Language, semantic contamination and mathematical proof

Matthew Inglis[1] & Juan Pablo Mejia-Ramos[2]

[1]Mathematics Education Centre, Loughborough University
[2]Graduate School of Education, Rutgers University

Abstract: The way words are used in natural language can influence how the same words are understood by students in formal educational contexts. Here we show that this so-called semantic contamination effect plays a role in determining how students engage with mathematical proof, a fundamental aspect of learning mathematics. Analyses of responses to argument evaluation tasks suggest that students may hold two different and contradictory conceptions of proof: one related to conviction, and one to validity. We demonstrate that these two conceptions can be preferentially elicited by making apparently irrelevant linguistic changes to task instructions. After analysing the occurrence of "proof" and "prove" in natural language, we report two experiments that suggest that the noun form privileges evaluations related to validity, and that the verb form privileges evaluations related to conviction. Implications of this finding for the linguistic content of university-level assessment materials are discussed.

Argumentation and proof are widely accepted as being central to mathematics (Heinze & Reiss, 2007; Hilbert et al., 2008). Consequently educators agree that argumentation and proof should be incorporated into mathematics learning and instruction at all levels (Hanna, 2007). At the undergraduate level, a large proportion of learning consists of reading and reflecting upon mathematical proofs. However, many studies have shown that students find engaging with proof difficult, regardless of whether such engagement takes the form of evaluating given proofs or constructing novel proofs (e.g., Knuth, 2002). In a different line of research, Segal (1999) demonstrated that many students simultaneously hold two different conceptions of proof when responding to argument evaluation tasks: conviction and validity, and argued that one major barrier to learning higher-level mathematics is an inability to decide which conception to use at any given time. In this presentation we report a study which seeked to investigate one possible contributory factor to this situation: linguistic confusion surrounding the notion of proof.

Halliday (1975) argued that mathematics is relatively unusual as its technical language, when naming new phenomena, regularly redefines words from natural language rather than coining novel technical terminology (e.g. group, knot, normal, function, etc.). This can give rise to what Pimm (1987) called *semantic contamination*: where the meaning or usage of a term from natural language influences how the term is understood by a learner in mathematical contexts.

Several examples of semantic contamination have been discussed in the mathematics education literature. In the context of advanced mathematics for example, Monaghan (1991) found that the natural language meaning of words and phrases associated with the limit concept ("tends to", "approaches", "converges", etc.) can impact upon students' concept images of the formal limit concept in calculus and analysis classes. Similarly, Tall & Vinner (1981) suggested that colloquial meanings of the term "continuity" influence how students engage with the formal mathematical concept. Difficulties which arise from such issues will need to be overcome if the learner is to successfully engage with the mathematical register (Pimm, 1987). In this presentation

we will argue that a similar phenomena influences how students engage with mathematical proof.

There are two main ways in which the concept of proof is referred to in natural (and mathematical) language: with the verb referent ("prove") and with the noun referent ("proof"). To investigate the way that these two referents are used in natural language we conducted a word frequency analysis of the British National Corpus (Burnard, 2000), a 100 million word corpora designed to be representive of modern English usage. We found that the verb referent is more commonly associated with informal day-to-day language than the noun referent. In contrast, the noun referent more commonly occurs in specialist technical language.

From this analysis we derived the hypothesis that, if semantic contamination were to influence students engagement with mathematical proof, we might expect the noun form ("proof") to more often evoke Segal's (1999) (more formal) validity conception of proof (because of its disproportionate occurrence in formal technical language), and the verb form ("prove") to more often evoke Segal's (more personal) conviction conception of proof (because of its disproportionate occurrence in everyday informal registers).

In the presentation we will report two experimental studies with undergraduate participants (N=220, N=241) designed to test this hypothesis. Participants were randomly allocated to one of two conditions, were shown a claim, a visual argument[1] in favor of the claim, and were asked "does the argument prove the claim?" (verb condition) or "is the argument a proof of the claim?" (noun condition). In both experiments the results supported our hypothesis. That is to say that participants tended to believe that the visual argument was not a proof of the claim, but did prove the claim.

That such an apparently irrelevant linguistic change can influence participants' responses to a straightforward argument evaluation task suggests that the two different proof conceptions identified by Segal (1999) are not merely of theoretical interest: they may well be unintentionally elicited by different linguistic cues during genuine instruction and assessment activities. We will conclude the presentation by discussing the implications of this finding for the design of materials in undergraduate teaching and assessment.

References

Burnard, L. (2000). *Reference guide for the British National Corpus*. Oxford: Oxford University Computing Services.

Halliday, M. A. K. (1975). Some aspects of sociolinguistics. In E. Jacobsen (Ed.), *Interactions between language and mathematical education* (p. 64-73). Paris: UNESCO.

[1] Visual arguments tend to be seen as highly convincing, but not formally valid (Knuth, 2002).

Hanna, G. (2007). The ongoing value of proof. In P. Boero (Ed.), *Theorems in school: From history, epistemology and cognition to classroom practice* (pp. 3-18). Rotterdam: Sense.

Heinze, A., & Reiss, K. (2007). Reasoning and proof in the mathematics classroom reasoning and proof in the mathematics classroom. *Analysis*, *27*, 333-357.

Hilbert, T. S., Renkl, A., Kessler, S., & Reiss, K. (2008). Learning to prove in geometry: Learning from heuristic examples and how it can be supported. *Learning and Instruction*, *18*, 54-65.

Knuth, E. (2002). Secondary school mathematics teachers conceptions of proof. *Journal for Research in Mathematics Education*, *33*, 379-405.

Monaghan, J. (1991). Problems with the language of limits. *For the Learning of Mathematics*, *11*(3), 20-24.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. London: Routledge & Kegan Paul.

Segal, J. (1999). Learning about mathematical proof: conviction and validity. *Journal of Mathematical Behavior*, *18*, 191-210.

Tall, D. O., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151-169.