss angular velocity again with him. In fact, although both students had been confident that the graph they drew represented Sandra's height versus time as she rode the ride, Oscar openly doubted if the graph that they had constructed actually was the right graph and that it would be better for them if they constructed the function using only the applet and what they knew about the dimensions of the wheel and it associated values.

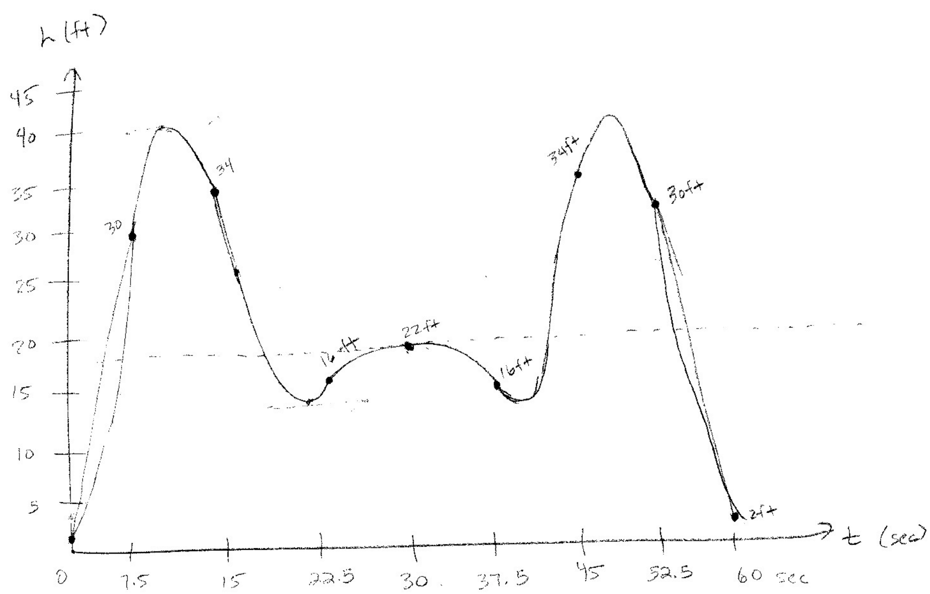

Figure 3

Here a tension arose regarding the relationship between the height versus time graph and the movement of the two wheels. So much so that Oscar decided to doubt the graph altogether and relate only to the phenomenon. This is further complicated by the fact that Andrew and Oscar have been asked to understand the function via the height versus time graph. Their first representation (Figure 2) was a simple translation from the applet onto the page. They even constructed it by placing a transparency over the applet and drawing upon the applet. In the case of the height versus time graph, the students were reducing the phenomenon to fewer salient features. Furthermore, with respect to angular velocity, in the first representation the angular velocity can be thought of as moving around the wheel and so angular velocity is more explicitly important in constructing the function. But as was mentioned earlier, the angular velocity is only implicit in the height verses time graph.

Andrew examined his newly notated graph and noted the minimums and maximums as being places where the velocity was zero. Furthermore, there were places where the change in height versus the time is increasing and decreasing which indicated that the velocity was not in fact constant at these places.

> Oscar: We know the velocities of the circles are constant. So the velocity increase or decrease depends on the height. If you're saying that they do change. And then you said…
>
> Andrew: But this is um…
>
> Oscar: But you said, um… the velocity increases as you move away from the center.
>
> Andrew: That's angular velocity right. Isn't angular velocity the one that increases as we move?

Andrew then gestured with his finger by moving the finger in a clockwise direction in front of him. As he is referring to angular velocity, Andrew then states, " Fast…alright, let's think about it." Andrew then placed his thumb and forefinger on the drawn graph in a manner that he used earlier to describe the changes in velocity that he observed on the newly notated graph. He then pointed to the center of the graph. As Oscar notes, Andrew has observed that the rate of change in the height decreases as Sandra's position on the double Ferris wheel nears the center axis of the wheel. Then Andrew states, "If this is the center….I don't know. I think the relationship that has her be closer to the center gives her smaller velocity. " He then pointed his pen at the paper. "…that has to do with angular velocity," indicating a local maximum on the height versus time graph.

This series of utterances and gestures helped to establish some of the problems that Andrew was having understanding the relationship between the angular velocity of the two wheels and the change in height verses the change in time. Andrew attempted to understand how it was that the rate of circular motion played into understanding the function. Secondly, the local maximum, at which the two participants agreed that the velocity slowed down, was seen as especially problematic because at this point the derivative of the height verses time function was obviously not constant and eventually hit zero. However, Oscar was still unable to consciously iterate that relationship, as evidenced by his statement, "I guess there's a relationship between the angular velocity," he pointed down at the paper with the height versus time graph and traces his pen across the curve, "and this stuff right here."

Even though Oscar was unable to iterate what the relationship was between the height verses time graph and the angular velocities of the two wheels, he does, at least tentatively, convince Oscar that angular velocity is important.

> Oscar: (He moves his hand upwards towards his chest and turns his finger) So Sandra's moving (he moves his finger up and down) in height (he moves his full palm up and down). Her height is changing. And then there's angular velocity (he moves his finger in a circular motion)….that's related….That corresponds to the circle….the circles….And then, there's the derivative (traces his finger along the graph)…Let's see (he places his thumb and forefinger over a portion of the height verses time graph) it's the derivative of the height, so I guess I'm trying to make sense of that…It's the velocity that she has (picks up his pen) on the height axis. So this is…(draws an x-y Cartesian coordinate axis) a Cartesian coordinate system, where this is the height here, she'll have some velocity (he draws an up and down arrow on the paper), either going up or down. Which would be the derivative of this…(points at the height verses time graph) I would see…Which is not the same as angular velocity….

Oscar recapitulated Andrew's gesture for the angular velocity, but added to that gesture the upward and downward movements in position with relation to the ground. Furthermore, he differentiated between the two motions by switching from the finger motion used in the angular velocity gesture to the full hand for the change in height gesture. His gestures differentiated between the two different types of movement, circular and up and down. His next gesture, which moved along the graph of the height versus time is accompanied by his discussion of the derivative. He drew a plane with an up-down arrow. At this point, Andrew seemed convinced that the angular velocity and the derivative of the height versus time graph were not the same thing.

This is the first time that Andrew and Oscar explicitly differentiated between the rate of change for the wheels and the rate of change for Sandra's height. This parsing is crucially important for the students and also for the purposes of the mathematics. For Andrew and Oscar it allowed them to consider separate different aspects of the applet for individual consideration and mathematization. From a mathematical standpoint, the separation of the two rates helped the student to distinguish the different functions that will be constructed and composed for the final height versus time function.

<u>The Covariation of the Small Wheel, the Big Wheel and the Height vs. Time Graph</u>

Andrew and Oscar's realization that the change in the height vs. time and the angular velocity were different did provide another step towards their construction of the function, but it also established a new dillema. They needed to better understand how the graph that they drew illustrated the movement of the two wheels.

This realization led the pair to return to the applet to see if they could somehow deal with their apparent problem. Andrew moved his chair next to the applet and followed the movement of the double Ferris wheel.

> Andrew: The only thing that I can see is that the velocity of the big circle cancels with the velocity of the small circle. Like the big circle is going (he moves his finger over the applet as Sandra moves along the wheel. He then takes his whole arm and uses the arm to trace the movement of the wheel.) And at one point the small circle is going the opposite way (he points to the intersection of the small wheel and the big wheel) [Interviewer: Okay.] And that's the only way that I can see that it would cancel out, but it doesn't happen…
>
> Int: Cancelling out? So explain what you mean by cancelling out?
>
> Oscar: She'll have zero velocity. That's what he means.
>
> Int: So, zero velocity. Explain how do you know where there is going to be zero velocity?
>
> Andrew: I'm trying to relate it but this happens (he moves from the table back to the applet and starts the applet again. He stops the applet) So this happens that in between one quarter and …(looks at the screen) somewhere in there right….(he points to the high point that Sandra reaches.)

> Oscar: The large lever arm is going going up (he gestures with his hand outstretched. He moves his hand upward pivoting from the elbow)… but she starts coming down…

The move back to the applet seemed to facilitate a better understanding of how the movement of the two wheels led to what appeared to be a point of zero velocity. Andrew and Oscar referred to this action as cancelling out. In both Oscar and Andrew's case, the motion of one wheel appeared to be in opposition to the motion of the other wheel. Andrew points to the spot on the applet where Sandra's height is at its maximum. Oscar mimics Andrew's movement with his arm. He then says that she starts coming down.

This interaction marks a shift in their participation and in both individuals' reasoning. Instead of deciding if angular velocity was important and related to the function, the pair attempted to explain how the movement of the two wheels lead to what they observed in the height versus time graph. The graph and the question of whether or not the angular velocity was necessary to construct a function of the height versus time led to a problem that the pair needed to solve. Imagining the arm's moving in opposite direction was highly problematic for both Andrew and Oscar. The movement of their gestures for both the large wheel (full arm motions) and the small wheel (single finger motions) move in the same counter-clockwise direction. Oscar hesitates as he describes how the two wheels could be "cancelling" each other out.

Andrew: This is the max remember (he points at a point on the applet, which corresponds to the local maximum on the height versus time graph). At this point, this wheel starts rotating (moves his finger around as the small wheel on the applet moves) downward while the other one (moves his whole arm) starts moving….(long pause) No, that doesn't make any sense does it?

At this point, Andrew realized that while Sandra is moving downward, the point where the wheel that Sandra is on and the big wheel is moving upward, but both wheels are moving in the same direction.

This eliminated the two moving in opposite directions as an explanation for the "cancelling out" that they observed in the height versus time graph. Oscar iterated as much, "They're always rotating the same time, but…they're always (rotates his finger in a circular motion)…they're rotating in the same direction." Oscar's gestures and his utterances indicates that he understood that they were moving in the same direction and moving with their separate constant velocities. He then offered an explanation for how the constant velocity of the two wheels can be the case and yet the graph of the height versus time can have a point that indicates zero velocity.

Oscar: They're both rotating in the same direction (circles his finger in the air in counterclockwise direction). But since they have different angular velocities, at some poi:::nt(he pauses) Like when, as far as height is concerned. The large lever arm is going way (he moves his

> forearm in a counterclockwise motion) and this (he moves his finger towards his arm in a counterclockwise motion) is coming down already. And that's where you say (points to the middle of the height versus time graph) that she has zero velocity.

At this point, Oscar's demeanor changed. He has found an explanation that was consistent with the applet, the graph and his understanding of the situation. It is clear that the explanation also resonates with Andrew, "Yeah, she doesn't really have zero velocity, like she's always moving right? [Int: Right] Its always relative between the angular velocity of the big wheel with the small wheel."

Andrew and Oscar created an explanation of the relationship between the height versus time graph and the movement of the two wheels on the applet. From a personal standpoint it helps to establish the role that angular velocity plays in their understanding. Elaborating the relationship between the two wheels and Sandra's height on the Ferris wheel allowed Andrew and Oscar to establish the independence of the rotation of the two wheels. This is key to developing a function for the height versus time. The students need to be aware that there are multiple independent but interrelated quantities that come into play. The importance of this knowledge is contextualized via how these quantities change and how it is that these changes contribute to Sandra's height. Although, Andrew and Oscar did not say so explicitly, their gestures and their utterances illustrate that they are coordinating first the change in height of the small wheel versus the change in height of the small wheel.

Finally, they coordinate how the differences in the angular velocities of the two wheels allowed for big wheel to be moving away from the ground while the Sandra's position on the small wheel moves towards the ground. This is a complicated covariant relationship in which the movement of one wheel combined with the movement of the second wheel leads to overall change in the height versus time graph. In actuality, the students needed to deal with 6 distinct co-varying quantities, the time, the angular movement of the small and big wheel, the height versus radians function of the small and big wheel, and the height versus time function for Sandra's ride on the double Ferris Wheel. Andrew and Oscar utilized the graph, the applet, and their gestures to work out this complex relationship. They demonstrated how understanding of the role of angular velocity in this situation and trigonometric situations in general requires working out not only the angular velocity itself, but how that velocity relates to the other quantities in this situation and the movement in the phenomenon. Consequently, Andrew and Oscar finally concluded that the angular velocities of both wheels play a large role in constructing the function and what exactly that role was.

Putting it All Together: Constructing the Function

Once Andrew and Oscar have distinguished the different quantities involved in the height versus time function and also distinguished how the wheels interacted, the students could work on developing the function by analyzing how the quantities combined and depended on each other. At first, Andrew and Oscar wanted to find an angular velocity for the movement of both wheels. They argued that the angular velocities of both wheels could be combined in such a way as to create a single angular velocity for the double Ferris wheel.

> Oscar: So that what you (Andrew) are talking about is equal to the angular velocity of the small circle and the big circle. So that's what we need to do.

The angular velocities of both wheels are not what need to be combined. But the students are clear that something needs to be combined. This highlights the difficulty of establishing how the different quantities involved in the double Ferris wheel combine. Not only in this problem did the students need to combine two height versus time functions, they also needed to keep the angular movement of both wheels separate and compose those functions with the function for the height versus time of each wheel.

At this point I intervened in the conversation and had the students construct the height versus time of Sandra if she was only riding the small wheel. When the time came to construct a function for Sandra's height versus time, the students had different ways of interpreting how they would find the function. Andrew decided to explain how to combine the sine waves for each wheel to generate a single wave for the height versus time (Figure 3) of Sandra's ride on the double Ferris wheel. Oscar, on the other hand, explained how he could imagine adding the height of the one wheel to the height of the second wheel at any given time and that would give you Sandra's height overall at any given time.

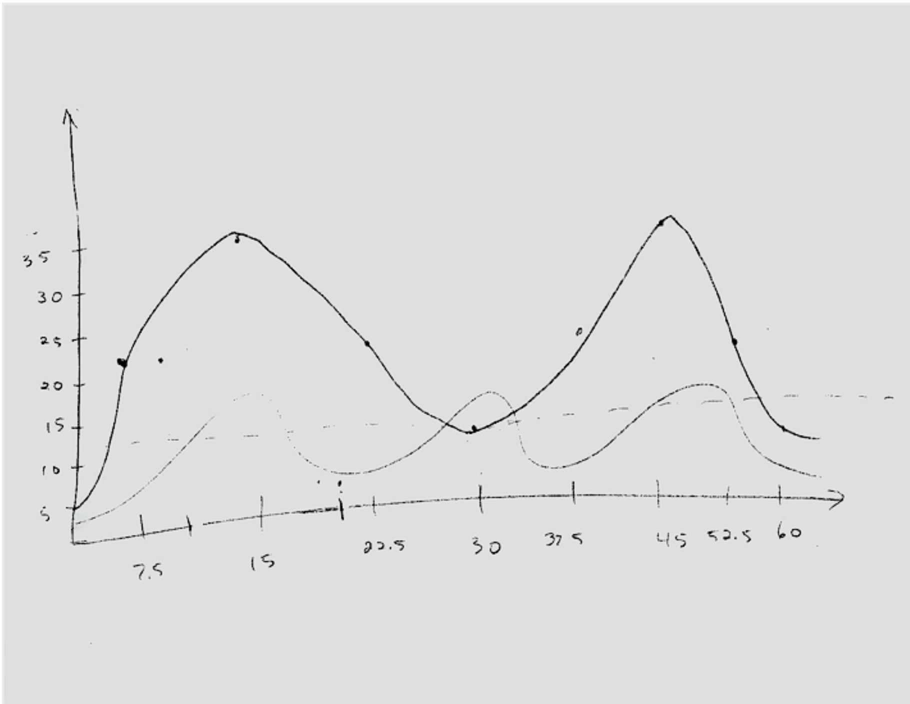

Figure 3

For both of these students, this was the final insight necessary for combining the multiple quantities into a single function. Both students had to deal with the relationship between the angular velocity and the height versus time functions. Combining the two height versus time functions requires the students to coordinate the heights of the two wheels as they move through time. Andrew's coordination using the two waves exemplifies considering the movement of the two wheels in conjunction with each other. The scale of the graph, which was only somewhat important for the purposes of drawing the height versus time graph, is very important in this instance. Andrew needs for the heights of the waves to be coordinated properly in order for the values for the height versus time function to be correct. Thus, Andrew needed to factor in, at least implicitly, the angular velocity for each of the wheels. As for Oscar, his explanation exemplifies a different, but no less complex understanding. Adding the heights for a given time can be interpreted as adding the height functions for each wheel, implicitly coordinating the other covariant quantities in the task. The two explanations highlight the different understandings for Sandra's height versus time that the two have elaborated as they construct the function.

Conclusions

The connection that Oscar and Andrew made between the angular velocity of both wheels and their construction of functions and graphs for the movement of the rider on the wheel was a key component in their eventual ability to model the double Ferris wheel situation. Their initial intuitions about the function being sinusoidal and so requiring angular velocity for the argument of the function proved to be correct. However, simply recognizing that angular velocity was necessary was insufficient to ultimately create and interpret the function of sandra's height versus time.

Four key insights contributed significantly to Andrew and Oscar establishing angular velocities role in mathematizing Sandra's ride on the double Ferris wheel. First, Andrew and Oscar recognized the independence of the two wheels. They concluded that the velocity of one of the wheels was not being added to the other. Second, they established that the rate of change in the graph of the height verses time was different from the angular velocity. Third, they distinguished the relationship between the motion of the two wheels and the graph of the height versus time. This allowed them to consider how it was that they could have points of zero velocity (rate of change in height verses time) but still could have constant motion in both wheels. Finally, they parsed out of the difference between the angular velocity and the rate of change in height verses time allowing the students to consider the function of height verses time distinct from the function for the angular movement which constituted the argument for the height verses time function.

Andrew and Oscar's work with the double Ferris wheel highlights the situated nature of student's understandings of complex phenomena. What is important in any problem situation and why it is important are crucial questions for any problem solver, but this particular episode also emphasizes the crucial role that when plays. At any given moment, for Andrew and Oscar different questions become important and so different features of the problem become important as well.

The question of what is important in this situation was not trivial even though the students had an immediate intuition about the sinusoidal nature of the graph and that angular velocity would be important. However, why and in what ways they were important arose as they dealt with the double Ferris wheel. In this situation, angular velocity operated as a way for the students to understand the circular motion of the wheel. In Andrew and Oscar's case, they needed to contend with two separate functions, one for each wheel, whose addition constitutes the function for Sandra's height versus time. Two other functions are also at play in this situation, the functions for the angular movement of the wheel. These two functions need to be composed with the functions of each wheel in order to come up with a function that modeled the height versus time. This particular situation requires the students' attention to four co-varying quantities, but issues of co-variation are also apparent in dealing with situations that only have single motion. Students in that case still must construct functions to model the sinusoidal nature of the phenomenon, and they will also have to construct functions that model the rate of change of those functions. Furthermore, they need to be able to compose those two functions to adequately model the phenomenon.

As for when and where issues of angular velocity were important, conversations about angular velocity arose as students engaged with their first inscription of the ride, their construction of the height versus time graph and construction of the function. In dealing with the initial inscription, angular velocity was a notion that was believed to have importance, but how it was important was still uncertain. As the students constructed the height versus time graph the angular velocity was implicit in their construction and interpretation of scale, but as for the quality and shape of the graph, its role was yet to be worked out. That working out took place during the construction of the function for the height versus time. This makes some sense if we consider where the issue of the angle and the change in the angle arise in trigonometry. The relationship between the change in the angle and the change in the height is explicit on the unit circle. If I imagine moving faster or slower along the edge of the circle or changing my angle in relationship to the center of the circle at a faster or slower rate, then I can also imagine that the rate at which I get higher or lower goes at a faster or slower rate as well. However, changes in radian measure are static points on a height verses time graph and in this case were used to fit points onto the graph. The students did not need to consider the rate at which each wheel was changing to get a sense of what the graph would look like. Only after they finished the graph did they need to work out how the angular velocity related to shape of the graph. This highlights the need to make explicit relationships between rate of change on the unit circle inscriptions and the rate of change on the height versus time graph. Understanding that shifts in the steepness of the curves of the graph represent increases in the rate of change of the angle on the unit circle allows students to negotiate between the two inscriptions and possibly relate them to phenomena.

Finally, in constructing the function for Sandra's height versus time, Andrew and Oscar needed to consider angular movement as a function. By the time they needed to construct the function, the students had already laid the groundwork for their understanding. But their work highlights the need for constructing meaning for that function. Understanding the function entails not just being able to construct it from a graph or even the phenomena, but it also means understanding how the angular movement function you are constructing contributes to the change in the height versus the change in the time.

Andrew and Oscar's mathematizing of the double Ferris wheel illustrates the role that function composition and covariation play in modeling situations with periodic motion. It also highlights the complex relationships that are implicit and explicit in trigonometric functions and inscriptions. When modeling these kinds of situations, teachers need to aid students in parsing out, understanding and mathematizing the multiple relationships that are implicit or explicit in any situation. I mention mathematizing with parsing and understanding because for Oscar and Andrew the act of mathematizing the phenomenon contributed significantly to their understanding of angular velocity. Engagement with constructing the function served as the ground for their insights into the co-variation of the rates of the two wheels and their understanding of the composition of the two functions. Furthermore, to gain a full understanding of the angular velocity in this situation, the students needed to contend with each of their representations. This underscores the role that situating the mathematics plays in students understanding of certain mathematical concepts.

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