No Teacher Left Behind: Assessment of Secondary Mathematics Teachers' Pedagogical Content Knowledge

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Abstract. The article reviews efforts of five iterations over three years in developing a written assessment of the mathematical pedagogical content knowledge (PCK) of middle and high school mathematics teachers. Of the 100 teachers to complete written items, half were already "Highly Qualified" according to No Child Left Behind Act of 2001 criteria and half were not. Content in the items addressed essential understandings for number and operations, algebra and functions, and proof. PCK measures included sub-scores on curricular content, discourse, anticipatory, and classroom action knowledge. We articulate the relationship between our conceptualization of PCK and other existing theories for assessing PCK.

Key Words: Pedagogical content knowledge (PCK), assessment, professional development.

Background

According to the No Child Left Behind Act of 2001, a teacher is "highly qualified" (HQ) in mathematics after "demonstrating competence" by completing 24 semester hours of college mathematics. However, those who devote their research to education have questioned this definition (Bolyard & Moyer-Packenham, 2008; Darling-Hammond & Youngs, 2002). Simultaneously, pedagogical content knowledge (PCK) has become one of the central constructs in research on the development of understandings for teaching mathematics (Ball & Bass, 2000; Shulman, 1986). Ball, Thames, and Phelps (2008) and Hill, Ball, and Schilling (2008) have developed typologies for mathematical knowledge for teaching that include components of subject matter knowledge and PCK. Within PCK, they have defined three types of knowledge: knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of content and curriculum (KCC). In particular, KCS "is knowledge that combines knowing about students and knowing about mathematics," while KCT "combines knowing about teaching and knowing about mathematics" (Ball, Thames, & Phelps, 2008; p. 401). In developing their theoretical framework, the Ball and colleagues acknowledge the possible weaknesses of their conceptualizations. One important weakness is that though the theory is steeped in knowledge of practice, there are potential problems due to the variety in curricula and associated classroom implementations. Additionally, it is not clear how cultural variability across and among teachers and students is accounted for in their model. The tests of PCK that Ball et al. have designed for elementary teachers have focused mainly on KCS and are based on the assumption that "common" student thinking and errors will occur regardless of curriculum or the context of its implementation. Also, while both groups pay particular attention to the implications of their theories to the moves teachers make in their teaching (i.e., for KCT), Hill, Ball, and Schilling assert that KCS is "separable from knowledge of teaching moves" (p. 378).

For this report, we conceive of PCK in terms of four components of professional understanding: curricular content knowledge, discourse knowledge, anticipatory knowledge, and action knowledge. These four areas align in many ways with the KCC, KCS, and KCT of Ball and colleagues. *Curricular content knowledge*, unlike what one is likely to learn in a college course, is substantive knowledge about topics, procedures, and concepts along with a comprehension of the relationships among them *as they are offered in school curricula*. Related to the KCC construct of Ball and colleagues, in its most robust form this part of PCK contributes to what Ma (1999) called "profound understanding of fundamental mathematics." *Discourse knowledge* is about the culturally embedded nature of inquiry and forms of communication in mathematics (both in and out of educational settings). While Hill et al. (2008), refer to the importance of teacher knowledge of standard and non-standard mathematical representations and communication, discourse knowledge does not appear explicitly in their model of PCK.

Anticipatory knowledge is an awareness of, and responsiveness to, the diverse ways in which learners may engage with content, processes, and concepts. Our view of anticipatory knowledge is similar to Ball and colleagues' "Knowledge of Content and Students" though our focus is on relational understandings teachers have more than declarative or procedural knowledge about students and content. *Action knowledge,* like Ball and Bass' (2000) notion of "knowledge for practice" and "Knowledge of Teaching and Content" (Hill, Ball, & Schilling, 2008) includes knowledge about how to adapt teaching according to content and socio-cultural context and *enact in the classroom* the decisions informed by content, discourse, and anticipatory understandings.

Theoretical Perspective

Our research is based on the above framing of PCK and on the radical constructivist view that teachers construct their own meanings for, and understandings of, classroom teaching experience. This is not to say that social interactions and shared repertoires related to teaching resources are unimportant (Birman, Desimone, Porter, & Garet, 2000). Key to any professional development program is an intention to build a "community created over time by the sustained pursuit of [the] shared enterprise" of fostering student learning (Wenger, 1998; p. 45). One step in examining the growth of PCK is to understand the perceptions and conceptions that a learner who is also a teacher might construct in developing PCK. Our multi-pronged approach to such investigation includes examination of individual teacher content, discourse, anticipatory, and action knowledge; awareness of the challenges to those views engendered by a professional development program; subsequent review of teachers' views and in-class actions; and analysis of learning outcomes for the students of those teachers. In particular, the research questions for the work reported here: How might teacher PCK understandings be characterized in order to track the effects of professional development through changes in PCK? Given these characterizations, what is the nature of PCK that in-service teachers bring with them to a college mathematics course-based professional development programs? The goal was to generate methods for authentic, rich description of PCK for in-service mathematics teacher-participants in a university-based professional development program. As we began developing machine-gradable written assessments of middle and high school teachers' mathematics PCK, the challenges of creating terse yet reliable items to assess action knowledge quickly led us to narrow our focus to curricular content, discourse, and anticipatory knowledge. We continue to work on creating worthwhile written assessments of action knowledge in (at least partially) machine gradable form. In the meantime, we rely on teacher-generated reflective writing, action research reporting, and our observation of classroom teaching as sources of information about the evolution of action knowledge.

Methods

Written Assessment of PCK

In each of five administrations, at least 15 in-service teacher-participants who taught grades 5 through 12 completed an assessment of 22 items. Each test-form of 22 items contained at least 14 items repeated from the previous test-form and 6 new items. Each item included a multiple choice format portion and some had extensions in other formats (e.g., short-answer, see Figure 2). We contend these items evaluated curricular content, discourse, and anticipatory knowledge – for details of these items and their development the reader is referred to Hauk, Deon, Judd, Kreps, & Novak (2009). We do not have a large enough data pool yet for powerful statistical analysis. Below we discuss low-n test-retest indicator statistics.

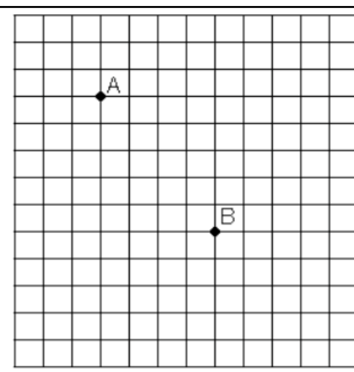
Item Triples. Each of the items on a test was accorded an ordered triple (c, d, p) for curricular content, discourse, and anticipatory knowledge components. This form of item coding arose from interviews with teachers and the early data from Ball and colleagues (2004) that indicated items will test multiple constructs. Rather than attempting to narrow what each item tested, we chose to code items for their multiplicity. A more complex analysis of items might have attempted to weight these components, but for the purposes of this study we used a binary scale: 0 if the aspect was largely absent from the item, 1 if there was a justified argument agreed to by at least three researchers for the likely presence of an aspect in the answering of the item. For example, the distributive property item in Figure 1 had a triple of (1, 1, 0) and the graphical item in Figure 2 had a triple of (1, 1, 1).

$$15(4 + 3) = 15 \times 4 + 15 \times 3$$

The equation above demonstrates which of the following?
(A) The distributive property of multiplication over addition
(B) The commutative property of multiplication
(C) The associative property of multiplication
(D) Additive inverse and additive identity

Figure 1. Item from *Praxis* (Educational Testing Service, 2003a).

In the given graph, the axes and origin are not shown, x is horizontal, y is vertical, and the axes have the same scale. If point B has (x, y) coordinates of (10, 3), which of the following coordinates could be point A?
(A) (–3, –3)
(B) (2, 10)
(C) (6, 8)
(D) (10, 6)



Suppose the item above was part of an Algebra 1 lesson:
a. What is the main mathematical idea in the item? How did you decide?
b. Create a follow-up item that would help an English language learner in developing further understanding of the main idea [Please give your new item in the space below and briefly justify its use].

Figure 2. Compound answer format item (multiple choice and short-answer).

Looked at in this way, each teacher-participant had an exam score triple: one percentage score for each of curricular content, discourse, and anticipatory knowledge. While the number of items that "loaded" on each component varied slightly from test-form to test-form, all items had a curricular content component, at least half (11 of 22) had a significant discourse component, and at least one-third (7 of 22) had anticipatory aspects.

Alignment with Colorado State Content Standards

The questions on the mathematics instrument were chosen to be representative of the six Colorado Mathematics Content Standards (see Table 1).

Table 1. Distribution of State Mathematics Standards on the Written Test.

Area of Understanding	*Approximate % of items on each test*
Number sense	30%
Algebraic methods	40%
Data collection and analysis	30%
Geometric concepts and relationships	30%
Problem solving	70%
Linking mathematical concepts & procedures	60%

Note. The first four percentages, for the content strands, sum to 100 because each item was aligned to one content strand; additionally, some items addressed a content strand and one or more of the state process standards (the last two on the list).

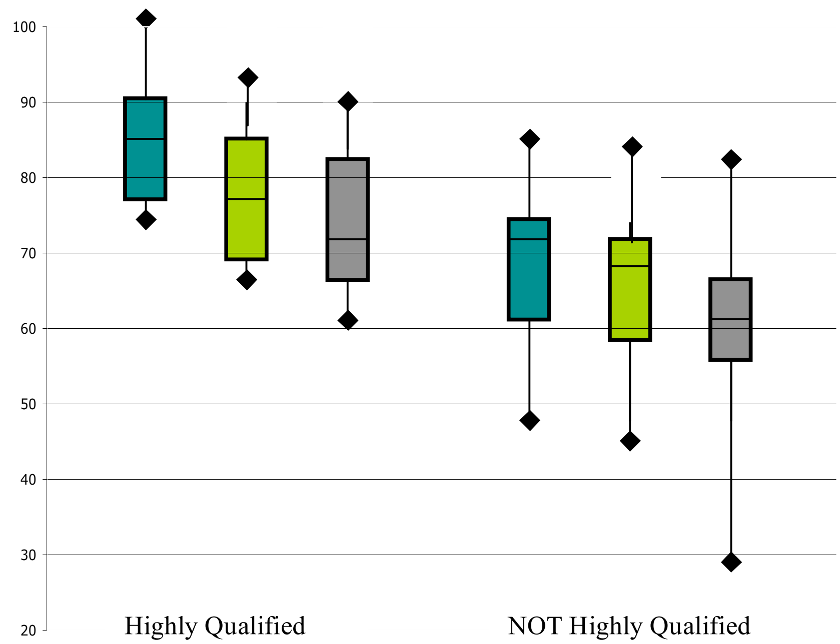
Interviews

In early test item development we conducted item-based interviews with teachers who had completed the PCK tests. All agreed to be interviewed. Due to time and resource constraints, we interviewed a total of 12 teachers (four in each category: less than 4, 4 to 10, and more than 10 years experience teaching). Together, the interviewed teacher-participants provided a diverse and representative cross-section of the 100 teachers who, over the five years of test development to date, have completed the PCK tests. The main thrust of the problem tasks in the interview protocol (two problem-solving activities and one problem-posing activity) was creating space for differentiating the four components of PCK (for more on these interviews, see Hauk et al., 2009).

Results

Written Test

Teachers with more mathematics courses in their backgrounds had higher average scores on the written instrument. Notably, the standard deviations for all aspects, particularly for *discourse* knowledge, were larger among the less mathematically experienced teacher-participants (i.e., were *not* "highly qualified"). Figure 3 gives a sample of post-test data for teachers who participated in a year-long professional development program.



	Highly Qualified: Mean (std.dev.)	NOT Highly Qualified: Mean (std.dev.)
Content	85% (10%)	71% (14%)
Discourse	79% (13%)	68% (14%)
Anticipatory	76% (14%)	62% (19%)

Figure 3. Box plots of indicative post-test data.

Interviews

We chose pseudonyms for teachers to code some of the information about them: single syllable names for the Not HQ interviewees, two-syllable for HQ participants and assigned names alphabetically according to level of mathematics preparation and experience, with Ann having the least and Quentin the most. Constant-comparative coding of the interviews resulted in 10 categories of interaction with the interview tasks, three in each of the four PCK components (some of the 10 categories aligned with more than one PCK component). We come back to this multiplicity of alignment in our discussion section below.

We used a constant-comparative analysis of teacher-participants' responses to the three problem task prompts to develop the discussion and illustrations offered below. We identified ten areas of response in the remarks and answers provided by teacher-participants. Subsequent analysis led us to associate subsets of these 10 categories with each of the four areas of pedagogical content knowledge identified above.

Curricular content knowledge was represented in three categories:

Method: The interviewee said how to go about answering/posing the problem;

Numeric answer: The interviewee provided an accurate numeric answer;

Units for answer: The answer given included use or discussion of units.

The third category, *Units*, also appeared to be associated with teacher-participants' comments around the syntax of the tasks and their solutions. All but Ann commented on the importance of students learning to "pay attention to how to say the units." Under discourse knowledge we grouped the category *Units,* with two additional categories:

Analysis of language: The teacher-participant made explicit comments about the general form of the language or syntax of the item;

Wording evaluation: The interviewee discussed how the wording or semantics influenced her or his own solving of the problem; including statements that the problem was worded “like something that would be in a book” or on “a standardized test.”

Anticipatory knowledge appeared to arise in responses classified in the following three categories. Note that in each case, evidence of epistemological knowledge of mathematics in social learning settings was a component:

Justification: The teacher-participant clearly articulated how the nature of the task (e.g., content, format, syntax) might affect students’ solution efforts. In particular, these comments extended beyond reference to a teacher-participant’s *own* solution efforts to include reference to different students perceiving the tasks differently;

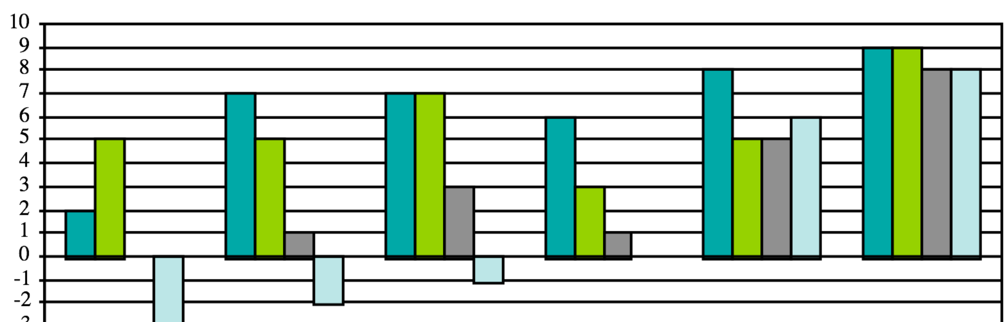
Anticipation: The interviewee provided a detailed explanation of how a “typical” student (most common) or how a variety of students might engage with the problem;

Internalization: The interviewee discussed how the item might be modified for use with her or his own students, might make specific suggestions on how to create alternate versions of the problem, or the participant made comparisons between the interview item and problem tasks with which they were already familiar.

Just as there was overlap between the categories assigned to curricular content and discourse knowledge expression, the anticipatory knowledge category *Internalization* also addressed aspects of knowledge for action because it included teacher-participants’ judgments about *use* (by them in teaching in their own classrooms). Consequently, we identified three task response categories associated with action knowledge: *Internalization*, along with:

Practice: Teacher-participants expanded on item use with a focus on what they might say or do in the classroom when teaching with or about the item’s concept(s). This included role playing scenarios offered by teachers from either previous experience or abstracted "typical" experiences they recalled (e.g., "Well, I'd say… then the student said… and they did…"; *Engagement*: Central to enacting effective learning opportunities in the classroom is the intention to search for and engage with students' ways of thinking (Lester, 2007). Because it represented a meta-cognitive and meta-affective level of interaction with tasks that was distinct from the other categories, coding in the Engagement category included *disengagement* (-1). For example, if a teacher-participant's own approach to the task was accompanied by an assertion that any student would "do it just like I did" the response was coded as –1. Also, some teacher-participants explicitly and firmly *dis*engaged from any discussion of teaching or learning in relation to a problem task by a clear refusal to analyze the task (e.g., turning a task paper over or pushing it away and saying "Okay, next!").

As an example, Figure 4 offers a representation of the interview coding for 6 of the interviewed teachers.



	Ann (22 yrs special ed; 3yrs math)	Bea (4yrs math)	Chris (9yrs sci; 1yr math)	Gracie (5yrs math)	Henry (6yrs math and science)	Iris (23 yrs math)
Content	2	7	7	6	8	9
Discourse	5	5	7	3	5	9
Anticipatory	0	1	3	1	5	8
Action	-3	-2	-1	0	6	8

Figure 4. Sample of interview PCK profiles.

This representation illustrates broader PCK for Henry and Iris and the sparseness of PCK for Ann, Bea, and Gracie. It is possible that Chris' (Not HQ) responses were richer in evidence of PCK than Gracie's (HQ), because Chris had more teaching experience (10 years including her nine years as a science teacher) and a greater variety of experiences than Gracie. The results of the interviews suggest that anticipatory knowledge is developed from learning and teaching IN or NEAR the content area rather than from learning or teaching ONLY in mathematics or in ANY subject. Also, as with the work reported by Hill et al., (2008), the interviews helped in identifying why teachers answered certain items the way they did and were key in member-checking the wording of compound format written test items (e.g., the multiple choice options and short-answer prompts in Figure 2).

Summary

Curricular Content Knowledge. In both interview and written tests, teacher-participants generally exhibited more content knowledge with more years of experience. This is most obvious in Figure 2 when comparing the bars representing the responses of the teacher-participants with the least and most mathematics teaching experience, respectively.

Discourse Knowledge. Regardless of whether or not they were highly qualified, teacher-participants with fewer than 10 years experience teaching in a mathematics or science field had lower overall discourse sub-scores on written items. The same was true for most of the interviews; for example, among those profiled in Figure 4, Chris and Iris each had 10 or more years experience teaching and had higher discourse knowledge scores.

Anticipatory Knowledge. Written test item scores increased as years of experience with mathematics or science teaching increased, "highly qualified" status notwithstanding (*n* is too small at this point to make definitive statements about correlations between years experience, HQ status, and differences in anticipatory knowledge). In the interview context, anticipatory knowledge (particularly the sub-category Anticipation) was higher among those with more experience teaching *mathematics* – for example in Figure 4, note that though Ann had many years of teaching experience in special education, she had taught mathematics for only three years. As another example, Gracie and Henry were the only two of the six interviewees in Figure 4 to comment on trouble communicating with their peers: Henry changed his undergraduate major because he said he "did not enjoy the nerdiness" of his colleagues, and he did not "talk like they did." In the same vein, Gracie said she had problems communicating with her less mathematically experienced teaching peers and wasn't sure "how to talk to them without talking down to them."

Action Knowledge. Only addressed explicitly in interviews, this was greatest for the more experienced "highly qualified" teachers (e.g. Henry and Iris in Figure 4). For example, Ann firmly refused to reflectively engage with all three interview tasks. Bea was equally firm in her assertions that her students' approaches would be just like her own. Chris and Gracie politely but clearly declined to engage with two of the tasks (one using letters to represent values and the other the distributive property), each asserting some version of "the problem has nothing to do with the kind of math I teach" (both taught algebra). The third task in the interview was a problem *posing* activity. The teacher-participants to manifest deep and rich pedagogical content knowledge on this task were those who were both HQ and experienced (e.g., Iris). Other teacher-participants' interactions with the problem posing task exhibited far less evidence of discourse knowledge, anticipatory knowledge, and knowledge for action than their interactions with the two problem *solving* tasks. The progression in Figure 4 from left to right for action knowledge, from Ann's negative score through Gracie's zero to Iris' eight, is an indication that the interview tasks did elicit valuable information about this component of PCK. In fact, the problem-posing task was the most challenging for all participants. That this is true was valuable information we took into the next round of data gathering, and lead to several extensions to multiple choice items that included problem posing (e.g., like the item in Figure 2).

Discussion

How might we characterize PCK in order to track its development?

In response to our first research question, we drew inspiration from Ball, Hill, and Schilling (2008) as well as Shulman (1986) to develop an instrument to measure PCK. While we agree with Ball and company's three components of PCK, as well as their theoretical foundations, we focus on slightly different aspects with additional particular attention to mathematical discourse. We have developed our own items to measure this, have examined *Praxis* items, and have reexamined items harvested from Ball and company's tests, coding them for the kinds of discourse involved. We now believe that we can track someone's performance, in terms of doing the items, according to which constructs load with which items. We have also expanded on Ball and company's test format. Due to the problematic nature of multiple-choice items (Hill, Ball, & Schilling, 2008), we have also included extended and short response items on our instrument. These items allow participants to provide us with much richer insight into their pedagogical mathematics understandings. Though time consuming to grade/code, a few such items on each assessment alleviates some of the validity concerns that Hill, Ball, and Schilling (2008) suggested about multiple-choice items. We continue to explore the possibility of creating machine-gradable items that are short-response; the English and Language Arts research community has already made headway in machine grading of written responses and we plan to look into that literature as we move forward.

What do our instruments and our constructs say about the nature of PCK that in-service teachers bring with them to a college mathematics course-based professional development programs?

One cohort of teacher-participants took three versions of our instrument over the course of two years: a pretest administered prior to participation in professional development; a first post-test administered one year later and a second post-test administered two years after the pretest and after another year of professional development. Each year, teachers were involved in 80 to 100 hours of professional development across a 2-week summer institute and 10 six-hour Saturday sessions during the academic year. On the pretest, participants were most successful at correctly responding to test items that loaded on curricular content knowledge and least successful at correctly responding to items that loaded on discourse knowledge. Participants demonstrated statistically significant gains ($\alpha= 0.05$) from the pretest to the first post-test, both overall and on their discourse knowledge scores, but not from the first post-test to the second post-test. We attribute the former finding to the effects of professional development, as opposed to a natural progression of knowledge (given that teachers with no significant professional development who took the same tests one year apart did not score differently between the pre- and post-test with any statistical significance). The latter finding may be attributable to a combination of several factors, including an increase in difficulty between the first and second post-tests.

Given our preliminary evidence, we make two claims: (1) our instrument measures the growth of some form of teacher knowledge, which includes knowledge of mathematical discourse; and (2) the professional development in which participants partook played a significant role in the development of this knowledge. However, we have yet to address, or present evidence in support of, whether what we are measuring is PCK. Our tests measure three constructs that we claim to be components in PCK: curricular content knowledge, anticipatory knowledge, and mathematical discourse knowledge. To define our first two constructs, we relied on the work of Ball and company. Our constructs and items align with conceptualizations of knowledge of curriculum and knowledge of content and students, respectively, so they are likely measuring similar areas within PCK. The claim that discourse knowledge is a component of PCK is slightly more challenging to defend, but we believe this to be evident in Shulman's (1986) conception of PCK, in our characterization of interview teacher-participants' understandings, and in Hill, Ball, & Schilling's (2008) failed items.

As Hill and colleagues note, Shulman (1986) claimed that PCK, in part, "is an understanding of what makes the learning of specific topics easy or difficult" (p. 9). Discourse knowledge applies here for a number of reasons, not the least of which is the potential challenge of learning mathematical topics for English language learners. English learners are not only being asked to learn one language, they are being asked to learn *three*: everyday English, the English of mathematics used in curricula, and the sometimes idiosyncratic dialect of English used in speaking about mathematics that is present in classroom discourse. An awareness and understanding of all of these languages and how to navigate them in spoken and textual discourse is a significant aspect of knowledge for teaching (Carr et al., 2009). Shulman also claimed that PCK is an understanding of "the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those frequently taught topics and lessons" (p. 9). Like teachers, students of all linguistic backgrounds may come to class with natural language understandings that conflict with the mathematical language of the class. This can interfere with students' understanding of the mathematics of the class (Civil, 2002). An understanding of student conceptions and preconceptions can be a discourse tool for a teacher to use in helping students develop their mathematical register (Wells, 2003) separate from their natural language register.

We also found evidence of the value of discourse knowledge in teaching from the interviews with teacher-participants: Chris, who was qualified in science but not mathematics, had higher discourse knowledge than Gracie, a teacher-participant who had a bachelor's degree and secondary credential in mathematics. Chris, however, had 10 years experience in the teaching of science (9 years) and mathematics (1 year), while Gracie had less than 2 years' experience in the classroom. Chris had an abundance of practice communicating scientific ideas, and when she was in a position to communicate mathematical ideas, she used mathematical language rather than natural or scientific language. This implies that discourse knowledge is used in the teaching of mathematics, making it a valuable component of PCK.

Multiplicity of Construct Loading

In their attempt to develop items that measured knowledge of content and students, Hill, Ball, and Schilling (2008) encountered difficulties with their conceptualization of KCS. The researchers claimed that KCS was separate from knowledge of curriculum and content, so they attempted to sanitize items of elements pertaining to this construct. Their confirmatory factor analysis of results on KCS items and curricular content knowledge items revealed that most of their items loaded on both constructs. While the researchers felt that their analyses confirmed the existence of some identifiable construct they called KCS, they admitted, "this knowledge may rely in part on teachers' underlying subject matter knowledge and is imperfectly discerned with the set of items used in [the] current instruments" (p. 385). Our conceptualization of discourse knowledge could explain this connection. The existence of an underlying discourse knowledge construct may also explain why some of Hill et al.'s items did not load strongly enough with

KCS. By taking the ordered triple approach to item coding, we aim to develop constellations of items that will co-load and are moving towards a semi-hierarchical non-linear model.

Implications

We see serveral implications for professional development of teachers from our work to date:

1. Though some groaned at taking a test, teachers valued having a way of tracking their professional learning across time; they wanted to know what the tests measured, how they did on the measures, and were particularly interested in the test as a record of learning from pre- to post-test. They also reported finding the tests challenging but "doable."
2. Taking many hours of college courses may support a teacher in developing an ability to *do* mathematics; however, robust PCK requires more than a degree in mathematics and five years' teaching experience (e.g., Gracie had both "qualifications" but had sparser PCK than Chris).
3. From interviews and some open-ended test items we found problem-posing to be an activity with the potential for stimulating reflective engagement with several facets of PCK simultaneously. This is an area for future research in teacher professional development.

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