A Confucian Approach to Teaching Algorithms in the Pre-Service Teacher's Program in the United States

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Abstract: As the influential international studies (e.g., TIMSS, 1999, 2003, 2007; PISA, 2003, 2006) showed that Asian students outperformed their American counterparts, Eastern methods of teaching and learning have attracted much attention from Western researchers (Stigler & Hiebert, 1999; Fan, Wong, Cai, & Li, 2004). This study aims to investigate how a Confucian teaching approach benefits pre-service teachers' learning of the base-n number system in the United States. The main differences between the Confucian Teaching Approach (CTA) and the Student-Centered Constructivist Model (SCCTM) (Kirshner, 2002, 2008) were investigated. Twelve pre-service teachers were selected from two sections of the same course for focus group interviews. The transportability in adopting the CTA in the U.S. for pre-service teachers' learning is discussed.

Keywords: Confucian Teaching Approach, Student-Centered Constructivist Teaching Model, Pre-Service Elementary Teachers, Base-n Number System.

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Introduction

Some large-scale international studies showed that Chinese students performed well in math and science. For example, in 2003 TIMSS, Hong Kong, Chinese Taibei, and Singapore ranked in the top four while the U.S. ranked 23rd, slightly higher than the average score in 8th grade math among 49 countries (Mullis, Martin, Gonzalez, & Chrostowski, 2004). In 2003 and 2006 the *Program for International Student Assessment* (PISA), Hong Kong ranked in the top three among participant countries regarding students' math performance. As a result, many researchers (e.g., Stigler & Hiebert, 1999) suggested that Eastern teaching approaches would be valuable to adopt for American classrooms. A broad understanding of Eastern teaching is rooted in Confucian Heritage Culture (Wong, 2004), loosely defined as traditional Chinese beliefs such as Confucianism, Taoism, and Buddhism. In this study, the Confucian approach is narrowed down to the two principles of Confucianism (providing hints and making students struggle mentally) regarding teaching and learning. These two principles are derived from Confucius' collection of sayings in *Analects* and have been practiced for more than two thousand years in China (Diwuhongyan, 2004). Based on these principles, Chinese math educators create their own way (e.g., Qifashi teaching) for math teaching during the past fifty years (Han, 2008). Researchers (e.g., Cao & Zhang, 2006; Wang & yang, 2000) have summarized and explored its historical roots, teaching beliefs, and the teaching strategies in Qifashi teaching. However, there is no consensus regarding a Qifashi teaching model in math education, as Han (2008) pointed out:

> The current research [on Qifashi teaching in math education] mainly focused on the fundamental teaching beliefs, basic requirements, main problems when implementing a lesson, and some teaching examples by using this method. Polya's heuristic methods were adopted as a tool in Qifashi teaching [in China]. …..There is no a consensus for a way to teach by using this method…. (pp. 53-54)

The constructivist approach, in contrast, is well cited in Western literature. During the past thirty years, Western researchers (e.g., Malone & Taylor, 1992; Simon, 1995; Solomon, 1998; Selley, 1999; Kirshner, 2002, 2008; Fosnot, 2005; Gagnon & Collay, 2006; Tobias & Duffy, 2009; Willis, 2009) have developed various instructional approaches and teaching models based on constructivist theory. Among these, I selected Kirshner's Student-Centered Constructivist Teaching Model (SCCTM) from his Crossdisplinary framework to compare to the Confucian Teaching Approach (CTA). Kirshner's Student-Centered Constructivist Teaching Mode was developed based on his Crossdisciplinary Framework (2002, 2008). In terms of constructivist pedagogy, Kirshner developed two forms, student-centered and teacher-centered teaching models. I selected the first one for this study. Kirshner's Student-Centered Constructivist Teaching Model (SCCTM) contains the following aspects:

> The teacher needs to have a model (always tentative) of the student's current conceptual structures, including the limitations of those structures relative to a mature understanding of the particular content to be taught . . . then the teacher helps mediate the student's engagement with the task by (1) monitoring the student's uptake of the task, making minor adjustments to it, as needed, (2) assessing the effectiveness of the task in stimulating development, as intended; this may involve rethinking and revising the model of the student's understanding, and/or the task environment, (3) responding to the students as they engage with the task to help them experience the discrepancies more fully, and (4) encouraging the student through the frustration that arises when conceptual obstacles are encountered. (Kirshner, 2008, p. 14)

Methodology and Research Questions

This is a qualitative study. Sixty college freshmen participated in this study. Most of them are females majoring in elementary education at a mid-sized public university in the mountain region. They enrolled in the same math course in two different sections. All participants were invited to attend a one-hour lesson taught in two different methods, CTA in one section and SCCTM in another section. Six of the participants in each section were randomly selected from the sixty participants for a one-hour focus group semi-structured interview (Morgan, 1988). As well, the participants in this study were asked to write a short lesson reflection regarding their thinking process or the times of struggle. As a Chinese native, I am familiar with both two teaching approaches. In China, I have taught K-12 and undergraduate math for more than 10 years by using CTA. In the USA, I took one course that mainly dealt with developing SCCTM lesson plans at LSU.

Two research questions are selected for this study.

1 What are the main differences of using the Confucian Teaching Approach to designing base-n number lessons compared to using Kirshner's Student-Centered Constructivist Teaching Model?

2 What are the successful and unsuccessful aspects of using the Confucian Teaching Approach compared to using Kirshner's Student-Centered Constructivist Model to teach the base-n number system in the pre-service teachers' class in the USA?

The same math problems (see Appendix) are adopted for the lesson plans for the two teaching approaches. Before attending the experiment classes, all participants have learned how to use toothpicks to represent base-10 numbers for addition and subtraction. Although they learned a variety of methods for addition and subtraction from their textbook (Beckmann, 2008), they do not have knowledge of addition and subtraction in the base-n number system. The teaching goal is to generalize rules and procedures from the base-10 system to the base-n system. The first group of problems aimed to review the algorithm they learned previously. The second group of problems contains three new forms (base-7, base-8, and base-6). The third group problem is a mixed form (base-7 and base-8). Both the second group and the third group of problems are new to the participants.

Data analysis

The Main Differences between CTA and SCCTM regarding task design

SCCTM requires teachers to have a mature model of the target concept and students' possible misconceptions. In the case of the base-n number system, a constructivist teacher should not only understand the place value in different number systems, but also he/she needs to envision the possible misconceptions that students might have when facing the problem. The target concept in this lesson is borrowing and carrying in the base-6, base-7, and base-8 systems. Teachers should have an overall version of the algorithm in the base-n system:

$$(a_1\, a_2\, a_3\, a_4 \ldots a_m)_n = a_1 n^{m-1} + a_2 n^{m-2} + a_3 n^{m-3} + a_4 n^{m-4} + \ldots a_m n^0$$

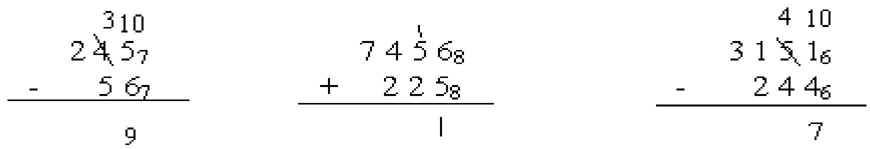
If n = 6, 7, 8, and m=4, this form can be specified as:

$$(a_1\, a_2\, a_3\, a_4)_6 = a_1 6^3 + a_2 6^2 + a_3 6^1 + a_4$$

$$(a_1\, a_2\, a_3\, a_4)_7 = a_1 7^3 + a_2 7^2 + a_3 7^1 + a_4$$

$$(a_1\, a_2\, a_3\, a_4)_8 = a_1 8^3 + a_2 8^2 + a_3 8^1 + a_4$$

Pre-service teachers previously learned that borrowing 1 (or carrying 1) should account for 10 to the next place (or previous place) in the base-10 number system. They may not have knowledge that borrowing 1 should account for 6 (7, or 8) in the base-6 (7, or 8) number system. They probably still use the rules they learned in the base-10 system to calculate the second group of the problems. For example:



This possible misconception also causes another error that the digits used in the base-7 and base-6 number systems is bigger than their bases.

Following the analysis of the concept, SCCTM teachers should intentionally design stumbling blocks for their students in order to provoke cognitive conflicts in students' minds. The lesson plan is tentative and may be revised as teachers' hypothetical learning trajectories do not fit students' actual activities in the class. The whole teaching for SCCTM teachers focus on conceptual understanding. Likewise, the aim of a CTA lesson design is to cause students mental struggle. CTA teachers do not need to envision students' hypothetical learning trajectories. Rather, they should focus on *Sheyi* (creating questions that seem conflicting), a technique for the task design that helps a CTA teacher put students in an conflicting situation. There are no teachers' predictions of their students' hypothetical learning trajectories in the CTA task design.

The successful aspects of CTA teaching

Since the main purpose of this study is to understand if CTA can be adopted to the US classroom, the data analysis will focus on the successful and unsuccessful aspects of the CTA. SCCTM served as a reference to support my arguments.

The successful aspects reflected in CTA teaching are the effective use of hints in class and the short lecture guidance. There are two kinds of hints provided in the CTA class: verbal hints and non-verbal hints. The verbal hints are derived from Polya's analogy. The verbal hints are used if a student is puzzled with the starting point—he/she may not know where he/she should go to start. Non-verbal hints are teachers' gestures used if a student is wrestling with the problem during the process of the problem solving.

The verbal hints I provided in CTA class are to ask students if they can draw pictures for the second group of the problems. Then I walked away. After awhile, I came back to see if the students had figured out how to draw a picture. If they were still wrestling, I asked them to go back to the previous problems and explain them to me. As they explained the problems, I reminded them to use similar thinking on the new problems. This teaching strategy worked very well. One of the participants reflected, "I didn't understand when to go to the next place in any base outside of base-10 until you asked me to draw a picture. Now I understand it much better."

Using Polya's analogy to provide verbal hints in CTA class is easier than the helping method suggested by SCCTM. Both experienced teachers and new teachers can learn the strategy for providing hints this way. For instance, the same hint drawing a similar picture can be given to the students who had different representations. However, in SCCTM class, I needed to check different types of the learning trajectories if students had different representations. This is difficult for a new teacher or a teacher who has just started to use SCCTM method for teaching.

The unsuccessful aspects of CTA teaching

Mental struggle did not benefit all students learning The first unsuccessful aspect is that at least half of the participants did not experience mental struggle from participants' written responses in CTA class. Among 30 participants, only nine of them claimed they had an Aha moment. Three of them felt the task was easy. Ten of them claimed they did not like the mental struggle. Four of them claimed they did not understand, even though I provided hints to them. Four of them did not respond to my request.

Participants who claimed they did not like mental struggle preferred to get help immediately as indicated in interview data. Once they had a question in CTA class, they tended to immediately ask their peers for assistance. One student said in her interview:

> I wasn't really sure where to start, so I asked somebody where they started, and [they] said to draw a picture. So I started drawing a picture, and I think I got it wrong once, and I asked them what they got and it was a different answer. So then I went back and figured out what I did wrong, and then I got the right answer.

Teacher-centered teaching did not fit in CTA class More than half of the participants in CTA class, no matter whether they experienced mental struggle or not, had a strong tendency to advocate group discussion.

Conclusions

This study aimed to understand if the Confucian Teaching Approach (CTA) can be adopted in the US pre-service teachers' training program. Meanwhile, the main differences between CTA and Student-Centered Constructivist Teaching Model (SCCTM) were analyzed and compared. The differences of the task design between CTA and SCCTM are as follows. SCCTM requires teachers to have a mature model of the target concept and students' possible misconceptions. A SCCTM teacher should envision hypothetical learning trajectories and intentionally design stumbling blocks for their students in order to provoke cognitive conflicts. The lesson plan is tentative and may be revised as teachers' hypothetical learning trajectories do not fit students' actual activities in the class. Likewise, the aim of a CTA lesson design is to cause students mental struggle. A CTA teacher does not need to envision students' hypothetical learning trajectories. Rather, he/she should focus on *Sheyi* (creating questions that seem conflicting), a technique for the task design that helps a CTA teacher put students in a conflicting situation. There are no teachers' predictions of their students' hypothetical learning trajectories in the CTA task design. To some extent, a CTA design is easier than a SCCTM design for new teachers and teachers who do not have experience with SCCTM design.

This study also concludes that providing hints, both verbal and non-verbal hints, worked very well in the CTA class. However, the key feature of CTA, mental struggle, cannot be adopted in the US class. In addition, US students do not advocate teacher-centered teaching.

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Appendix:

Teaching materials used in the two classes

1 Explanation by using toothpicks (the first group of problems)

$$\begin{array}{r} 149 \\ -\ \ 83 \\ \hline \end{array} \qquad \begin{array}{r} 142 \\ +\ \ 85 \\ \hline \end{array}$$

2 A variety of the problems (the second group of problems)

$$\begin{array}{r} 245_7 \\ -\ \ 56_7 \\ \hline \end{array} \qquad \begin{array}{r} 7456_8 \\ +\ \ 225_8 \\ \hline \end{array} \qquad \begin{array}{r} 3151_6 \\ -\ \ 244_6 \\ \hline \end{array}$$

3 An advanced problem (the third group problem)

$$\begin{array}{r} 3546_7 \\ -\ \ 767_8 \\ \hline \end{array}$$