Making Actions in the Proving Process Explicit, Visible, and "Reflectable"

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ABSTRACT. We describe the practices of a team of U.S. university teacher/researchers who were invited to attempt to alleviate students' proving difficulties in an undergraduate real analysis course by offering a voluntary "proving skills supplement." We analyze what happened in the supplement and why it happened in terms of our theoretical perspective concerning actions in the proving process. This perspective includes that the proving process is a sequence of actions, some of which are not visible or are difficult to recall, and that understanding the justification for an action differs from a tendency to execute it autonomously. Also, the real analysis course and that teacher's somewhat traditional style of teaching are briefly described, and a comparison is made between proofs co-constructed in the supplement and proofs assigned in the real analysis course. Finally, some student difficulties, views of the supplement given by three students, and the effect of the supplement are briefly discussed.

Keywords: real analysis, proof, supplemental instruction, behavioral schemas, actions

1. Introduction

It is well known to mathematicians who regularly teach proof-based courses, such as abstract algebra or real analysis, that undergraduates have difficulty constructing proofs. Such difficulties have been reported in the literature (A. Selden & J. Selden, 2008). In addition, there have been papers at the undergraduate level that have described university teachers' classroom practices (e.g., Arcavi, Kessel, Meira, & Smith, 1998; Yackel, Rasmussen, & King, 2000; Weber, 2004). This paper contributes to this literature by describing the practices of a team of U.S. university teacher/researchers who were invited to attempt to alleviate students' proving difficulties in an undergraduate real analysis course by offering a voluntary "proving skills supplement." That supplement has been conducted twice, once in Fall 2008 and again in Spring 2009, and it is currently being implemented again in Spring 2010. Here we mainly report on what happened, and why it happened, in the second version of the supplement -- a design experiment (Cobb, Confrey, diSessa, Lehrer, & Shauble, 2003).

We describe our theoretical perspective concerning behavioral schemas and actions in the proving process. We then briefly discuss the real analysis course, that teacher's somewhat traditional teaching, and make a comparison of actions used in proofs co-constructed in the supplement with actions used in proofs assigned for homework. Finally, we report on some student difficulties, views of the supplement's effectiveness as given by three students, and the effectiveness of the supplement.

2. Our Theoretical Framework

The first, third, and fourth authors constituted the teaching team for the second version of the supplement. Our instructional decisions were informed by our theoretical framework.

2.1 Actions in the proving process

We see much of the conscious part of the proving process as a sequence of mental and physical actions, such as writing or thinking a line in a proof, drawing or visualizing a diagram, reflecting on the results of earlier actions, or trying to remember an example. As a person gains experience, much of proof construction appears to be separable into sequences of small parts, consisting of recognizing a situation and taking a mental or physical action. Actions which once may have required a conscious warrant can become automatically linked to triggering situations. We view such small, automated <situation, action> pairs as persistent mental structures that we have called *behavioral schemas*. Such behavioral schemas are a form of procedural knowledge, that is, knowing how to do something, as well as "knowing *to* act in the moment" (Mason and Spence, 1999). We do not see behavioral schemas as necessarily part of fixed sequences that one might regard as procedures or as the implementation of algorithms. (For a detailed description of behavioral schemas, see J. Selden & A. Selden, 2008 or Selden, McKee, & Selden, 2010.)

A number of helpful actions, such as looking up definitions and relevant theorems, drawing a sketch, or constructing an example, are not visible in a final written proof. Often the reasoning behind choosing certain actions, such as deciding to use cases, is also not visible in a final written proof. We are attempting to make such actions and the reasons for them visible or noticeable by orchestrating actual or vicarious student experiences of these actions. Our intention was that the supplement students, through reflection and practice, would become better able to carry out appropriate actions autonomously, and ultimately easily.

However, students can sometimes develop detrimental behavioral schemas. Changing detrimental behavioral schemas often requires more than just understanding the need for a change because such schemas amount to ingrained habits of mind (Margolis, 1993; Selden, McKee, Selden, 2010). For example, it is often not enough to show algebra students that $\sqrt{a^2 + b^2} \neq a + b$ by presenting them with a counterexample, as understanding the reason why such schemas are incorrect may not be enough to influence their behavior. It appears that often students must actually carry out the action correctly a number of times.

We see our perspective on behavioral schemas, or habits of mind, as consistent with the views of psychologists like Bargh (1997) who have discussed the automated nature of actions in everyday life. However, to our knowledge, these psychologists do not employ a theoretical framework such as ours.

2.2 The formal-rhetorical and problem-centered parts of a proof

We divide a proof into its formal-rhetorical and problem-centered parts. The *formal-rhetorical* part of a proof is the part that depends only on unpacking and using the logical structure of the statement of the theorem, associated definitions, and earlier results. In general, this part does *not* depend on a deep understanding of, or intuition about, the concepts involved or on genuine problem solving in the sense of Schoenfeld (1985, p. 74). In constructing this part of a proof, we believe students need to develop a number of beneficial behavioral schemas. That is, they need to know how to act, and to actually act, in various proving situations. We suspect this is learned primarily from practice at constructing proofs and are attempting to facilitate learning through such practice.

We call the remaining part of a proof the *problem-centered* part. It is the part that *does* depend on genuine problem solving, intuition, and a deeper understanding of the concepts involved. Here, too, behavioral schemas appear to play a major role. For example, a student can come to recognize situations in which drawing a sketch or diagram might be useful and link such situations to the act of starting a drawing.

Our distinction between the formal-rhetorical and the problem-centered parts of a proof is similar to Weber and Alcock's (2004) distinction between syntactic and semantic proofs, except that we refer to parts of proofs rather than to whole proofs. Many real analysis proofs, such as the product of continuous functions is continuous, have formal-rhetorical and problem-centered parts, both of which are nontrivial for many students. In facilitating the construction of such proofs, we have found it helpful to encourage students to construct the formal-rhetorical part first. Doing so can be very useful in revealing the "real problem" to be solved, that is, the problem-centered part (Selden & Selden, 2009).

3. Descriptions of the Real Analysis Course and the Supplement

The real analysis course is offered regularly at a doctoral-granting southwestern U.S. public university. It is a one-semester, three-credit junior level class whose contents include sets and functions, supremum and infimum, sequence convergence, Cauchy sequences, accumulation points, limits of functions, continuity, and closed sets.[1] It serves three populations: mathematics majors, pre-service secondary mathematics teachers, and beginning mathematics graduate students needing remediation.

In an interview, Dr. R, the real analysis teacher, said that the course "tries to be all things to [all] students and it is virtually impossible in three hours per week to accomplish all of that." In response to why she had invited us to provide a supplement Dr. R said, "Students in my opinion learn to do proofs by doing proofs rather than [by] reading them or doing exercises" and that this cannot always be done in the normal class setting. She went on to mention our "proofs" course designed to improve beginning mathematics graduate students' proving abilities (described in Selden, McKee, & Selden, 2010). She thought the same thing would be helpful for her real analysis students.

The real analysis course met twice a week for 75 minutes and was taught in a somewhat traditional manner by Dr. R, who was not one of the researchers. There were 18 students enrolled in the course. While the course instruction was primarily teacher-directed, class discussion, questions and input from the students were encouraged. In fact, Dr. R reports that the students' inputs were paramount to her moving the instruction forward. Proofs were regularly presented in a manner somewhat similar to the instruction in the supplement, using the side board for definitions or scratch work. In particular, Dr. R made an effort to emphasize the fact that proofs are generally not produced from the top down.

The students who attended the supplement did so on a voluntary basis, and every effort was made to conduct it at a time when almost every student in the real analysis course could attend. The supplement was offered once a week for 75 minutes, making a total of one-third of

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[1] The book used for this course was Gaughan's *Introduction to Analysis*, 5th edition.

the class time for those students who chose to attend. The number of students attending the supplement ranged from 6 to 10. Each week, one proof problem was selected or created to resemble an assigned homework problem that would be graded in detail and that could subsequently be improved and resubmitted for credit. The proof problem was selected or created to call for actions similar to those required to prove the corresponding assigned homework problem. The selected problem was *not* a template for the corresponding homework problem. (See Figure 1.)

The instruction in the supplemental class had certain similarities with those of Dr. T (described in Weber, 2004), except that our students co-constructed all the proofs themselves. Also, Dr. T did not use diagrams or graphs in presenting sets, functions, and limits of sequences. But our supplement students attempted to use them whenever they thought these would be helpful, especially in constructing the problem-centered part of a proof.

Our instruction was consistent with a constructivist approach in that every effort was made to maximize the students' opportunity to perform the necessary actions themselves, thereby facilitating their construction of knowledge from their own experiences. It also had a Vygotskian perspective, because on the one hand, we served the role of more knowledgeable others, and on the other hand, we commented on the mathematical community's genre of proof, which the students began to adopt. The fourth author wrote the statement of the selected supplement theorem on the board, after which the students themselves, or a supplement teacher if need be, offered suggestions about which actions to do next. For each suggested action, such as writing an appropriate definition, drawing a sketch, or introducing cases, one student was asked to carry out the action at the blackboard. The other students were thus able to experience that action vicariously. This process was introduced with the intention that the students would reflect on what had occurred and later perform this or a similar action autonomously on their assigned homework. All students were encouraged to participate in co-constructing the proof, but of course not every student could perform every action.

The supplement students were encouraged to first co-construct the formal-rhetorical part of the proof. This consisted of first writing the hypotheses at the beginning of their proof. Then, leaving a space for the body of the proof, they would write the conclusion at the end of their proof. Next students would unpack the conclusion and write the relevant definitions, such as that of sequence convergence, on the side board, which had been set aside for "scratch work." Then the students would change the notation in the definition to "match" that of the theorem to be proved. They would then examine this definition to see where to start and end the body of the proof. For example, if the proof problem were one of showing a sequence $\{a_n\}_{n=1}^{\infty}$ converges to A, then they would write "Let $\varepsilon > 0$" immediately after the hypotheses, leave a space for the determination of N, write "Let $n \geq N$," leave another space, and finally write "Then $|a_n - A| < \varepsilon$" prior to the conclusion at the bottom of their proof. This brought them to the problem-centered part of the proof, where some "exploration" or "brainstorming" on the side board would ensue. The entire co-construction process, and accompanying discussions, was a slow one – so slow that only one theorem was proved and discussed in detail in each supplement class period.

During the entire co-construction process, student discussion and questions were actively encouraged, and we agree with Yackel, Rasmussen, and King (2000) that "when the classroom norm is that of making sense of other student's reasoning, class discussions often form the basis for students to further their own mathematical development." Towards the end of each supplemental class period, students were given a handout that went through *a* proof of the supplement theorem and described the actions of a hypothetical proof co-construction trajectory (Simon, 1995). Of course, this handout was not identical to the proof the students had produced, but was close enough and sufficiently detailed so that the actions were exposed.

4. Data Collection

The supplement was videotaped and field notes were taken. The supplement teachers and Dr. R met following each supplemental class period to review what happened and plan for the next supplemental class period. To inform her own instruction, Dr. R was particularly interested in student misconceptions or difficulties that had surfaced during the supplement classes. Further, in her lectures, she would point out some proving actions in order to reinforce the supplemental instruction; for example, she would unpack the conclusion of a theorem on the sideboard. In addition, all real analysis students' homework and tests were photocopied for later analysis. The first author attended one-quarter of Dr. R's classes and took field notes.

A semester after the real analysis course was over, the first two authors interviewed Dr. R, as well as three of the supplement students who had regularly attended. These interviews were videotaped.

5. Example of a Supplement Proof and a Paired Real Analysis Homework Problem

In Figure 1, we give an example of a supplement proof and a paired real analysis homework problem. The supplement proof is on the left, and the homework problem, taken from the textbook, is on the right. The actions in the proving process are numbered in bold square brackets (e.g., **[1]**). These indicate the order in which the actions could have been implemented, and for the supplement proof, they represent both the hypothetical co-construction trajectory and roughly the order in which the actions actually occurred. Notice that the supplement proof is not a template for the homework problem, but the lists of actions for the two proofs are very similar.

Problem from the Supplement:

Theorem. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be sequences, both converging to P. If $\{c_n\}_{n=1}^{\infty}$ is the sequence given by $c_n = \begin{cases} a_n, & n \text{ even} \\ b_n, & n \text{ odd} \end{cases}$, then $\{c_n\}_{n=1}^{\infty}$ converges to P.

Proof. [1] Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be sequences, and P be a number so that $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ both converge to P. Suppose $\{c_n\}_{n=1}^{\infty}$ is the sequence given by $c_n = \begin{cases} a_n, & n \text{ even} \\ b_n, & n \text{ odd} \end{cases}$. **[6]** Let $\varepsilon > 0$. **[9]** There is a positive integer N_a so that for every positive integer i, if $i \geq N_a$, then $|a_i - P| < \varepsilon$. **[9]** Also There is a positive integer N_b so that for every positive integer j, if $j \geq N_b$, then $|b_j - P| < \varepsilon$. **[7,10]** Let $N = \max\{N_a, N_b\}$. Let $n \geq N$ be a positive integer.
[11] Case 1: Suppose n is even. Then **[8]**$|c_n - P| =$ **[12]**$|a_n - P| < \varepsilon$, because $n \geq N_a$.
[11] Case 2: Suppose n is odd. Then **[8]**$|c_n - P| =$ **[12]**$|b_n - P| < \varepsilon$, because $n \geq N_b$. **[3,4,5]**
In either case, $|c_n - P| < \varepsilon$. **[2]** Therefore, $\{c_n\}_{n=1}^{\infty}$ converges to P.

Actions in the Proving Process:
[1] Write first line
[2] Write last line
[3] Unpack conclusion
[4] Write definition of convergence
[5] Change notation in the definition of convergence
[6] Let $\varepsilon > 0$
[7] Let N=? Let $n \geq N$ be a positive integer.
[8] $|c_n - P| \leq \ldots < \varepsilon$.
[9] Use definition of $\{a_n\}$ and $\{b_n\}$ converging to P
[10] Find N using N_a and N_b.
[11] Break into cases because of how c_n is defined.
[12] Algebra

Paired Homework Problem from the Textbook:

Theorem. $\{a_n\}_{n=1}^{\infty}$ converges to $A \Leftrightarrow \{a_n - A\}_{n=1}^{\infty}$ converges to 0.

Proof. [1] ($\Rightarrow$) **[2]** Suppose $\{a_n\}_{n=1}^{\infty}$ converges to A. **[7]** Let $\varepsilon > 0$. **[10]** By definition, there exists a positive integer N such that for all $n \geq N$, $|a_n - A| < \varepsilon$. **[8]** Let $n \geq N$ be a positive integer. **[4,5,6,9]** Then $|(a_n - A) - 0| =$ **[11]** $|a_n - A| < \varepsilon$. **[3]** Therefore $\{a_n\}_{n=1}^{\infty}$ converges to 0.
[1] ($\Leftarrow$) **[2]** Suppose $\{a_n\}_{n=1}^{\infty}$ converges to 0. **[7]** Let $\varepsilon > 0$. **[10]** By definition, there exists a positive integer N such that for all $n \geq N$, $|(a_n - A) - 0| < \varepsilon$. **[8]** Let $n \geq N$ be a positive integer. **[4,5,6,9]** Then $|a_n - A| =$ **[11]** $|(a_n - A) - 0| < \varepsilon$. **[3]** Therefore $\{a_n\}_{n=1}^{\infty}$ converges to A.

Actions in the Proving Process:
[1] Break into two parts for the “if and only if”
[2] Write first line
[3] Write last line
[4] Unpack conclusion
[5] Write definition of convergence
[6] Change notation in the definition of convergence
[7] Let $\varepsilon > 0$
[8] Let N=? Let $n \geq N$ be a positive integer.
[9] $|(a_n - A) - 0| \leq \ldots < \varepsilon$.
[10] Use definition of a_n converging to A
[11] Algebra
[12] Repeat for the opposite direction

Figure 1. Example of a supplement proof and a paired real analysis homework problem.

6. Preliminary Observations

6.1 Students' general difficulties

We have noticed four student difficulties that were general in the sense that they were not associated with specific aspects of mathematics, but were not as general as, say, lack of sufficient discipline to do homework or attend class. These are surprising because they occurred in a relatively small student population, almost all of whom were moderately successful in Dr. R's and our judgment. That is, they appeared to be neither the most, nor the least, successful students.

When it was necessary to look for the statement of an appropriate definition or theorem in their textbooks or notes, we found several students were not even turning the pages. In addition, some students were unable to copy a definition accurately onto the sideboard, occasionally while holding the open book. Some also had difficulty altering the notation in a definition or theorem to match that of the current argument. Finally, when explaining their own co-constructed proofs, a few students had difficulty reading a proof aloud, in the sense that it was hard for them to articulate some of the words and symbols. This interests us because much thought in the proving process appears to involve inner speech, and we suspect that inarticulateness in normal speech may correspond to similar inarticulateness in inner speech, and thus, in thinking.

We suspect that students who eventually become skilled at constructing proofs overcome such general difficulties. Thus we suggest that competence at constructing proofs should also improve other, nonmathematical kinds of reasoning, and wonder whether this might partly account for the popular idea that studying mathematics "improves reasoning." Inglis and Simpson (2008, 2009) have examined whether the study of advanced mathematics contributes to the development of abstract reasoning abilities, which we do not see as entirely equivalent to reasoning. They provided evidence for the hypothesis that such study contributes to the development of certain logical skills, in particular, to the ability to recognize certain invalid inferences.

6.2 Students' mathematical difficulties

Early in the supplement our students often started constructing a proof by attempting to unpack and use the hypotheses immediately, rather than looking at the conclusion to see what was to be proved. (For an elaborated example, see the case of Willy in Selden, McKee, & Selden, 2010.) Gradually this difficulty was alleviated by having the students start co-constructing the formal-rhetorical part of proofs. However, it seems to take a long time, perhaps half a semester, for students to become reasonably fluent with autonomously constructing that part of a proof.

In a beginning real analysis course in which proofs often involve both universal and existential quantifiers, one of the benefits of starting by co-constructing the formal-rhetorical part of a proof is that the variables are automatically introduced in the order of their occurrence in what is being proved. Some of our students questioned whether it was necessary to preserve the order of the variables, even after seeing examples of statements for which changing the order of the universally and existentially quantified variables changed one statement from true to false. They did not seem to grasp this point until at least the midpoint of the course.

We also noticed that many students had difficulty when we encouraged them to "explore" or "brainstorm" about the problem-centered part of a proof, and a few did not seem to have the concept of "exploring" or "brainstorming" in mathematics. In addition, they were sometimes unable to make an appropriate sketch at the appropriate time in the proof construction process.

Further, in showing a sequence $\{a_n\}_{n=1}^{\infty}$ converges to A, they often had difficulty choosing an appropriate natural number N and showing for $n \geq N$ that $|a_n - A| < \varepsilon$.

6.3 Students' views of the supplement

The three interviewed supplement students all responded very positively concerning the supplement and what they had learned therein. When asked how the supplement had impacted how they constructed proofs in their subsequent courses, they replied that they *now* know where to start a proof, know how to unpack the conclusion, know how to use definitions, and know how to use "fixed, but arbitrary" [in proofs of convergence and continuity where one begins "Let $\varepsilon > 0$"]. When we asked one student how we could improve the supplement, he suggested that the supplement meet twice a week. Later he informally mentioned that he would like us to provide a similar supplement for his introductory abstract algebra course.

During the semester, there were occasional informal comments indicating that students enjoyed attending the supplement. Several times during the semester, a student, who for some reason could not attend the supplement, stopped by to collect one of the handouts. Also, one student, who was not recognized by us or Dr. R, came to the supplement because he had heard about it and was interested.

6.4 Effect of the supplement

Dr. R and the supplement students believed that the supplement was helpful in developing the students' proving skills. We have analyzed supplement students' real analysis homework in an effort to gain more insight into this belief. In addition, we have compared some of the homework that the supplement students turned in to Dr. R with that of those who did not attend the supplement. Preliminary evaluation of homework papers indicates students who attended the supplement wrote their proofs in a more concise, clear manner.

In describing the attempts of the supplement students to produce a proof on tests, Dr. R said "I would see the first line [the hypotheses], I would see the last line [the conclusion]… I can see the technique… some more obvious than others but most definitely it was on the test." As the semester progressed, students attending the supplement knew what to do next with less prompting or help from the supplement teachers.

7. Discussion

It is well accepted that reflection is a part of constructing knowledge. But, when speaking of reflection as part of constructing knowledge, mathematics educators are generally referring to conceptual knowledge. However, in this paper, we have proposed that reflection can be applied to a type of procedural knowledge, that is, knowing *how* to do something. In particular, it can be applied to actions in the proving process. Such actions often occur in constructing the formal-rhetorical part of a proof, but can also occur in the problem-centered part.

Specifically, we suggest that reflection (or even rehearsal), along with practice, can be used to strengthen the link between certain proving situations and appropriate corresponding actions in the proving process, turning them into behavioral schemas. These can then become a routine part of the proving process, especially the formal-rhetorical part. Doing this can unburden one's working memory so that one can concentrate on the problem-centered parts of proofs (Selden & Selden, 2009).

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