CONTRIBUTED RESEARCH REPORT

Student Motivations for Mathematical Understanding in an Inquiry-Based Calculus Classroom

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Student motivation has long been a concern of mathematics educators. We present Contextualized Motivation Theory (CMT) as a means for understanding the complexities of student motivations in an inquiry-based university honors calculus class. This qualitative, grounded theory study is part of a longitudinal project in calculus learning and teaching. Here, we characterize motivation, defined as an individual's desire to act in particular ways, through analysis of students' extended, collaborative problem solving efforts. Students persisted beyond obtaining correct answers to build understandings of mathematical ideas. Analysis of extended student collaborations suggests a supporting "web" of motivations, existing simultaneously, from which a learner chooses to act upon at any given time. Students chose to act upon various intellectual-mathematical motives and personal-social-emotional motives. CMT positions personal agency as central, characterizes the social nature of motivation, and encompasses conceptually driven conditions that foster student engagement in mathematics learning.

Introduction

Teachers' perceptions about student motivation have been shown to influence classroom activities and lesson plans (Middleton, 1995). Based on these findings, Middleton (1995) suggested that teachers and teacher educators need to better understand student motivation in learning mathematics. The National Council of Teachers of Mathematics (NCTM) has also recognized the desire and need for more information about student motivation in the mathematics classroom. In 2011, the NCTM yearbook will comprise manuscripts that explore motivation as an important element in student learning.

In this paper, we present a qualitative analysis of student motivation based on data collected from a university experimental calculus class. Specifically, our purpose is to build theory illuminating some of the complexities of motivation that are manifest through students' mathematical actions.

Theoretical Perspective

Motivation may be defined as an individual's desire to act in particular ways (Weiner, 1992). Our perspective on motivation in learning is grounded in the exercise of personal agency (Walter & Gerson, 2007) wherein the learner purposefully chooses to act upon some motives and not upon other motives. This contrasts with a perspective that neglects choice and suggests that motives completely determine our actions (Deckers, 2001).

Literature

Middleton and Spanias (1999) reported some deficiencies in extant research literature on motivation. First, motivation is sometimes reported as an ancillary affect in studies designed to examine other factors. For example, Francisco (2005) conducted clinical interviews with five students and discovered sources of motivation in mathematics as an additional insight. Second, most studies have "used self-report measures as indices of motivation without actually looking at and listening to children who are engaged in mathematical activity" (Middleton & Spanias, 1999, p. 83). Sullivan, Tobias, and McDonough (2006) claimed motivation "may be as much a product of group or cultural factors as individual goals" (p. 91), but their conclusions were based in student self-reports and were distanced from an analysis of students' mathematical work in classrooms. Third, motivation research studies tend to be atheoretical. Finally, the majority of research data are gathered under models of mathematics instruction that are not driven by students' conceptual development (Middleton & Spanias, 1999). Recent work partially addresses these deficiencies. Noting that "motivation has not been a popular topic of study lately" (p. 165), Hannula (2006) examined needs, goals, and emotion in relation to motivation. He suggested that "emotions are the most direct link to motivation" (p. 167). To further address noted deficiencies and to expand our perceptions, more research is needed to paint a clearer picture of student motivation in the mathematics classroom.

Research Questions

What insights can we gain about the nuances of student motivation by studying the actions of students as they engage in mathematics problem solving in an inquiry-based calculus classroom? More specifically, what motives do students have for understanding mathematics?

Research Methods

A grounded theory approach (Corbin & Strauss, 2008) is appropriate for building contextual theory about motivation in learning through analysis of video data of students' mathematical problem-solving. Thus, self-report is not the only index of motivation. This qualitative study is based on data collected from a teaching experiment at a large private university in the western United States. Two mathematics education professors designed and taught experimental honors calculus courses for three semesters. Classes were task-based and conceptually driven. Each class session was videotaped and the videos were transcribed. In vivo open and focused coding (Charmaz, 2006) of selected episodes supported the development of Contextualized Motivation Theory (CMT). The analysis presented here is primarily based on video data collected at the beginning of one semester while students were working on the Cat Task (Speiser & Walter, 1994). The Cat Task asks students to find how fast a cat is moving at two different instants given a series of photographs taken at intervals of 0.031 seconds. The task was chosen to elicit the need for derivative as a mathematical tool in problem-solving and to prompt students' conceptual development of the derivative without teacher lecture.

Data and Analysis

The transcript excerpt presented here is from a focus-group episode on the first day students worked on the Cat Task. Prior to this, students measured the distance the cat moved between frames and graphed a displacement-time graph using graphing calculators. Students discussed the possibility of using the derivative function on the calculator to find the instantaneous velocity of the cat at each point. However, at this time, students were unsure about their understanding of derivative. Referring to the definition of derivative in the book, Daniel states that he doesn't fully understand derivative or the associated notations (26:44).

Time	Speaker	Transcript	Open and Focused Codes
26:44	Daniel	I still don't know the derivative of "Q" though or what "Q" is. [Looks at p. 76 in book—recreated below] $Q'(a) = \lim_{b \to a} \frac{Q(b) - Q(a)}{b - a}$	-Temporally extended desire to know -Poses problem -Derivative -Book notation
26:49	Riley	"Q" is the quantity of the function. [Quietly, matter-of-fact]	-Response - Reads book
26:52	Andrew	Thank-you [chuckles]…what does that *mean*? [Garbled words]…we're trying to figure out what the book's talking about. 'Cuz I mean, we already know how to get the average, and it took us like ten minutes to figure out what he [book author] was talking about.	-Desires meaning for book language and notation -Question for understanding -Tries to "figure out" -"Ten minutes", persistence -Book notation -"Already know" -Average [rate of change]
27:09	Justin	To find out we already *knew* what he was talking about.	-We, social -"Already knew" average
27:13	Andrew	Huuh—that's horrible. [Laughs, a moment's silence]…ok, I'll try an' figure out what he's talking about.	-Laughs, emotion -Tries to figure out book definition of derivative -persist -extend understanding

According to Andrew (26:52), their group determined how to find an average rate of change in the motion of the cat, before they spent "ten minutes" trying to "figure out" that the notation in the book was "talking about" the "same" average rate of change. Next, they endeavored to make sense of the book's definition of derivative (27:13). Later, when talking to the instructor, Daniel stated that they thought the equation in the book could help them be precise, to "understand exactly, like how to get the instantaneous velocity" (30:25) of the cat. The group decided they had already found an answer to each question, but were "trying to think how much farther [they] could go" (35:58). Notice that the students were doing mathematics as a problem solving enterprise before looking at the book for information, solution approaches or notations that characterized their own mathematical work.

In this short episode, we see confirming evidence that students choose to exhibit "intellectual passion" (Polanyi, 1958/1974, p. 142) in pursuit of clarity, meaning, and coherence in their mathematical work. Complete analysis of extended student collaborations over several days grounded the development of CMT which suggests that these students' motivations may be characterized by a "web" of intellectual-mathematical motives and social-personal-emotional motives. This "web" comprises motives that exist simultaneously and in varying degrees of importance from which a learner chooses to act at any given time. Intellectual-mathematical motives that emerged as important to these students included: seeking for meaning and making sense of mathematical problems, resolving inconsistencies in mathematical ideas among group members and building shared meaning, desiring to know and extending understanding, responding to challenges or self-posed problems, persistence in satisfying curiosity and discovering new truths, and seeking perfection and precision. Social-personal-emotional motives included: feeling good about achievements, saving embarrassment, lessening frustration, enhancing self-efficacy, and emulating the use of agency by others.

CMT, as a perspective on students' motivations for understanding mathematics, positions personal agency as central, characterizes the social nature of motivation, and encompasses conceptually driven conditions that foster student engagement in mathematics learning with intellectual passion. Implications of CMT for building best practices in mathematics classrooms depend on teachers, teacher educators, and researchers recognizing the productive role that student personal agency plays in enriching motivations upon which students might choose to act for building meaningful understandings of mathematics.

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