CONTRIBUTED RESEARCH REPORT

Evolution of Mathematical Discourse with the Mediation of Electronic Environment: The case of Tangent Line

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This study focuses on a teaching experiment applied in a Year 12 class for the introduction of the derivative and the tangent line of function graph. This experiment was based on research results concerning students' perspectives on tangents when they had met the notion of tangent in different mathematical contexts (Geometry and Analysis). For the experimental needs an electronic environment was developed, utilising Dynamic Geometry software. The analysis focuses on the evolution of classroom mathematical discourse with the mediation of the electronic environment and with specific examples. Here I focus on an incident of the experiment. This incident exemplifies: how an image of a curve magnified in order to look straight in the electronic environment did not act as visual mediator for a student towards a claim that a curve has a tangent; the conflict in mathematical discourse about tangents; and, its resolution through the discussion of a particular example.

Introduction

The work discussed in this report is a part of a doctorate study (Biza, 2008) on upper secondary and first year mathematics undergraduate students learning and understanding of tangent line to a function graph. The aim is the investigation of the perspectives on tangents that are built through the transition between mathematical contexts (e.g. from Geometry to Analysis); and, the ways in which the tangent line could be introduced in order to take into consideration these perspectives. This paper focuses on the second aim and presents some results on a teaching experiment applied in a Year 12 class for the introduction of the derivative and the tangent line of function graph. For the needs of this experiment a selection of examples was prepared and an electronic environment was developed utilising Dynamic Geometry (DG) software.

Theoretical Background

Previous research has highlighted the strong influence of the perspective on a tangent to a circle to the general perspectives on tangents (Biza, Christou & Zachariades, 2008; Tall 1987; Vinner, 1991). Some of these results are attributed to the insufficient emphasis in the classroom on the transition from the *global* view of the graph that characterise the geometrical approach to the *local* view in the analytical approach (Castela, 1995). For example, students who have met the concept of tangent in different mathematical contexts (Euclidean and Analytic Geometry and Analysis) demonstrated several *Intermediate* perspectives about tangents between the *Analytical Local* perspective that is in accordance with the definition and uses of the tangent line in Analysis (e.g. limiting position of secant lines, slope, and derivative) and the *Geometrical Global* perspective according to which the tangent preserves geometrical properties being applied globally at the entire curve (Biza et al., 2008).

Local straightness (Tall, 1989) or *micro-straightness* (Maschietto, 2008) is a property that characterises a graph of a differentiable function. This property refers to the fact that, if we focus close enough to a point of a function curve, a point in which the function is differentiable, then this curve looks like a straight line. Actually, this "straight line" is the tangent line of the curve at this point. This property can be visualised with zooming tools in appropriately designed software (Tall, 1989) or in a graphic calculator (Maschietto, 2008) and lead the students inside the *local/global game* that is so important for a transition to Analysis (ibid).

In addition to the above, examples and counterexamples have a very important role in the introduction of new concepts (Watson & Shipman, 2008) and can help learners encounter conflict the resolution of which may lead to modification of their knowledge (Zaslavsky & Shir, 2005). Furthermore, classroom discussion could identify students' insights and turning points in their personal construction about specific concepts and their definition (Zaslavsky & Shir, 2005).

As changing in thinking about tangents on a classroom experience is the focus of this study, I will use Sfard's communication approach according to which thinking can be regarded as a special case of communication activity (2008). In this activity, the four interrelated features that distinguish the mathematical discourse are: *word use* (e.g. words that signify quantities and shapes); *visual mediators*, namely visible objects that are operated upon as a part of the process of communication (e.g. mathematical formulae, graphs, drawings, and diagrams); *narrative*, namely a sequence of utterances framed as a description of objects, of relations between objects, or of processes with or by objects, and that is subject to endorsement or rejection (e.g. definitions, proofs and theorems); and, *routines*, namely repetitive patterns characteristic of a given discourse (ibid, p. 133-134). In this report I will refer only to the feature of the *visual mediators*. I will focus on the role of the inscription of the *local straightness* in the electronic environment as a *visual mediator* or not as it appeared in the mathematical discourse of the classroom.

Methodology – The Context of the Study

The participants of this study were 15 Year 12 students (aged 17-18 years) of a Greek secondary school who had taken mathematics as a major subject. By the time the research took place, the students had been taught in an introductory Analysis course: functions, limit and continuity, but not derivatives. In addition, the previous years they had encountered the tangent line to the circle and other conic sections.

The teaching experiment was supported by an electronic environment developed in the framework of EU funded project called *CalGeo* (Biza, Diakoumopoulos & Souyoul, 2007) and utilised DG software named EucliDraw (http://www.euclidraw.com). In addition to DG facilities, this software offers a function editor/sketch environment as well as some tools appropriate for Analysis instruction. Indicatively, I refer to the magnification tool that can magnify a specific region of any point on the screen in a separate window. This magnification can be repeated as many times as the user specifies through a magnification factor and the graph and its magnification are presented at the same time on the screen (Figure 1).

The design of the experiment was based on the research results, presented earlier, about students' understanding of tangents. The aim of the experiment was the reconstruction of previous, restricted perspectives about tangents and the creation, through the introduction of derivative, of a more general understanding about tangency. To this end the experiment intended to: deploy the dynamic visual graphics in the electronic environment (e.g. the magnified image) and the symbolic expressions (e.g. the limit of rate of change) as *visual mediators* of the discourse on tangency; and, shift the students' *word use*, *narratives* and *routines* (Sfard, 2008) towards those related to the general definition of tangency. In accordance with these aims, the experimental instruction had the following stages:

1. Examination of generalisable properties of tangency (e.g. the tangent line as the limiting position of secant lines and the linear approximation of the curve) in the case of circle.
2. Examination of the above properties in the case of semicircle.
3. Introduction to the definition of the derivative and the tangent to a function graph.
4. Establishment of these definitions in classroom discussion through critical examples of function.

The incident I am going to describe in the next section was happened at the very beginning of the second stage.

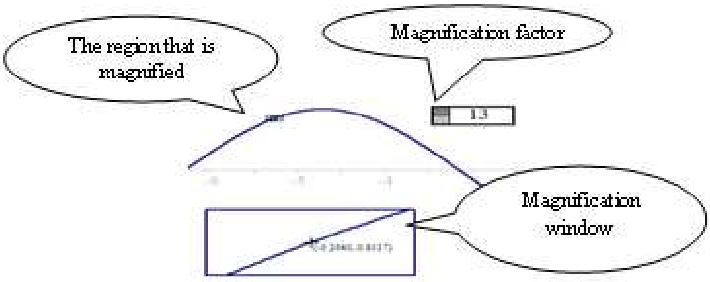

Figure 1: The magnification tool in the electronic environment

One incident from the teaching experiment: Is there a shared meaning of the magnified image of the graph?

In the first stage, the students had been introduced to the local straightness as a property of a circle and had seen in the electronic environment the connection of this property with the existence of the tangent line. The incident occurred at the moment when the classroom had been invited to investigate this property in the case of the semicircle in order to generalise it later in the case of the function graph. The graph and the visual inscription on the screen are presented in Figure 2.

In my request to the students to comment on the configuration in the environment (Figure 2) one student responded: "[The tangent] seems to coincide [with the semicircle] because if the magnification number is big, we cannot see the difference. That is because of the low resolution of the screen". Through this response we could say that the student had not connected the local straightness with the properties of the tangent as I had intended in the previous part of the experiment. Actually for him the inscription of the magnified graph was connected with the technical inherent restrictions of the electronic environment and did not have any mathematical meaning.

This response was welcomed but unexpected – according to the instructional design – and led to a slight change of the initial lesson plan. Firstly, we discussed the differences between the images of the lines in the magnification window in Figure 2. The secant line *AB* (which was green) did not coincide with the curve whereas the tangent through *A* (which was red) did. If the screen resolution was an issue, both lines and circle should match. Then, I proposed to the classroom to examine the same situation if the point *A* is a vertex of a parallelogram. In this case the line could not coincide with the curve regardless of how big the magnification factor was (Figure 3). During the comparison of these two cases and trying to explain the differences between these two inscriptions the same as above student mentioned: “this happened because the line is the tangent of the circle”

This incident indicated the conflict between the meaning I (as an instructor) had given into the image in the magnification window, and used in my discourse, and the meaning of the student(s). The discussion in the classroom made this conflict transparent and led to changes in the initial lesson plan. We now aimed at the refutation of this perspective and drove towards a shared interpretation of the inscription in the magnification window. As long as there was a tangent and the curve became straight in the magnification window the image acted as a natural illustration without any connection with the curve’s properties. As a result this image did not act as a *visual mediator* to support a claim that a line is a tangent or not – as I had presupposed that the students were going to do. The connection started to be made only when we considered a case in which the property could not be applied (Figure 3).

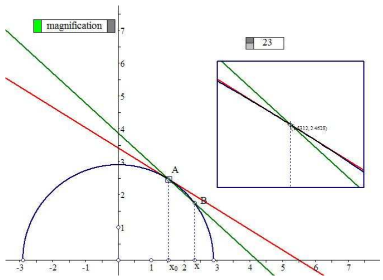

Figure 2: Magnification on a circle point

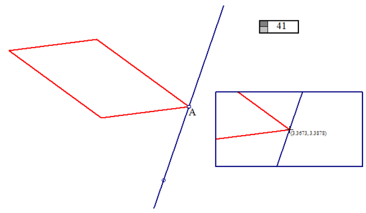

Figure 3: Magnification on a parallelogram vertex

Concluding remarks

This incident exemplifies important issues about the introduction of mathematical notions in the classroom. These include the evolution of classroom mathematical discourse with the mediation of electronic environments and through appropriated selected examples. Although, the electronic environments usually offer perceptual configurations of mathematical notions, the interpretation of these configurations is not necessarily shared knowledge in the classroom and a discursive conflict is likely to occur. This conflict can be resolved through discussion and the use of appropriate examples. In this process the role of the teacher is very important to understand the classroom situation; to take decisions and possibly change the initial plan in the moment; to select the appropriate examples; and to orchestrate the discussion in order to allow the emergency of students’ insights and turning points in their personal perspectives.

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