Building Classroom Mathematical Knowledge: Extending a framework for the co-construction of knowledge to the classroom context

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Abstract

Discussions are central to most instructional forms, particularly in learner-centered classrooms. The nature of student involvement and quality of instruction may be revealed through analysis of classroom discussion; however, student involvement also complicates discussion structure and discourse analysis. While studying unfacilitated discussions among teachers in a professional development program, Belnap and Withers (2008) developed a framework describing how individual contributions construct a discussions content. I plan to extend this framework to the classroom context, in order to reveal the nature of student involvement in classroom discourse.

This discussion will focus on preparing for such a study. Questions include: What critical differences are there between professional development and classroom contexts? How should these be addressed in study design? and What literature may inform this effort?

Many contemporary views of mathematics (Davis, 1992) and learning, such as constructivism (Cobb, 1994; Cobb & Yackel, 1996; Ernest, 1996; Sfard, 1998; Zevenbergen, 1996), the social perspective (Cobb, 1994; Cobb & Yackel, 1996; Lerman, 1998, 2000; Sfard, 1998), socioconstructivism (Lerman, 1998, 2000; Cobb & Yackel, 1996; Cobb, Jaworski, & Presmeg, 1996), and agency (Walter & Gerson, 2007) advocate learner-centered instruction. Consequently many modern instructional contexts require students to take an active role in learning by engaging in educational tasks, participating in the construction of knowledge, and contributing to the discourse of the learning environment in substantial ways.

In a learner-centered mathematics class students are active participants in doing mathematical activity: making and testing conjectures; solving problems; collaborating with their peers; and presenting ideas, proofs, and solutions. All of these activities require active student participation in classroom discourse.

Determining the effectiveness of an interactive class or the degree to which a class is "student-centered" is challenging. Cursory observations of superficial activity (i.e. whether students are working in groups, taking notes, or answering questions) may not reveal the quality of engagement or learning.

Consider the impact that task selection has on students working in a group. A group working a large number of routine and repetitive problems may learn only algorithmic steps or basic skills, whereas a group working on an open-ended project may connect multiple ideas and apply them in a novel situations.

Consider the impact that question selection has on the quality of an interactive lecture. Contrast what students gain when they are asked questions that only require students to recall a definition or give the next step in an algebraic procedure, with what they gain when asked questions requiring them to recognize patterns, make a conjecture, or justify their choices or reasoning.

In both of these examples, recognizing the quality of the discourse goes beyond a

superficial view of the instructional form. It's not enough to simply know that the students were working in groups; we need to know what they were working on and what type of involvement the task elicited. It's not enough to simply know that a teacher lectured or even that students were answering questions; we need to understand the nature of those questions and the types of students' responses. Determining the quality of discourse involves answering deeper questions: a) what are the students' roles and degree of involvement in classroom discourse, and b) what is the nature and significance of their contributions. Toward this end, the purpose of this paper is threefold. I will first describe an analytical framework developed initially to study discussion among practicing teachers; next, I will illustrate some of its uses in examining the content development of a discussion; and finally, I will discuss ways to extend this framework to the classroom setting.

Theoretical Framework

Discourse

I view discourse as the mutual construction of both individual and social knowledge. As a social activity, discourse is shaped by participants' interactions. Participants exercise agency, making willful choices and actions which develop both social norms and social knowledge. On the other hand, individuals construct their own knowledge from their participation in the discourse; opportunities for participation are provided or constrained by the norms, practices, experiences, and information taking place during the discourse. Thus each individual's knowledge and the social discourse are mutually altered through social activity.

From my perspective, discourse is a social activity based on (but not limited to) discussion; it involves not only spoken dialogue, but all surrounding activity, such as written and non-verbal communication, physical actions, and tasks. Discussion is a central activity to discourse, shaped by individual contributions. While not all-encompassing, discussion is

central to discourse in most learning environments, serving as a means of both communicating information and constructing new ideas or understanding (TruxawnD:2007,Wells:1996). The text of a discussion (i.e. the content and meaning of spoken dialogue) can be seen: (a) as a product of the discourse; (b) a record of individual involvement in the discourse; (c) a form of social knowledge constructed by individual contributions; and (d) a pool of available knowledge or information from which participants can construct their individual knowledge.

The nature of student involvement and quality of instruction is partially encoded in a discussion's text. So, understanding how an individual student contributes to a discussion can help us understand the nature, depth, and quality of the student's involvement in the learning process. To ascertain this, we can ask questions about the discussion's text, such as: How is the learner contributing to the discussion? What is the nature of his/her contributions? What role is he/she playing in the discussion? What significance and impact do his/her contributions have on the developing content?

Instructors who do *not* follow a transmission (telling) model of instruction may involve students in various ways, such as: drawing them into whole class discussion; having them present, teach, or explain at the board; or dividing the class into micro-discussions (e.g. group work). In doing so, the teacher relinquishes control over various conversational aspects: topics discussed, ideas presented, organization of ideas, logical progression, or who contributes and when. In this setting, conversations become structurally complex and difficult to analyze.

I faced this challenge while studying discussions among participants in a professional development program (Belnap & Withers, 2008), the Video Observations with Peer-feedback Sessions (VOPS) program. Participants in this program were graduate student instructors with varied teaching experiences, teaching assignments, and backgrounds. All were pursuing a masters in Mathematics Education. As part of the study, students met weekly for 50-

minute sessions, each consisting of 30-40 minutes of discussion, catalyzed by watching a 15-20 minute video clip of one of the participants teaching an actual class.

As we studied the resulting discussion texts, we observed that these unfacilitated (or monitored) conversations would often develop in very complex and non-linear ways. Topics could arise, shift, or reemerge at anytime, even subtly. Furthermore, the development of different ideas could overlap or happen simultaneously.

To address these problems, we developed an analytical framework utilizing the techniques of Grounded Theory (Strauss & Corbin, 1998) along with independent coding and negotiation to develop, conceptualize, define, and refine a framework describing both how the content of a conversation develops and how individual contributions construct it.

The Co-construction of Knowledge: An Analytical Framework

The analytical framework provides a view of how individual contributions link to build the content of a discussion. Table 1 provides a textual segment of a conversation between three graduate student instructors, Lyndsay, Maud, and Sarah, taken from one session, VOPS 9 (30 Nov 2006). I will utilize this as a context for both discussing the framework and defining important terminology.

The analytical framework describes the active and interactive process through which a discussion's content develops. Content develops as individuals take *turns* speaking. The statements an individual makes in one turn can be parsed into segments whose meaning (content) has one or more functions, describing how it affects the growing content; we call these segments *moves*.

The textual segment in table 1 consists of 19 different speaking turns. With her first turn speaking, Maud makes two moves; first, she makes a partial statement regarding her need to find more applications for conic sections (M216); and then she breaks off from this to make a separate comment (M217), mentioning a problem that she encountered in

Table 1: Conversational Moves for Fibers 39—41 from VOPS9 (30 Nov 2006). * M229a and M229b happened simultaneously.

Mv/Code/Target	Speaker	Text
216, Inform	Maud	I'll have to research more applications on–
217, Initiate	Maud	Oh, and guys, remember that article we read a while back, in 661? And it– I don't remember much of the article, just this one thing where they had 1^2 =–or $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, anyway, I used that yesterday to teach them about mathematical induction. We proved that the sum of n odd integers equals, um, n^2. Now, I don't know if they were excited about it– well, the first class was excited about it, the second class was pretty bored.
218, Initiate	Lyndsay	I don't know how you showed it, but you can show that–
219, Extend 217	Maud	I actually did it on graph paper. I'll show you– (Removes a notebook and starts flipping through pages)
220, Continue 218	Lyndsay	Just with like, squares? Growing squares?
221, Req.Clarify 220	Maud	(While still flipping pages) Wait, doing what?
222, Continue 219	Maud	On graph paper (shows a drawing/diagram from her notebook)
223, Req.Clarify 220	Sarah	(Speaking to Lyndsay) How do you explain the difference?
224, Clarify 220	Lyndsay	Oh, you just make it squares.
225, Respond 224	All	(chuckling)
226, Extend 224	Sarah	Oh, and then you added the, uh–
227, Invalidate 226	Lyndsay	And I didn't get to that conclusion that you could add them up, but I was–
228, Extend 224	Lyndsay	Cuz the difference between squares is odd numbers, in grey (pulls out a book and searches for a diagram; then she shows everyone the diagram)
229a, Confirm 228	*Sarah	Nice.
229b, Respond 228	*Maud	Oh.
230, Req.Clarify 228	Maud	Wait, how do you (Lyndsay gives Sarah takes the book to examine the diagram) get the orange one?–
231, Clarify 228	Sarah	Oh, they lie on top of each other. Oh–
232, Confirm 231	Lyndsay	Yeah.
233, Continue 231	Sarah	–ok, got it. (Shows and hands it to Liz.) I was trying to think about like a 1, not 2 dimensional one.
234, Extend 227	Lyndsay	I was just looking through that last night. But I didn't quite derive any conclusions. (long pause)
235, Respond 228	Maud	(Pauses while Liz hands book to Maud, who examines the diagram for a few seconds and then nods with realization)
236, Qualify 219	Lyndsay	So, you know, graph paper would probably work just as fine. Depending on the student.
237, Confirm 228	Maud	Anyway, that's cool.
238, Extend 217	Maud	Cuz we did like one, and then one around it– so there was one and then one, two, three. So, three plus one adds up to four, then you add five around, and then all ...

a research article and used when teaching a class the day before. Even though M216 and M217 are both given during the same turn speaking, M217 is a separate move, because it presents new content, unrelated to that presented in M216.

Each move has two main characteristics: its content and the content's function(s). A move's *content* refers to the meaning and ideas contained in the actual spoken text. A move's *function* describes its content's role in shaping the growing text, such as how it may change existing content.

Moves M216, M217, and Lyndsay's M218 all have a similar function. They each add new content to the conversation, introducing new ideas (not previously discussed). By contrast, Maud's move in M219 serves a slightly different function, altering the content of M217, extending it by providing more information.

In general, a move's function has two main dimensions: an *action* and a *target*. The *target* is the move receiving the action. For M219, the action is altering existing content and its target is M217. Most targets precede the moves acting upon them (e.g. M217 precedes M219); certain functions, however, may have no target (e.g. M216, M217, and M218) or may even anticipate a target (e.g. questions, which request content, which can be considered an anticipated target).

A function's action describes how a move's content affects a target. M219 *alters* the content of M217 (extending it); M221 *requests* content (requesting clarification of M220); and M227 *invalidates* the claim started in M226. Overall, we identified five essential content-altering actions: a) adding new content, b) validating content, c) altering existing content, d) requesting content, and e) re-voicing content.

Distinguishing moves by function, we identified 13 different move (or function) categories: *initiate* (*initiation*), *inform* (*information*), *substantiate* (*substantiation*), *assert* (*assertion*), *extend* (*extension*), *modify* (*modification*), *clarify* (*clarification*), *request* (*request*), *incomplete* (*incompletion*), *terminate* (*termination*), *respond* (*response*), *restate* (*restate-*

ment), and *continue* (*continuation*). We named each category with both a noun and verb, emphasizing the close relationship between the content and its contextual action; using these names, we can both talk about the move as an object and describe its active function. Two of these (substantiate and assert) have three sub-categories each, discussed later. Contrasting the characteristics of these 13 categories with their role in the discussion led us to cluster them into five themes or groups, according to their action type: *Anchoring* moves, *Valuing* moves, *Altering* moves, *Soliciting* moves, and *Contentless* moves.

Anchoring Moves.

Moves M216, M217, and M218 are examples of *anchoring moves*. Anchoring moves provide the initial content off of which discussion builds, essentially adding novel content to the discussion. They are critical points in the conversation, serving as *potential* launching points for the conversation. in this role, they usually have no target. M218 is an anchoring move, because even though Lyndsay is talking to Maud (socially responding to M217) about a similar topic, she introduces a separate approach to proving Maud's stated conjecture, discussing her own experience the night before. Since M217 was about Maud's use of the problem, M218 is a new anchoring move, related to but not building on M217.

Anchoring moves consist of: *initiate* (*initiation*) and *inform* (*information*) moves. Some anchoring moves, like M217 and M218, are picked up in the conversation, initiating discussion; this idea, *conversational uptake*, is what discriminates different anchoring moves. If content building off of an anchoring move includes a significant move by a different speaker, we call the anchoring move an *initiate* move, because it initiated significant discussion. Otherwise, it serves only to convey information, and so we call it an *inform*–I will discuss significance of moves in the Data Analysis section.

In our example, M216 never leads to significant discussion, so it is coded as *information*; on the other hand, both M217 and M218 are developed by at least one person other

than the initiator, making each an *initiation*—Note that M236 by Lyndsay targets M219, which targets M217 by Maud; and M222 by Maud targets M220, a continuation of M218 by Lyndsay.

Valuing Moves.

Valuing moves address the value, validity, or correctness of existing contributions. While they may provide content, they are more focused on assessing, supporting, refuting, or otherwise dealing with the credibility of prior contributions.

M227, M229a, M232, M236, and M237 exemplify this. As Sarah starts to draw a conclusion (in M226) about what she thinks Lyndsay did in her class, Lyndsay *invalidates* it in M227. The information she presents in M227 is content refuting Sarah's conclusion of, "then you added..." M229a (by Sarah) and M232 (by Lyndsay) both confirm the content of their targets (M228 and M231 respectively), making them valuing moves, even thought they provide no supporting content. M237 shows a positive valuation of the idea Lyndsay discussed in M228, also without any additional content. M236 deals with the credibility of M219. Referencing the approach Maud mentions in M219, Lyndsay acknowledges that Maud's approach *may* work, "Depending on the student." This comment, although not overtly discrediting, puts limitations on the applicability of Maud's comment, affecting its credibility; M236 *qualifies* M219.

Valuing moves cluster into subcategories along two main dimensions: *synthesis* and *effect*, as shown in table 2. Their *effect* on the target's content can be: a) overtly reinforcing, positively affirming the content (e.g. M229a, M232, and M237); b) discrediting, negatively impacting the content (e.g. M227); or c) limiting, providing limitations on the content without overtly reinforcing or discrediting it (e.g. M236).

Teh presence (or absence) of some form of synthesis or significant content (e.g. logic, reasoning, evidence, or examples) supporting the effect, further divides valuing moves. *Sub-*

stantiate (*substantiation*) moves provide this content support (e.g. M227 and M236), while *assert* (*assertion*) moves do not (e.g. M229a, M232, and M237).

Table 2: Subcategorization and Dimensions of Most Valuing Moves

(Sub)Category Names	Synthesis Provided	Effect on Target
Substantiate (Substantiation)	Yes	Various
Justify (Justification)	Yes	Reinforcing (+)
Qualify (Qualification)	Yes	Limiting (L)
Invalidate (Invalidation)	Yes	Discrediting (−)
Assert (Assertion)	No	Various
Confirm (Confirmation)	No	Reinforcing (+)
Vacillate (Vacillation)	No	Limiting (L)
Deny (Denial)	No	Discrediting (−)

Altering Moves.

Altering moves are content-developing moves. Their primary function is to develop the content of existing contributions by adding to or modifying their targets' content. Altering moves include: *extend* (*extension*), *modify* (*modification*), and *clarify* (*clarification*) moves. *Extensions* add new dimensions, ideas, or detail to the target, which can be seen as an extension of the target's content without a change in function. Like extensions, *modifications* serve the same function as their target, but make significant changes or replacements to the target's content; in a sense, modifications edit their target in a way significant to the content. By contrast, *clarifications* exemplify or clarify their target, by providing detail, illustration, explanation, or instances (examples).

Our example text provides examples of both extensions and clarifications. In M218 and M220, Lyndsay mentioned a way of proving a conjecture, but it was unclear to Sarah; at her request (in M223), Lyndsay added some details to *clarify*, making M224 a clarification of M220. In M226, Sarah starts adding more details to what Lyndsay did in the approach in M224, details that serve the same purpose as M224 (i.e. to clarify M220), so M226 *extends* M224. Then, in M228 Lyndsay further clarifies the approach she used in class, showing a

textual illustration; so M228 *extends* M224. Finally, in M231, Sarah adds details to *clarify* M228.

All altering moves describe substantive change to the conversation's content; so they do not include statements that simply restate, reword, or paraphrase prior content, unless it significantly changes the current meaning of the content in one of the ways described.

Requesting Moves.

Request (*request*) moves are solicitations for contributions. They may be posed as questions (e.g. M221, M223, and M230), but need not be; for example, a teacher saying "Tell us more," or "Please explain," following a student's comment would be a request.

Among move types, requests are unique because they overtly anticipate content that is not yet provided (and which may not ever *be* provided). M223 and M230 are immediately answered by M224 and M231 (respectively), while M221 was not.

Based on conversational norms, the anticipated target is usually quite specific, as requests often specify (by their wording) the type of response desired; this includes specifying the type of content, the target for that content, and the essential action of the responding move. For example, Maud asks a question in M221, soliciting information; the most appropriate type of response to this question would be information clarifying what Lyndsay was describing in M220; and so M221 *requests* a *clarify* move for M220. Similarly, M223 *requests* a *clarify* move for M220.

Requests solicit, but do not control the type of response actually given; a response or desired response might not ever *be* provided, which could be problematic for mapping the conversation. To avoid this, we code requests by the type of move they solicit (i.e. by the most appropriate type of response, not the elicited response); consequently, when linking contributions, we consider the target of the request to be the target of the anticipated response. We consider the target of M223 to be M220 because M223 tries to elicit content

targeting M220; it attempts to build onto M220.

Contentless Moves.

Contentless moves are moves counted as if they did not have a role in developing the discussion's content. Contentless moves include moves that do not directly develop a discussion's content, but also include those that do, if they need to be considered contentless for various reasons (e.g. to not overrepresent the number of certain contributions in a discussion). Contentless moves include: *incomplete* (*incompletion*), *terminate* (*termination*), *respond* (*response*), *restate* (*restatement*), and *continue* (*continuation*).

The first of these are contentless because they do not provide or shape the discussion's content, even though they can play important roles in a discussion. *Incompletions* are incomplete contributions; they are either too short to determine their function, or do not sufficiently complete their action to warrant coding it otherwise. A *termination* is a move whose sole purpose is to explicitly terminate the content development of some developing strand of the conversation. *Responses* are statements that simply express agreement, acceptance, or acknowledgment, but otherwise do not contribute to the discussion as the other categories describe (e.g. M229b); statements that do not clearly assert validity, support, or truth are identified by this category (instead of assertions) to avoid over representing valuing moves by including common social responses, such as, "Yeah."

Regarding our example text, the following examples are of note. First, M226 is *not* an incompletion because there was enough content that its function was clear, to extend M224. Second, a tone of realization in Sarah's voice (M231), followed by a definitive response by Lindsay in M232 made M232 a clear confirmation, instead of defaulting as a response.

Even though these next two categories may play important conversational roles and often contain important content, for research purposes, we include them as contentless moves because their impact on content development has already been performed or counted

with their targets. *Restatements* are re-voicings, which restate, reword, or paraphrase a prior contribution *without* significantly adding to or modifying the content; counting restatements as contentless avoids counting both it and its target as contributing important content to the discussion, when there is actually no significant difference in the content.

Continuations simply continue a prior contribution from the same contributor, without a change in function; these represent a natural continuation which accounts for interruption, disruption, or other discursive breaks. Considering continuations as contentless avoids over counting a speaker's single contribution by how many turns it may have taken to utter it.

Consider these examples, which contrast *continuations* and *extensions*. Although Maud is talking to Lyndsay in M219 (prompted by Lyndsay's comment in M218), Maud's comment is providing more information regarding what she said in M217; so M219 *extends* M217. From the video, it is clear that Maud's comment interrupted Lyndsay's contribution in M218, so M220 *continues* M218. Catching Lyndsay's comment in M220, Maud breaks off what she is saying in M219 to ask a question in M221. Immediately after posing this question, she finishes (in M222) what she started in M219, by sharing her graph paper diagram; so M222 *continues* M219; in essence, she interrupted herself.

Content Fibers: An Example

The coding of the example text in table 1 accomplishes two main things. First, it describes the role of each move's content. Second, it shows how the moves link together, with linkages specified by each move and its target.

Using this information, we can make a visual map of the conversation's content development, shown in figure 1. If we represent moves by labelled rectangles, we can show the progression of the discussion by arranging them chronologically from top to bottom. If we vertically align each under its target (so that its target is at the head of its column) then the

moves form descending chains, each representing the development of a single idea. We call these conversational *fibers*; a *fiber* is an anchoring move and all moves directly descending (resulting) from it. In our example, we see three conversational fibers, illustrated visually in figure 1: an undeveloped first, consisting of M217; a slightly more developed second, focusing on what Maud did in her class with the $1+3+5+\cdots+(2n+1)=n^2$ conjecture; and a well developed third, focusing on Lyndsay's approach to and work on solving the same conjecture.

Data Analysis

The framework describes both the content development of a discussion and how individuals' contributions construct the content; so, it allows us to address the questions we posed earlier, namely: How are learners contributing to the discussion? What is the nature of those contributions? What role are they playing in the discussion? What significance and impact do their contributions have on the developing content? As an illustration, consider each of these questions in the context of VOPS9 (30 Nov 2006).

In the analyses that follow, I consider moves *significant* if they seek to further develop the content of a discussion; these are moves directed at shaping, expanding, or changing the discussion's content. I consider significant moves to consist of: a) anchoring moves, b) substantiations, c) altering moves that target (and requests that solicit) either significant moves or the continuation/restatement of significant moves.

I argue that anchoring moves are significant in that they provide new content and open up possible venues for discussion. Substantiations are significant in that they access the validity, value, or perception of content and provide content or synthesis of content to support that position. Altering moves that target other significant moves shape, extend, or modify significant content in substantial ways. Finally, Requests for significant content show an attempt to develop the content in substantial ways, representing an important form

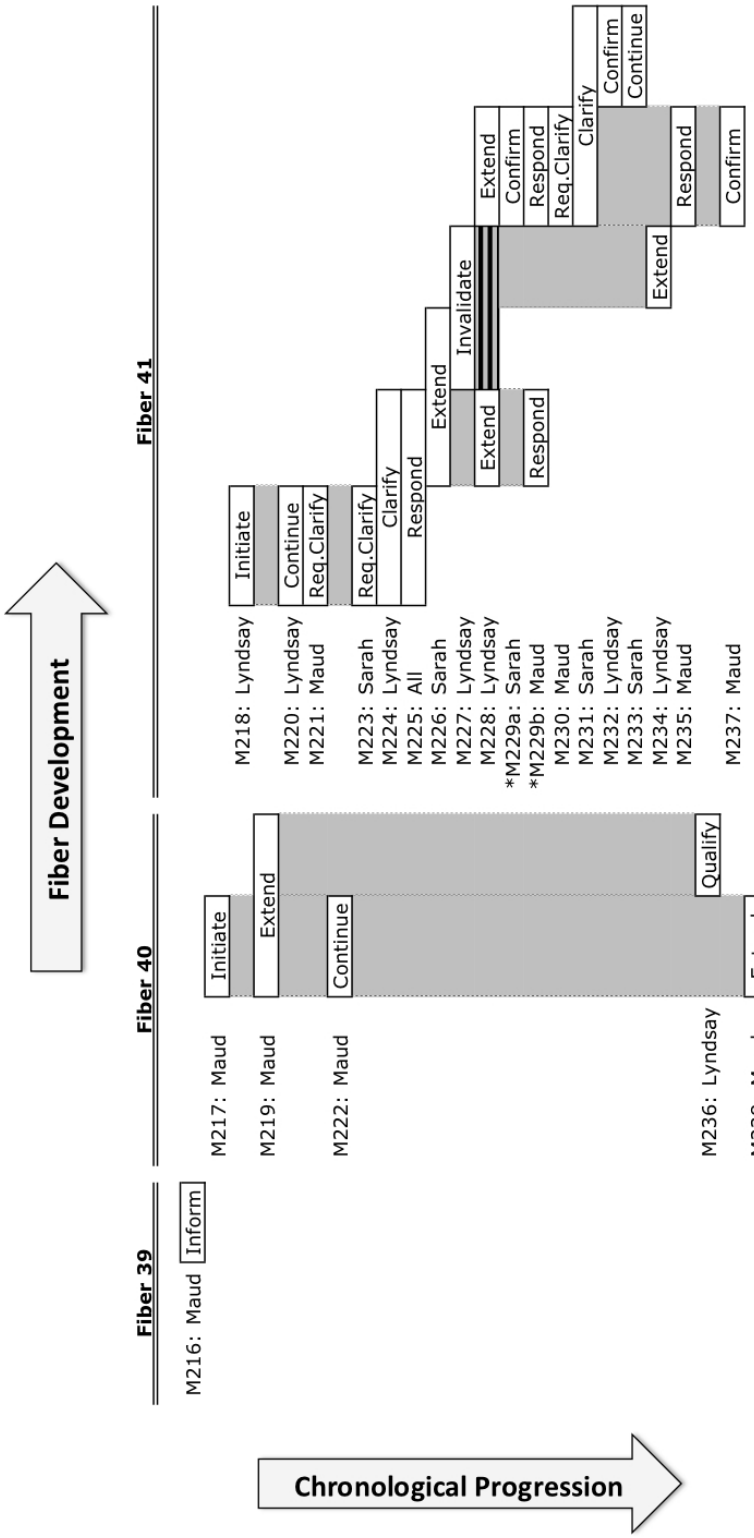

Figure 1. Chronological Content Development of Fibers 39—41 for VOPS9

of inquiry that (like anchoring moves) can open up possible venues for discussion.

How are learners contributing to the discussion?

We can get a general overview of participants' involvement in the discussion by looking at how many fibers they contributed to. Table 3 breaks down VOPS9 by its fibers, showing all fibers and (more specifically) significant fibers (i.e. fibers that are on-task for the session by discussing teaching-related issues or discussing the video viewed). In the appropriate rows, the table gives the number of fibers to which each participant gave at least one significant contribution or gave at least one contribution of any type.

Table 3: Number of fibers and significant fibers from VOPS9 (30 Nov 2006) to which participants made at east one contribution, by contribution type.

	Facil.	Liz	Lyndsay	Maud	Sarah
Fibers (N=44):					
Significant contributions	14	10	13	31	17
Any contributions	18	11	16	37	24
Significant Fibers (n=35):					
Significant contributions	11	10	9	25	13
Any contributions	13	11	12	30	19

Looking at this data, we get some general information about the participants' involvement. All participants were actively engaged in the discussion, each contributing to about one-third of the fibers (or more); they all made meaningful contributions to meaningful discussion fibers.

Among these participants, it is of note that the facilitator engaged in less than a third of significant fibers. This not only shows the facilitator surrendering conversational control, but indicates that participants took ownership in the conversation, directing most of the conversation.

Aside from the facilitator, the table suggests some important differences among the other participants' involvement. Maud contributed to almost all fibers. She made important

contributions to over 50% more fibers, as well as significant fibers, than anyone else; she contributed to over 85% of significant fibers and made relevant contributions to over 71% of the significant fibers. This suggests she may have taken a leading role in the session's discussion. By way of contrast, Liz and Lyndsay participated in a third or fewer significant fibers.

This analysis leads us to other important questions regarding the discourse. These include: Why is there such a drastic difference between Maud's and Liz's involvement? Is this due to established conversational norms, personality, background, or experience? Is it because Maud's clip was featured? Are there other reasons accounting for these differences?

What is the nature of those contributions?

To understand the nature of each individual's contributions, we can examine individual moves. We obtain this detailed look by counting the number of contributions of each type, from each participant. For the 35 significant fibers of VOPS9, table 4 shows these counts. Discrepancies between the participants' counts and the total number of moves for each category are due to moves given by all, multiple, or unidentified participants. The table also provides the number of fibers represented (in parenthesis) for each entry, if it differs from the number of moves. For example, Lyndsay gave a total of seven substantiations, occurring in six different fibers (indicated in parenthesis), whereas Maud gave a total of nine, each in a different fiber.

By nature, the contributions to VOPS9 were varied, but limited. Moves in VOPS9 focused mainly on initiating and building content, justifying content, clarifying and ensuring agreement. Very few moves focused on presenting opposing points of view, argumentation, deliberation, or inquiry.

Most participants gave very similar types of moves. As seen by the number of initiation moves, all of the participants took an active role in opening up discussion fibers by

Table 4: Number of moves (and corresponding fibers) of each type made by participants during the 35 significant fibers of VOPS9 (30 Nov 2006)

	Facil.	Liz	Lyndsay	Maud	Sarah	Total
Anchoring Moves						
Information	0	0	2	5	4	11
Initiation	7	5	2	6	3	23
Valuing Moves						
Substantiation	2	5(4)	7(6)	9	4	27(19)
Justification	0	3	4(3)	2	3	12(10)
Qualification	1	0	2	6	0	9
Invalidation	1	2	1	1	1	6
Assertion						
Confirmation	0	0	3	7	4(3)	15(12)
Vacillation	0	0	0	0	0	0
Denial	0	1	0	0	0	2
Significant Altering Moves						
Extension	1	4	3(2)	10(9)	4(3)	22(14)
Modification	0	1	0	0	1	2
Clarification	2	1	1	7(6)	2	13(9)
Soliciting Moves						
Significant Requests	2(1)	0	0	4(3)	2	8(5)
Other Requests	3	0	0	4	1	8
Contentless Moves	11(7)	8(5)	6(5)	24(16)	15(10)	72(24)
Total Significant Moves	14 (11)	16 (10)	15 (9)	41 (25)	20 (13)	106 (35)

Each is the number of moves of the type by that individual; numbers in parenthesis represent number of fibers to which they contributed, if less more than one given per fiber.

making initiating moves. Each participant initiated some conversational fibers; the facilitator, Liz, and Maud initiated nearly the same amount.

All participants listened to and built off of the ideas of others. They made moves altering their peers' content, usually by adding content and details. These moves reveal participants' efforts to understand and clarify the ideas being discussed.

Participants also made efforts to explain and support their own and others' contributions. They all provided similar numbers of justifications and invalidations, justifications being most prevalent. This shows positive support and explanation for others' ideas.

The disproportionate number of justifications also suggests two possibilities: a) participants agreed on most points discussed or b) there was a hesitancy to initiate disagreement. Further observation reveals, there was very little discussion and refinement of ideas; there were almost no modifications and each individual contributed on average only about 1.5 moves per fiber. Additionally, there was little disagreement and resolution of differences of opinion; 45.7% of fibers received no substantiating move of any type. Out of the 44 valuing moves provided, only 17 moves countered the ideas being discussed by any means; most of those given were limiting (qualifications), not overtly opposing (invalidations and denials). This may be due to shared values, positions, and ideas; however, it may also suggest a tendency against conflict or argument, a hesitation to take opposing positions, or the existence of other socially hidden elements (e.g. hierarchy or power constructs). In spite of this, all participants did voice disagreement at least once directly supporting their position.

Examining the moves in VOPS9 shows a profound lack of inquiry. Of the 106 significant moves, only eight (7.5%) were significant requests (i.e. moves requesting significant content). Of these, five requested clarification. Only three represented an attempt at inquiry, one requesting substantiation and the other two requesting initiations. So inquiry was rare and not a driving characteristic of VOPS9.

What role are they playing in the discussion? (contrast each individual's participation)

The counts in table 4 not only reveal the nature of individual contributions, but show us individuals' roles in the conversation. As noted, participants contributed similar types of moves. Certain individuals stand out for their involvement, namely the facilitator and Maud.

The facilitator's role could be most concisely described as initiating content and soliciting new content from participants. The facilitator initiated seven (20%) significant fibers. He also engaged other participants in doing the same. One of his two requests was a re-

quest for initiations, resulting in four participants initiating six related significant fibers (five conversational and one informational).

In other ways, the facilitator took the backseat, avoiding content control and allowing participants to direct the conversation's content. Aside from anchoring and soliciting moves, the facilitator made few significant moves—only two substantiations, one extension, and two clarifications. He did not control the conversational content, allowing it to develop at the participants' discretion; as a result, other participants took an active role in the conversation.

Among the other participants, Maud's involvement stands out. Maud was the most involved in the session's discussion, both in the spread of her involvement (contributing to 25 of the 35 significant fibers) and in the number of moves she made (making 38.6% of the significant moves in all significant fibers). Although the discussion had no one clear discussion leader, Maud played a leading role in the conversation.

Maud's role could be described as initiating content, conveying information, evaluating/refuting contributions, and integrating and building ideas. Aside from the facilitator, Maud initiated more significant fibers than any other participant. She initiated six conversational fibers, but she also raised five informational fibers, conveying information she felt important, without it being discussed by others.

Maud also played the role of evaluator or refuter. She made more valuing moves than any other participant (i.e. 16 out of the 42, or 38%). Like other participants, she justified ideas; however, unlike others, she made a number of moves to dispute content. In these instances, she often did not open herself up for deliberation. Instead she mostly made evaluative comments that stayed on the fringes of conversation, limiting (qualifying) ideas, but not overtly attacking or negating (invalidating) them. In this indirect way, she showed more disagreement with ideas than any other participant; this disagreement did not, however open-up discussion. She also made several assertions (44% of her valuing moves

and 47% of all assertions in significant fibers), confirming ideas without justifying them.

Finally, Maud furthered the development of existing content. She was the most involved in these efforts, making 45% of all content extensions and 54% of all clarifications in significant fibers.

Although Maud was the most extensively involved in VOPS9 and played leading roles in the conversation, she did not dominate the discussion. The other participants took on similar roles, just not in as many fibers. Sarah initiated content (including both conversational and informational fibers), justified and evaluated content, and extended and built existing content. She did these things in similar proportions to Maud, but just in about half as many fibers.

Liz and Lyndsay also made important contributions to the discussion. While their participation was not far below that of Sarah's, they did contribute the least to the conversation, staring fewer fibers and giving the least moves overall. It appears that they were either holding back, choosing not to contribute, or were unable to contribute more for some reason. Recognizing this leads us to wonder why the difference? Why did they not contribute more? What could have drawn them more into the conversation? Why is Maud's involvement so high? Are the differences due to personality, experience, motivational, or other social factors that affected this participation?

What significance and impact do their contributions have on the developing content?

The result of all these moves is a conversation that is characterized by the conveyance, slight building, and justifying of a wide variety of ideas and opinions, with almost no discussion, change, and deliberation of ideas. In fact, it was common for individuals to open up an idea and justify it; then others would add to it; and then the topic would shift.

VOPS9 involved a large number of ideas, but little time was spent deeply investigating them. During the 22 minutes of discussion of VOPS9, participants discussed 23 significant

conversational fibers, which leaves less than a minute per fiber, on average (not counting the insignificant fibers discussed). These fibers ranged from two to ten significant moves in length; on average, they included four significant moves (both median and mean) given by two or three different speakers.

The content of VOPS9 arose mainly from spontaneous comments and thoughts rather than inquiry into teaching. Most fibers began with an unsolicited anchoring move. During fibers, few inquiries were made, usually requesting unsubstantiated acceptance of ideas or clarification of content.

So the main focus seems to have been on conveying content and acceptance of those ideas. Thus, in spite of the widespread involvement, the conversational content or discussion does not appear to have been particularly rich or deep. Of course, ascertaining this is partially unaddressed, because the level of the framework we have discussed in this paper does not look at the relationship among fibers; thus we cannot discuss the higher structure of the discussion or its overall depth and cohesiveness. Within fibers, however, we can say that most ideas were not well discussed.

Discussion

As demonstrated, the Co-construction of Knowledge Framework revealed content structure and participants' roles in discussions among teachers in a professional development (PD) setting. It is plausible that this framework could also provide important insights into classroom discussions. For instance, it may reveal the roles of teachers and students in building class content, the extent and significance of their involvement, and the nature or qualities of their contributions.

In this RUME preliminary research session, we discussed several possible implementations for the study and applications of such a framework. These include: a) understanding classroom discussion structures; b) identifying the impact of teacher actions on discussions;

c) changing student patterns of discussion; and d) helping teachers understand and utilize discussion.

The framework could be used to better understand classroom discussion structures. There are a variety of different forms of instruction presently used in teaching. Questions we may address with this framework could include: What does discourse look like in an effective mathematics classroom? What does it look like in classrooms with different delivery methods? What roles do teachers and students typically play in regard to building content? What does effective discussion look like in the different instructional forms? What ways can students be engaged?

In addition to understanding discussion, we could look at how teachers impact the discussion structure and student involvement. Along these lines, we could address questions like: What can teachers do to effectively engage students in content development? What impact do teacher interventions have on discussion structure? How can teachers change the nature of students involvement and patterns of interaction?

Finally, from the position of PD, we can look at how discussing and showing discussion structure to students or simply talking about quality involvement may change patterns of student participation. We could also look at how examining discussion structure could empower teachers professionally to decisively change their classroom discussion and students' involvement patterns.

Anticipating the study and use of this framework to understand these various topics and address these various needs, it is important to recognize the underlying context. The classroom context differs from the PD setting (i.e. the context in which the framework was developed) in at least four key ways.

First, the formal classroom environment has a long history, and there are strongly rooted cultural norms of participation associated with it. These play a larger part in dictating student and teacher roles. These norms may differ greatly from those common in

PD settings and *do* differ greatly from those established in VOPS.

Second, the content of most mathematics classes is very different in nature than what would be discussed in the PD setting. The study of math is highly professionalized. Consequently, mathematics has been highly studied, refined, and carefully constructed, proven, and axiomatized. The knowledge base for teaching is not nearly as determined, structured, and established. Thus the content for most mathematics classes is very different than that of PD programs.

As a result, the problems that would arise in mathematics classes would be very different, by nature, than those that would arise in the PD setting. Although approaches and reasoning may vary, most class mathematics problems have distinct, correct, and verifiable answers. By contrast, problems discussed in PD, regarding teaching, have solutions shrouded in complex contexts and impacted by numerous social and cultural issues; answers often are not clear, fully generalizable, or even certain.

Finally, mathematics classes often center around the acquisition of specific content and skills, both of which are clearly established and provable. This makes them certain and reliable. By contrast, many aspects of teaching can be vague, abstract, and complex, due to the social nature of the activity and changing aspects of the classroom environment.

The framework described focuses on how the content of a discussion develops. Because of the contextual differences between the mathematics classroom and professional development setting, however, it is plausible that changes are required in the framework. After all, we can already anticipate that such differences would likely impact answers to questions such as: What needs to be discussed and when? How much justification is needed? What constitutes justification? What constitutes valid answers?

At the same time, while contextual differences have the potential to change the discourse, they might not require changes in the framework. It is possible that what changes from one context to the next is not how individuals *can* respond to existing content (i.e. the

framework itself), but the *patterns* in which they employ those actions to build the content (i.e. the discussion structure).

This uncertainty is why more research is warranted. Extending the framework to the classroom setting may require adaptations or further refinements to accommodate such differences. My intent is to conduct classroom research to see how the framework can be extended or adapted to usefully describe classroom discourse. This involves addressing the following questions: How can the framework usefully describe classroom discourse? What fundamental differences in the contexts affect its implementation? What must be done to accommodate these differences?

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