Instructor Responses to Prior Knowledge Errors Made by Calculus Students

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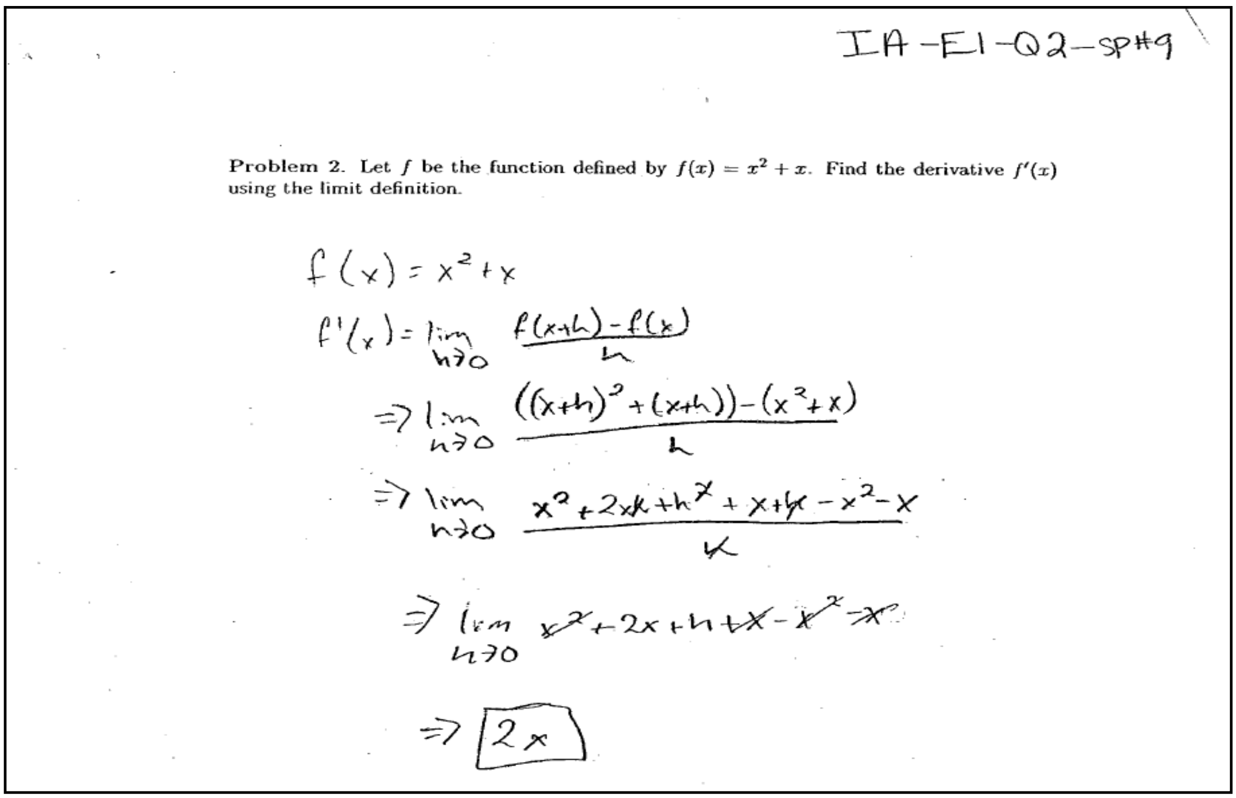
Introduction and Background

As mathematics educators many of us have encountered the following type of calculus error that, though frustrating, is quite intriguing and may prompt us to think carefully about the type of errors that our students make.



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Problem 2. Let f be the function defined by $f(x) = x^2 + x$. Find the derivative $f'(x)$ using the limit definition.

$f(x) = x^2 + x$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\Rightarrow \lim_{h \to 0} \frac{((x+h)^2 + (x+h)) - (x^2 + x)}{h}$

$\Rightarrow \lim_{h \to 0} \frac{x^2 + 2x\cancel{h} + h^{\cancel{2}} + x + \cancel{h} - x^2 - x}{\cancel{h}}$

$\Rightarrow \lim_{h \to 0} \cancel{x^2} + 2x + h + \cancel{x} - \cancel{x^2} - \cancel{x}$

$\Rightarrow \boxed{2x}$

The above example of a calculus student's work illustrates one of the many errors I have witnessed as a graduate teaching assistant. These mistakes gave motivation to the current study of prior knowledge errors. *Prior knowledge* is operationalized as any skills and/or understandings that students must possess to successfully complete a Calculus I course. As seen above, it seems that students can often comprehend calculus and carry out the common processes learned in a calculus course; however, they may not have overcome their deficiencies in prior knowledge. This creates a unique consideration for instructors who much assess the abilities of calculus students. One goal of this study is to better understand the views that calculus instructors have about student prior knowledge errors. Additionally, the aim is to uncover how those views influence the assessment techniques that calculus instructors apply to class assignments and exams.

In contrast to the motivation of this study, White and Mitchelmore (1996) aimed to investigate calculus students who possessed *instrumental understanding*. Instrumental understanding refers to procedural knowledge unsupported by conceptual knowledge. Interestingly, they found that among the calculus students they studied, instrumental understanding was largely the result of "an underdeveloped concept of a variable (p. 91)", a concept many would deem as prior knowledge. Though inquiries into prior knowledge are plenty within the field of undergraduate mathematics education (Cox, 2000; Hailikari, 2008; Trigueros, 2003), the current study will open a new door to begin exploring understanding that is characterized by conceptual knowledge unsupported by procedural knowledge. More specifically, how instructors respond to such understanding is in question. To achieve the goals of this study the following research questions will be addressed:

1. How do Calculus I instructors define prior knowledge?
2. How do Calculus I instructors view prior knowledge in the context of a calculus course?
3. How do Calculus I instructors grade prior knowledge errors?

Methods

This exploratory study was conducted in two parts. Part I consisted of student exams and instructor interviews collected during the summer of 2008. A total of 31 students from two

Calculus I classes agreed to allow copies of their three class exams to be analyzed throughout the semester. The two instructors were individually interviewed following each exam. The interview protocol consisted of two sections. Section I of the interviews included open ended questions aimed at determining the participants' beliefs about their role as an instructor, the importance of prior knowledge within the context of a calculus course, their definition of prior knowledge, and what grading techniques they consider when they encounter prior knowledge errors. A task-based approach was used to conduct Section II of each interview. In this section the instructor was asked to answer the following about each of the exam questions:

1. What work/explanations were expected of the students on each question?
2. Have you done similar questions in class/review?
3. How did you decide what point value to assign to each portion of each question?
4. What were the most common errors for this problem?
    a. How would you classify these errors: prior knowledge or calculus?

In addition to the above questions, examples of student errors were presented to the instructor to allow them to expound on the type of errors they identified and how they decided to allot points.

As the goal of this study is to understand instructors, it was important to extend the study. Thus, Part II of the study consisted of five instructor interviews during the fall of 2008. These instructors were all tenured faculty members who taught the Calculus I course within the last five years. The interview protocol was very similar to that of the Part I interviews with minor changes made to Section I based on the responses from Part I participants. Section II was task-based; however, instead of discussing their own exams the instructors were asked to review several examples of student work chosen from the Part I student exams. These examples fit into the following categories of question type: Finding the derivative using the limit definition, finding intervals of continuity, finding the derivative using rules, finding tangent lines, applying the process of implicit differentiation, solving related rates problems, and solving optimization problems. These types were chosen because they each reflected questions that were common among most University of Oklahoma Calculus I exams. The set of specific examples that were chosen included a range of errors from prior knowledge to calculus to simple arithmetic mistakes. For each of the 22 student error examples presented, the instructors were asked to address the following questions:

1. If possible, identify any errors.
2. Classify the error as a calculus error or prior knowledge error.
    a) Classify, if possible, the type of prior knowledge error.
3. How would you score this question given the stated point value?

Emerging Themes

Open coding of the instructor responses during Part II interviews yielded some preliminary findings concerning the first two research questions. Through initial review of expanded field notes, the responses identify algebra, trigonometry, and an understanding of functions as necessary prior knowledge to succeed in a calculus course. Preliminary analysis also gave way to several emerging themes that speak to the second research question. A description of selected themes along with an illustrative quote from the instructor interviews follows.

<u>Ideal vs. Reality of Student Preparedness</u>

The participant's ideal class would consist of students who have all the necessary prior knowledge mastered. However, they have found that time must be spent on reviewing prior knowledge material which takes away from the amount and depth of calculus concepts they are able to cover.

"In an ideal world your students are very well prepared and you spend your time focusing on the concepts…What happens in practice is, students learn a lot of basic algebra in a calculus course…They are supposed to know this stuff beforehand but that's often not the case. "

~Instructor E

Understanding Without Performance

Students can understand some of the concepts of calculus without being able to complete a problem because of lack of prior knowledge skills such as algebra or trigonometry.

"…ideally they should be comfortable… doing basic algebra…[often] it ends up being algebra that's the reason they get a problem wrong and not so much a misunderstanding of calculus."

~Instructor C

Understanding Requires Performance

To develop a comfortable, fluent, and/or deep understanding of calculus, the algebraic processes used in calculus must be mastered.

"Deep understanding requires comfort with the algebra and an ability to work lots of examples…students will lose the concepts without lots of practice."

~Instructor F

Comparison of Instructor and Student Educational Background

Instructors thought of how their educational background differed from that of their students when trying to find reason in student prior knowledge deficiencies, student overall preparedness, and student work habits.

" I am always sensitive to the fact that I shouldn't come off like 'well in my time it was like this but in your time…'. We all struggle through these things. But what is clear to me is that these students simply do not practice in high school or in college. I give them literally six or seven

homework problems and they complain that it is too much work. And I'm saying this without exaggeration. I use to get 60 to 70 problems a week. Sometimes 100. I'm not saying that's what they should be doing – no I take that back I am saying that's what they should be doing…They just don't drill enough."

~Instructor F

The above themes say a great deal about the feelings that instructors have about their students and the prior knowledge skills their students possess or lack. To begin addressing the last research question, the task-based portions of the interviews are currently being analyzed. Using the theoretical framework of sensible systems (Leatham, 2006), the instructors' comments that developed the above themes will be compared to their actions when presented with actual student work.

Implications

Though still in progress, the results of this research are likely to be useful to instructors as well as administrators. Teachers who understand how their practices, specifically in terms of assessment, are influenced by their views have a unique opportunity for reflection. In turn, necessary adjustments in assessment policies can be made to fit within their belief systems. Administrators often have little awareness of instructor assessment techniques within a college or university setting. After gaining insight about instructor beliefs and policies, they may find it necessary to provide professional development to instructors whose views are out of sync with department goals for student performance.

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