Addressing Impulsive Disposition: Using Non-proportional Problems to Overcome Overgeneralization of Proportionality

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Impulsive disposition is an undesirable way of thinking where one spontaneously applies the first idea that comes to mind without checking its relevance. In this research, we explore (a) the possibility of helping pre-service teachers improve their disposition, from being impulsive to being analytic, in one semester, and (b) the effect of using non-proportional situations. This study involves two sections of a mathematics course for pre-service teachers for Grades 4-8. The lessons were designed whenever possible to elicit students' impulsive disposition so that they could become cognizant of it and make conscious attempts to overcome it. Some test items were designed to be superficially similar but structurally different to those they had experienced in class or homework. Pre-post-end test results show that pre-service teachers' tendency to overuse ratios and proportions can be reduced in one semester and that the use of non-proportional problems can minimize impulsive responses.

Introduction

Cuoco, Goldenberg, and Mark (1996) have advocated *habits of mind* as an organizing principle for mathematics curricula where students have opportunities to "think about mathematics the way mathematicians do" (p. 377) and learn to be pattern sniffers, experimenters, describers, tinkerers, inventors, visualisers, conjecturers, and guessers. Harel regards habits of mind as interiorized *ways of thinking*—conceptual tools that are necessary for constructing and advancing mathematics (Harel, 2008a). Harel (2007, 2008b) differentiates ways of thinking from *ways of understanding*—products of doing mathematics which include axioms, definitions, theorems, proofs, problems, and solutions—in order to highlight the importance of helping students develop desirable ways of thinking as well as desirable ways of understanding.

From a constructivist perspective, students build on their existing knowledge (Piaget, 1975/1985; von Glasersfeld, 1995). Students with different ways of understanding are likely to "acquire" a particular mathematical concept at different rates. Ways of thinking can also influence what students learn and how fast they learn. Undesirable ways of thinking, which tend to impact learning negatively, include beliefs such as "doing mathematics means following rules laid down by the teacher, knowing mathematics means remembering and applying the correct rule when the teacher asks a question, and mathematical truth is determined when the answer is ratified by the teacher" (Lampert, 1990, p. 31) and dispositions such as "doing whatever first comes to mind, waiting to be told what to do, or diving into the first approach that comes to mind" (Watson & Mason, 2007, p. 207).

Impulsive Disposition

Many pre-service teachers for Grades K-8 (kindergarten to 8th grade) seem to have an *impulsive disposition* which we define as a proclivity to proceed spontaneously with an action that comes to mind without checking its relevance or appropriateness. Students with an impulsive disposition tend to solve a problem by applying a recently learned idea. For example, after learning how to use ratio to compare *squareness* of rectangles in one lesson (a square has a length to width ratio of 1), 9 out of 22 students used ratios to compare palm-size in the next lesson. As shown in Figure 1, a pre-service middle school teacher misapplied the strategy and did not seem to have attended to the meaning of ratio nor the measure for quantifying palm-size. Lim (2008a) describes such a phenomenon as *recency effect*—applying “a recently learned idea to a problem situation without checking its validity” (p. 98).

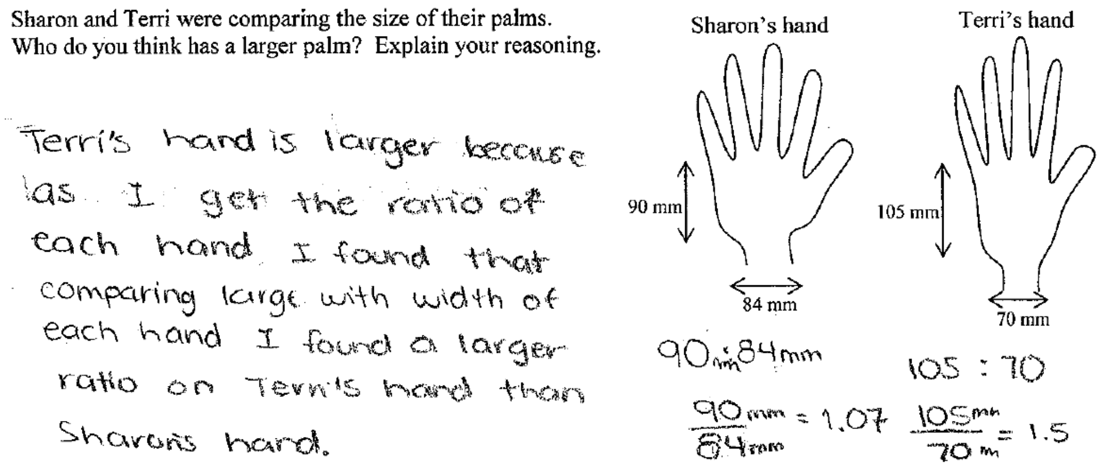

Figure 1: A pre-service teacher’s use of ratio to compare palm-size

Theoretical constructs such as intuitive cognition, intuitive rules, spurious correlations, metacognition, cognitive style, habits of mind, ways of thinking, and overgeneralization of proportionality are related to impulsive disposition. The disposition of spontaneously proceeding with an action that comes to mind is related to intuitive cognition, which according to Fischbein (1987) has the characteristics of being self-evident, intrinsically certain, and implicit. An intuitive idea, when triggered, generally compels a person to respond without doubts about its appropriateness. Tirosh and Stavy (1999) have identified various intuitive rules to account for students’ tendency to respond in a certain way to a wide variety of conceptually unrelated situations. Students are impulsive because they tend to approach a problem with conceptual tools that are familiar to them, a phenomenon typically known as the *Einstellung* effect or spurious-correlation effect. Ben-Zeev and Star (2001) explained the spurious-correlation effect using a two-phase process: A person first conceives an association between a problem feature and an algorithm for solving the problem, and then uses the algorithm upon perceiving the feature in another problem. Students who lack the ability to monitor and regulate their cognitive processes are unlikely to overcome their impasse due to being fixated on a particular procedure. Schoenfeld (1985) found that approximately 60% of students’ solution attempts were somewhat like “read, make a decision quickly, and pursue that direction come hell or high water” (Schoenfeld, 1992, p. 356).

When one consistently responds to situations with an impulsive disposition, then one’s impulsive disposition is considered a cognitive style–“a person’s typical or habitual mode of problem solving, thinking, perceiving and remembering” (Riding & Indra, 1991, p. 194). The term *conceptual tempo* refers to a cognitive style that is along the impulsivity-reflectivity dimension (Kagan, 1965). Nietfeld and Bosma (2003) describe *impulsives* as “individuals who act without much forethought, are spontaneous, and take more risks in everyday activities” (p. 119) and *reflectives* as “more cautious, intent upon correctness or accuracy, and take more time to ponder situations” (p. 119).

Impulsive disposition is more useful as a theoretical construct than impulsive tempo in mathematics education because the latter is a cognitive-personality trait that is rather stable across time and across situations whereas the former is a cognitive tendency which, depending upon the circumstances, may or may not actualize. Conceptualizing impulsivity as a disposition allows teachers to help students improve their disposition from being impulsive to being analytic. In addition, students can become progressively more analytic as they expand the repertoire of situations in which they analyze the problem instead of seeking a procedure or formula to apply.

Impulsive disposition is considered a habit of mind because it has the *habituated* characteristic as well as the *thinking* characteristic of a habit of mind (see Lim & Selden, 2009). Impulsive disposition characterizes one's *way of thinking* (Harel, 2008b) in the context of problem solving. Harel (2008b) defines way of thinking as "a cognitive characteristic of a mental act [and] such a characteristic is always observed from observations of ways of understanding" (p. 269). He defines ways of understanding as "a particular cognitive product of a mental act carried out by an individual" (p. 269). A way of thinking differs from a way of understanding in that the latter "conveys the reasoning one applies in a local, particular mathematical situation" whereas the for former "refers to what governs one's ways of understanding, and thus expresses reasoning that is not specific to one particular situation but to a multitude of situations" (Harel and Sowder, 2005, p. 31).

In this study, we regard impulsive disposition as a way of thinking, and the association between proportion and missing-value problems as a way of understanding. Consequently, overgeneralization of proportionality can be viewed as an instantiation of impulsive disposition.

Overgeneralising Proportionality

An important aspect of understanding proportionality is "knowing what it is *not* and when it does *not* apply" (Lamon, 2007, p. 647). Unfortunately, the emphasis on proportional strategies in mathematics classrooms has led students to develop a disposition to associate certain characteristic of problem formulation with the use of proportional strategies. In their study involving Belgium students in Grades 2-8, Van Dooren, De Bock, Hessels, Janssens, and Vershaffel (2005) found that the inappropriate use of proportional methods is most prominent in Grade 5 where students are extensively taught to reason proportionally.

> [T]he inappropriate use of proportional methods would considerably increase in Grades 4 and 5 (when students [in Belgium] are intensively trained in proportional reasoning and receive a lot of missing value problems with a proportional structure). … After this peak, the percent of proportional errors gradually decreased, indicating that—besides the ability to actually calculate the missing value in a proportion—certain students from sixth grade on also gradually acquired another essential subskill of proportional reasoning, namely, the ability to distinguish between situations in which a proportional method is applicable and situations in which it is not. However, considering that still more than one fifth of the answers in eighth grade contained a proportional error, this subskill still seemed not yet fully acquired in eighth grade. (p. 80-81)

The inappropriate use of a proportion to solve a missing-value problem (MVP) that does not involve a direct-proportion situation is also common among pre-service teachers (Cramer, Post, & Currier, 1993; Lim, 2008b; Monteiro, 2003). In Monteiro's study involving 19 pre-service elementary teachers, the use of a proportion ($a/b = c/x$) accounts for all 11 wrong solutions for an additive-relation problem ($y = k + x$) and 6 out of 8 wrong solutions for an inverse-proportion problem ($y = k/x$). Lim (2008b) found that, after a one-semester course on rational numbers and algebraic reasoning, pre-service K-8 teachers (n = 124) performed better on the post-test than on the pre-test for all four direct-proportion items, but worse for all three non-direct-proportion items. These findings inspired us to find ways to help pre-service teachers overcome their impulsive disposition of using proportional strategies to solve MVPs without analyzing the problem situation.

De Bock, Van Dooren, Janssens, and Verschaffel (2002) identified four factors to account for students' tendency to overgeneralize proportionality in the context of scaling a figure: (a) intuitiveness of linear relationships to the extent that it becomes "inaccessible for introspection or reflection" (p. 327), (b) conviction in the applicability of linearity to the extent that correct non-proportional solutions were immediately rejected, (c) shortcomings in geometrical knowledge, and (d) inadequate habits and beliefs about solving word problems. Such factors contribute to a *deficient mathematical modeling process*—one that "mainly occurred on the basis of 'reflex-like' recognizing, and is almost immediately translated in calculations" (p. 329). An effective instructional strategy will therefore need to help students deepen their mathematical understanding as well as develop an analytic disposition towards problem solving.

Lim (2009a) explored the use of superficially-similar but structurally-different missing-value problems to draw student attention to the mathematical structure underlying those problems. Consider the following pairs of missing-value problems.

- Two different candles, P and Q, lighted at the same time were burning at different, but constant, rates. When candle P had burned 16 mm, candle Q had burned 10 mm. When candle Q had burned 35 mm, how many mm, how many mm would candle P have burned?
- Two identical candles, A and B, lighted at different times were burning at the same constant rate. When candle A had burned 20 mm, candle B had burned 12 mm. When candle B had burned 30 mm, how many mm would candle A have burned?

Out of 28 pre-service 4-8 teachers, 22 used the same strategy to solve both problems; 17 of whom used a proportional strategy and 5 students used an additive strategy. These problems are effective in helping students realize the danger of indiscriminately applying a proportional strategy. These problems also highlight the importance of analyzing and identifying co-varying quantities and invariant relationship in the problem. Whereas the first problem involves an invariant ratio of 1.6 (candle P burned 1.6 times as fast as candle Q), the second problem involves an invariant difference of 8mm (candle A burned 8mm more than candle B), as shown in Figure 2.

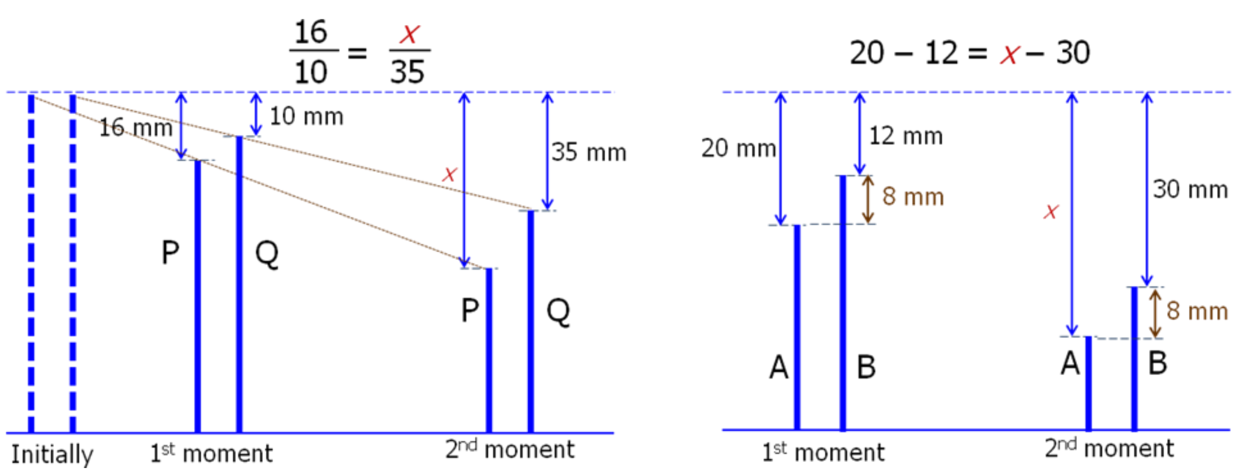

Figure 2. Using diagrams to contrast invariant ratio and invariant difference

Lim (2009) created several missing-value problems, all involving the context of burning candles, to highlight that (a) a ratio is invariant in a proportional or multiplicative-comparison situation, (b) a difference is invariant in an additive-comparison situation, (c) a product is invariant in an inverse-proportional situation, and (d) a sum could be invariant in certain situations.

A Quasi-Experimental Study

A two-part study involving two sections of a mathematics course for pre-service 4-8 teachers was conducted to (a) investigate the short-term impact of using non-proportional MVPs to minimize pre-service 4-8 teachers' improper use of proportional strategies, and (b) explore the possibility of helping these students improve their disposition, from being impulsive to being analytic, in one semester.

In Part I (first three weeks of semester), two pairs of MVPs were used as in-class activities for the treatment group (N_T = 35). Each pair consisted of a proportional MVP and a non-proportional MVP (e.g., *It takes approximately 30 hours for 4 workers working simultaneously to paint an entire motel. Approximately how long will it take 6 workers to repaint the entire motel?*). In the control group (N_C = 42) all four items were proportional MVPs. In both classes, a multiple-choice pre-test was administered on the first day of class and a structurally equivalent post-test was administered in the third week of class. Each test consisted of six non-proportion MVPs, three non-ratio comparison problems (CP), two proportion MVPs and two ratio CPs. For non-proportion MVPs and non-ratio CPs, certain answer choices are considered impulsive. The two proportional MVPs and two ratio CPs did not have impulsive answer choices and were therefore grouped together as one problem type.

In the 2 (class) × 3 (problem type) × 2 (test) analysis of variance with the percentage of correct responses being the dependent variable, there were main effects of Test (p = .013) and of Type (p < .001). There was, however, no main effect of Class nor were there any interactions involving Class. In the 2 (class) × 2 (type) × 2 (test) ANOVA with the percentage of impulsive responses being the dependent variable, there was an Class × Test interaction effect (p = .025), with a substantial pre-to-post increase of 15.5% in impulsive responses for the control class but only a small increase of 1.8% for the treatment class. These results suggest the use of non-proportional items may decrease the number of impulsive responses but may not necessarily increase the number of correct responses.

In Part II (last twelve weeks of semester), both groups underwent the "same" intervention. The lessons were designed whenever possible to elicit students' impulsive disposition so that they could become cognizant of it and make conscious attempts to overcome it. Some test items were designed to be superficially similar to but structurally different from those they experience in class or homework in order to penalize them for being *impulsive*. For example, students were asked to "write the terminating decimal 0.252525 as a fraction in its simplest form" in a take-home examination, after they had recently learned the procedure for converting a repeating decimal such as 0.151515… into a fraction. The lessons were designed and implemented according to Harel's *DNR-based instruction* (2007, 2008). The goal was to help students inculcate efficacious ways of thinking while developing mathematical understanding. The lessons were aimed at helping pre-service teachers develop two specific ways of thinking: *quantitative reasoning* (Sowder et al., 1998) and *referential-symbolic reasoning* (Harel, 2007). At the end of the semester, the pre-test was administered again (i.e., end-test) as part of their final examination. To avoid teaching to the test, the contexts of the non-ratio and non-proportion problems used in the lessons were generally different from those used in the end-test, and the solutions to both the pre-test and post-test were not discussed.

Table 1 shows the percentage of correct scores and percentage of impulsive scores for various problem types across the three tests for the two classes (N_T = 31 and N_C = 35 because a few students had dropped). In both classes, there was a progressive improvement for proportional MVP and ratio CP, as well as for non-ratio CP. As for non-proportional MVP, the gain was marginal from pre-test to post-test but large from post-test to end-test for the treatment class. There was an initial drop from pre-test to post-test followed by a substantial gain in the end-test for the control class. In terms of impulsive responses, there was a decrease from pre-test to end-test for non-proportional MVP for both classes. For non-ratio CP problems, only the treatment class showed a decrease from pre-test to end-test.

Table 1 *Comparisons of Mean Percentage of Correct Responses and Impulsive Responses*

	Control Class (N_C = 35)			Treatment Class (N_T = 31)		
	Pre-test	Post-test	End-Test	Pre-test	Post-test	End-Test
Correct Responses						
Prop MVP & Ratio CP (4 items)	61%	70%	78%	57%	68%	76%
Nonprop MVP (6 items)	41%	34%	63%	41%	46%	69%
Non-ratio CP (3 items)	51%	52%	57%	45%	55%	65%
Impulsive Responses						
Nonprop MVP (6 items)	41%	57%	25%	37%	42%	22%
Non-ratio CP (3 items)	36%	44%	36%	47%	42%	34%

In the 2 (class) × 3 (type) × 3 (test) ANOVA with the percent of correct answers as the dependent variable, there were main effects of Test ($p < .001$) and of Type ($p < .001$). In the 2 × 2 × 3 ANOVA with the percent of impulsive responses as the dependent variable, there was main effect of Test ($p < .001$), but not of Type ($p = .214$). Items were less likely to be answered impulsively in the end-test (29.6%) than in the pre-test (40.2%) or post-test (46.4%). In both analyses, there was no main effect for Class nor were there any interactions involving Class, possibly because the treatment class and the control class had the same intervention from Week 4 to Week 15.

The significant improvement from pre-test to end-test for both groups suggests students' overgeneralization of proportionality can be minimized in one semester. Nevertheless, the reduction in impulsive disposition is context-dependent and may not transfer into other mathematical domains such as impulsively using the Pythagorean theorem or the quadratic formula. Impulsive disposition can be regarded as a negative habit of mind. Being a habit, it is not easily addressed and may require several semesters over multiple domains to overcome.

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Acknowledgement

This research was supported in part by the Teachers for a New Era, a project on teacher education funded by the Carnegie Corporation of New York, which has provided partial faculty support for this study.