Preservice Mathematics Teachers' Understanding of Geometry

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Introduction

Geometry has long been a standard course in the high school mathematics curriculum. In his classic work, Fawcett (1938) described the need for secondary students to pursue geometry, both as a prerequisite for post-secondary education as well as for general education. Trafton and LeBlance (1973) highlighted a shift towards including geometry at the elementary school level, justifying such a recommendation by citing its importance in further mathematics, in general education, and in practical applications. With the publication of *Principles and Standards for School Mathematics* (NCTM, 2000) the call for geometry to be studied by all students at all grades is complete, and geometry now holds a significant place throughout the K-12 span. In addition, geometry is now a common presence in the state-level curriculum standards that have become ubiquitous in the past decade. One consequence of this increased prominence of geometry in K-12 schools is that teachers at all levels need experiences studying geometry in order to attain the content knowledge necessary to be effective instructors (e.g. Usiskin, 1987; Swafford, Jones & Thornton, 1997). Thus, many teacher education programs include a course in geometry for their students. This study looks at two separate courses, one for preK-8 teachers, and one for grades 7-12 teachers. The researchers address the issue of whether pre-service mathematics teachers at both the elementary and secondary level possess a strong enough understanding of geometry to teach the subject well, and whether the required geometry course they take enhances their geometric competence.

Historically, as more students encountered geometry, and educational researchers began to think about how children learn mathematics, some focused on the learning of geometry in particular. The core framework that guides the current study is the work of D. Van Hiele Geldof (1957) and P.M. Van Hiele (1957) who posited a theory that students' level of geometric understanding can be explained by looking at a structured heirarchy of stages. The Van Hiele theory draws heavily on the work of L.S. Vygotstky. As cited in Byrnes (2001), Vygotsky describes the process where students have growth in their understanding of concepts only when taught within their zone of proximal development. The zone of proximal development is that region when instruction is most advantageous because the student is most open to exploring new ideas. This idea forms a key piece of the Van Hiele theory, especially as it relates to the process by which instruction in geometry may help students advance from a given level to the next higher level.

The Van Hieles built on Vygotsky and explained that students have five levels of understanding that are achieved sequentially. Teachers must present the material within a student's current level to enable the student to master the content at that level and move on to the next level. Once that material is mastered, the student's zone of proximal development now encompasses the next level. This paper seeks to determine if evidence exists to support additional content work in geometry for preservice elementary and secondary mathematics educators. The study investigates the van Hiele Level of Understanding Geometry in a pre-test/post-test manner, both before and after completion of the geometry course required by the respective teacher education programs.

Background and Conceptual Framework

Early Background: Vygotsky

Vygotsky's Theory, developed in the 1920s and 1930s, describes learning as the process by which a student achieves a mature understanding of a concept or function. This process, as described by Byrnes (2001), consists of taking "pseudoconcepts" and turning them into the more mature "true" concepts. Vygotsky (1962) writes, "… a concept is more than the sum of certain associative bonds formed by memory, more than a mere mental habit; it is a complex and genuine act of thought that cannot be taught by drilling but can be accomplished only when the child's mental development itself has reached the requisite level" (p. 82). A key element to Vygotsky's Theory is the need for teachers to provide instruction to students, called scaffolding, which is within their zone of proximal development. Although the Van Hieles did not use the phrase "zone of proximal development," a clear connection can be drawn between this concept and the complex theory developed by the Van Hieles.

The Van Hiele Theory

The van Hiele model, developed by two Dutch mathematics educators, has been used since the early 1980s to explain why students have difficulty with high school geometry in general and with higher order cognitive processes in particular. The van Hieles (van Hiele-Geldof, 1957; van Hiele, 1957) theorized that there are five ordered levels of understanding that each person must go through to gain complete understanding (scientific insight) into geometry. The theory posits that students who are being taught at a van Hiele level higher than they have achieved, or are ready to achieve, do not attain the level of success in high school geometry that is necessary for further study of geometry and of other subjects that depend on geometric knowledge.

The theory has three aspects, (1) there exist levels of understanding, (2) there are properties that are inherent to each level, and (3) movement to a level needs to be from the previous level. The van Hiele-Geldofs identified the levels using numbers 0-4. Other educators (Hoffer, 1981; Usiskin, 1982; Senk, 1989), recognized that some students do not have a full understanding of geometry even at the recognition level and therefore determined that there is a need for the identification of a level prior to the Van Hiele-Geldof Level 0. These researchers have numerically identified the levels as 1-5 and retained a sixth level, 0, for those who do not have an understanding at least at the recognition level. This study will also utilize the 1-5 numbering schema to allow utilization of Level 0.

The van Hiele Levels

Level 1, labeled by some as recognition and others as visualization (Hoffer, 1981; Usiskin, 1982; Fuys, 1985; Burger & Shaughnessy, 1986; Crowley, 1987; Senk, 1989), is defined as the ability to learn the names of figures and to recognize shapes by their physical appearance but not by their individual parts or properties. The students can learn the geometric vocabulary but do not yet have full understanding of the definition.

Level 2, labeled as analysis, is defined as the ability to identify the characteristics of figures so as to create classes of figures but is unable to explain the relationship between different properties and still does not understand definitions. For example, given a grid of parallelograms the student could use methods such as coloring equal angles to determine that the opposite angles of intersecting lines are equal and, with enough of these examples, the student can determine that for all parallelograms the opposite angles of a parallelogram are equal.

Level 3, labeled as ordering, logical ordering, abstraction or informal deduction (Hoffer, 1981; Usiskin, 1982; Fuys, 1985; Burger & Shaughnessy, 1986; Crowley, 1987; Senk, 1989), is defined as the ability for the students to start establishing an interrelationship between properties, either within a class of figures, or among a class of figures. Students understand the definitions of geometric terms and can follow and give informal arguments. Formal proofs can be followed but the student is unable to reproduce the proof when starting from a different or unfamiliar premise.

Level 4, labeled as deduction, is the point at which a student can understand and see the role of undefined terms, postulates, definitions, theorems, and proof. At this level the possibility of developing proofs from more that one premise is realized and students understand the difference between necessary and sufficient information.

Level 5, labeled as rigor, is defined as the ability to transfer understanding and compare different axiomatic systems. This level is not examined in the current study.

The van Hiele Properties

Within each of the van Hiele levels there exist properties that are common to the levels. These properties are identified as: (a) fixed sequence, (b) adjacency, (c) distinction, (d) separation, and (e) attainment. Fixed sequence is the inability of the student to progress to level n+1 without first having attained level n. Adjacency is the ability to recognize that the properties of an object, which are intrinsic at one level, are extrinsic at the next. Distinction is the ability to use and understand the vocabulary associated with the level. Separation is the inability of two people who are at different levels to understand each other. Many researchers (Usiskin, 1982; Mayberry, 1983; Senk, 1989) believe that it is this property that explains why most secondary school geometry students fail to succeed.

The fifth property, attainment or advancement, outlines the learning process that leads to complete understanding at the next higher level. The key element of this property is that understanding depends on the content and methods of instruction more than on age (Crowley, 1987). The current investigation did not include the design or creation of curricula or instructional techniques. However, the Van Hieles described in great detail what such work might look like and a future direction for research could design and evaluate geometry courses specifically tailored to the van Hiele hierarchy. Mistretta (2000) found limited success in this endeavor. Another group of researchers, although primarily interested in gender differences in learning geometry (Akkaya et al, 2009), also found some success in designing instructional programs consistent with the Van Hiele recommendations.

Geometric Expectations for Students

The NCTM *Principles and Standards for School Mathematics* (2000) provides a description for the Geometry Standard that states: "Instructional programs from pre-kindergarten through grade 12 should enable all students to – (a) analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; (b) specify locations and describe spatial relationships using coordinate geometry and other representational systems; (c) apply transformations and use symmetry to analyze mathematical situations; and (d) use visualization, spatial reasoning, and geometric modeling to solve problems" (p. 41). Grade spans from the NCTM *Principles and Standards for School Mathematics* (2000) closely approximate the van Hiele model.

Past studies have shown that students who have not attained a van Hiele Level 3 of understanding of geometry before taking a secondary school geometry course have a level of understanding too low to insure success (Usiskin, 1982; Senk, 1989). Therefore, the expectation of the successful completion of a course in formal geometry at the secondary school level can only be realized if the students have attained Level upon completion of elementary and middle school (Usiskin, 1982; Burger & Shaughnessy, 1986; Crowley, 1987). Once the student is at that level they can mature to the level of understanding identified as deduction upon completion of a secondary school geometry course. It is also reasonable to assume that for students to reach these respective levels of understanding the teachers of elementary and secondary mathematics need to have attained a level of understanding geometry at or above these levels to be able to assist students by providing the scaffolding needed for the students to achieve these levels.

The Impact of Changes in the Requirements for Teaching Certification

Swafford, et al. (1997) stated, "The common belief is that the more a teacher knows about a subject and the way students learn, the more effective that individual will be in nurturing mathematical understanding." (p. 467). Currently, at a state supported university in the Northeastern region of the United States preservice Elementary Education undergraduates are required to complete a course in elementary descriptive geometry and preservice Secondary Education Mathematics undergraduates are offered a rigorous course in Euclidean geometry. Recent changes to the teacher certification requirements no longer require prospective teachers to complete a college geometry course to attain a "Highly Qualified" endorsement. This situation creates the possibility that teacher preparation programs may no longer require these content courses in geometry. This is most disconcerting if, as the CDASSG Project found (Usiskin, 1982), a majority of these potential teachers do not attain a satisfactory level of geometric understanding even with the geometric course, much less without it.

Further Rationale and Related Literature

The main catalyst for this investigation is a question posed by Glenda Lappan, the former president of NCTM. Lappan (1999) stated that "research shows that we can improve students' knowledge and ability to visualize and reason about the spatial world in which they live," but she questions if our students are achieving this knowledge and these abilities. The Third International Mathematics and Science Study (TIMSS) and the National Assessment of Education Progress (NAEP) have collected data that show that student performance in geometry, at all levels, is quite alarming (Lappan, 1999). The TIMMS results for geometry at the grade 8 level showed that 24 nations scored significantly higher than students in the United States, while only four nations scored significantly lower. One reason for these results may be teachers' content knowledge in this area of mathematics. In fact, many researchers agree (Usiskin, 1987; Swafford et al., 1997; Clements, 2003) that the level of understanding that students achieve for any concept is limited by the level of understanding of their teacher. Usiskin (1987) put it quite eloquently, "We cannot expect elementary school teachers to teach a broadened curriculum in mathematics if, at the college level, they have only taken a course in the teaching of arithmetic." (p. 20).

The Cognitive Development and Achievement in Secondary School Geometry project (CDASSG), developed and implemented by Zalman Usiskin and Sharon Senk, while testing the van Hiele theory on a population of 2700 secondary geometry students, found that the theory is a good predictor of students' success in geometry courses (Usiskin, 1982; Senk, 1989). The study showed that students who had a van Hiele Level ≤1 had only a 35% chance of experiencing success in proof, one of the indicators of success in a secondary class, had about an even chance (38% - 60%) of experiencing success in proof. Whereas, students who had a van Hiele Level of Understanding Geometry ≥3 had a 75% chance of experiencing success in proof.

The Usiskin (1982) study was also able to conclude that students in junior high [middle] school are not learning even the simplest geometry notions and therefore will not expand their geometry understanding in high school.

Mayberry (1983) found that in-service and preservice elementary teachers have demonstrated low levels of geometric understanding and Hershkowitz & Vinner (1984), showed that in-service teachers and their students tend to exhibit similar patterns of misconceptions when assessed on their knowledge of basic geometrical figures and the attributes of those figures. The ability to change this situation is possible. A study, conducted in 1997, of in-service middle-school teachers (Swafford, Jones, & Thornton, 1997), that measured the content knowledge and van Hiele level of understanding, before and after a 4-week intervention program found significant positive changes.

Method

This study investigated the level of understanding geometry of preservice elementary and secondary mathematics teachers enrolled in the Elementary and Secondary Education programs at a state supported university, both before and after the completion of the geometry course required by their program. The data were used to determine if there is statistically significant evidence that these future teachers have a level of understanding geometry at or above their expected audience. The study also investigated change in the van Hiele level of understanding geometry for preservice elementary and secondary mathematics teachers to determine if the current undergraduate curriculum is realizing an increase in the subjects' level of understanding geometry as theorized by the van Hieles.

Descriptions of Geometry Courses in the Study

The 100-level geometry course for preK-8 teachers is designed to prepare the preservice teachers to instruct students in geometry based on a preK-8 curriculum that models the expectations outlined in the NCTM Standards. The emphasis is on geometric exploration activities, problem solving and informal deductive reasoning using many of the manipulatives used to teach geometric concepts in grades preK-8. Specific areas covered include two- and three-dimensional figures, systems of measurement and volume, area and volume, symmetry, constructions, and translations.

A 400-level geometry course is offered to the students fulfilling the requirements of the preparation program for secondary teachers. This course covers topics in constructions, Euclidean properties, Ceva's and Menelaus' theorems with applications, Desargues', Pappus' and Pascal's theorems, isometries, axiomatic approaches to one of the geometries, algebraic models for geometry, Klein's Erlanger program, and classical construction problems. Indicative of its upper level rigor, this course holds a pre-requisite of 3 semesters of calculus.

Sample Population and Test Tool

The sample population consists of students currently enrolled in these two geometry courses during a single semester. The tool used as the pre- and post-test is the Van Hiele Test developed and copyrighted in 1982 by Zalman Usiskin and Susan Senk for the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) Project. Each question was written as a means to identify the behaviors, using quotes from the van Hieles, of the students at each level. The test was first piloted as an oral exam administered by three project personnel in three states. The responses that were obtained from this pilot study were used to identify 5 multiple-choice questions for each level for an instrument consisting of 25 questions. The instrument was then administered at four schools, to entire classes, to gauge the length of the test and to ensure that the questions reflected the appropriate van Hiele level. The CDASSG Van Hiele Test has also been used in studies to measure the geometric understanding of in-service middle grade teachers (Swafford, Graham, & Jones, 1997), to assess the readiness for geometry in mathematically talented middle school students (Mason, 1997), and to investigate the relationship between achievement in writing geometry proofs and a students' van Hiele Level of Understanding (Senk, 1989).

The pre-test was administered during the first class of the semester in each of three classes, two sections of the 100-level elementary descriptive geometry course, and one section of the 400-level geometry course. The test is designed to take 35 minutes. The post-test, which was identical to the pre-test, was administered at the end of the semester after the last class of formal instruction.

Results

Presentation of Data and Analysis: Identification of van Hiele Level

The van Hiele Test is a 25-question multiple choice test. There are two cases to choose from, as well as two criteria to choose from, to determine student understanding at each level. The method of scoring these cases and criteria is complex, so we followed the method used by the developer (Usiskin, 1982). The test is organized in blocks of five questions that were created using behaviors described by the van Hieles. The questions are arranged sequentially, in blocks of 5 questions, each block dedicated to measuring one of the five van Hiele levels.

Usiskin (1982) found that the behaviors provided by the van Hieles for the first three levels were of sufficient number and detail so that questions to test these levels were easy to develop. Usiskin also concluded that, although more challenging to create, questions that test for understanding at Level 4 could still be developed. However, because the behaviors identified by the van Hieles for the fifth level seemed vague and open to interpretation, this study does not examine the fifth level.

Explanation of the Two Cases

A student's level is determined by looking at the highest block of five questions in which the student correctly answers the required number of questions. However, in the event a student "misses" a level, two cases exist. As an example, in the Classical case (C), if the student correctly answers the required number of questions in the first two blocks and also correctly answers the allotted number of questions in the fourth block (but does not correctly answer enough questions in the third block) then the student is identified as not fitting the criterion and therefore not fitting the model. In the Modified case (M), this same student would be identified as having an understanding at Level 2. A great deal of detail on the different scoring methods can be found in the CDASSG Project (Usiskin, 1982). Based on Usiskin's work we use the Modified Case in the current study.

Explanation of the Two Criteria

To determine how many questions, in a block of five questions, must be answered correctly in order to be identified as having achieved that level of understanding, the developers of the van Hiele Geometry Test specify the use of one of two criteria. The criterion to use is based on whether the researcher wants to reduce Type I error or Type II error. Type I error is defined by Coladarci, Cobb, Minium, & Clarke (2004) as "getting statistically significant results when you shouldn't" (p 230). For the purpose of the test this would be assigning a subject a level of understanding higher than achieved. This erro is minimized by requiring that at least 4 of 5 questions (4) in a block should be correctly answered to achieve that level. The second type of error, Type II, is described by Coladarci et al. (2004) as "failing to claim that a real difference exists when in fact it does" (p. 231). For the purposes of the test this would be placing too stringent a requirement for attaining a level of understanding. This error is minimized by requiring that only 3 of 5 questions (3) in a block should be correctly answered to achieve that level. The decision to use the more stringent 4 of 5 correct answers as the criterion versus the 3 of 5 correct answers stemmed from giving priority to reducing the Type I error. This would translate to reducing the likelihood that a student attained a level not because of understanding the question but because of guessing correctly.

Specifics of Data Analysis

Given the nature of the sample, the standard deviation of the mean is an estimate so a sample t-test, as explained in Chapter 13 of *Fundamentals of Statistical Reasoning in Education* (Coladarci, Cobb, Minium, & Clarke, 2004), was used to determine statistical significance. For each summary table that follows, "n" is the total number of people who took each test (the sample size). A null hypothesis, H_0, and an alternate hypothesis, H_1, are specified for each of the analyses conducted. The level of significance determines the limit at which the mean of the sample is far enough below the null hypotheses that it falls within the most unlikely of all possible sample means. Three sample statistics are calculated. The mean of the sample, $\overline{X}$, the estimate of the population standard deviation, *s*, and the t-ratio, t. Since each of the analyses are concerned with determining statistical significance at or above the mean, a one-tailed analyses was conducted. Critical *t* values are for one-tail analysis and are taken from Table B in Appendix C of Coladarci, et al (2004). If the calculated *t* value is less than the critical *t* value, then the null hypothesis should be rejected because there is no statistical significance that the expected mean was realized.

The Measured van Hiele Level (Pretest and Posttest)

One of the key items of interest is the van Hiele level of preservice Elementary and Secondary instructors. The expectations for students who have completed grade 8 correlate with the behaviors that the van Hieles identify as Level 3, *informal deduction*. The geometry expectations for students who have completed grade 12 correlate with the behaviors that the van Hieles identify as Level 4, *deduction*.

Preservice Elementary Teachers (Pretest and Posttest)

Two research questions for preservice elementary teachers follow:

1. Are Elementary Preservice Teachers, prior to taking their program-required geometry course, at or above the van Hiele Level 3 of

Understanding Geometry as expected of their audience, students completing grade 8?

2. Are Elementary Preservice Teachers, after completing their program-required geometry course, at or above the van Hiele Level 3 of Understanding Geometry as expected of their audience, students completing grade 8?

For each of these questions a null hypothesis and a corresponding alternate hypothesis were developed:

$H_0 : \mu = 3$ Preservice Elementary Teachers, prior to (or after) completing their program required geometry course, are at or above the van Hiele level of understanding geometry expected of students completing grade 8

$H_1 : \mu < 3$, Preservice Elementary Teachers, prior to (or after) completing their program required geometry course, are below the van Hiele level of understanding geometry expected of students completing grade 8.

The hypotheses were tested for significance using one sample, all students taking elementary descriptive geometry who are also enrolled in the Elementary Education program. The results of the statistical analysis are presented in Table 1.

Table 1

One-Sample *t* Test of Elementary Level Participants

		n	*df*	$\overline{X}$	*s*	*t*
Students enrolled in an education program and taking the 100-level geometry course	pretest	31	30	1.806	1.302	-5.105^{*}
	posttest	27	26	2.259	1.130	-3.407^{*}

Note: * = $p < .01$

From this information it is determined that H_0 should be rejected for both the pre– and post-test samples. This indicates that the M4 van Hiele Level of Understanding Geometry of the elementary education students, both *prior to* and *after* completing their required course, is statistically significantly lower than Level 3, *informal deduction*, expected of students completing grade 8.

Preservice Secondary Mathematics Teachers (Pretest and Posttest)

Two research questions for preservice secondary mathematics teachers follow:

1. Are Secondary Preservice Teachers, prior to completing their program required geometry course, at or above the van Hiele Level 4 of

Understanding Geometry as expected of their audience, students completing grade 12?

2. Are Secondary Preservice Teachers, after completing their program required geometry course, at or above the van Hiele Level 4 of Understanding Geometry as expected of their audience, students completing grade 12?

The null and alternate hypotheses for these two questions are:

$H_0 : \mu = 4$, Preservice Secondary Mathematics Teachers, prior to (or after) completing their program required geometry course, are at or above the van Hiele level of understanding geometry expected of students completing grade 12

$H_1 : \mu < 4$, Preservice Secondary Mathematics Teachers, prior to (or after) completing their program required geometry course, are below the van Hiele level of understanding geometry expected of students completing grade 12.

The hypotheses were tested for significance using two samples. The first sample consists of all students taking the 400 level geometry course. The second sample consists of students taking the course who are also enrolled in the Secondary Mathematics Education program. The results of the statistical analysis are presented in Table 2.

Table 2

One-Sample *t* Test of Secondary Level Participants

		n	*df*	$\overline{X}$	*s*	*t*
All students taking the 400-level geometry course	Pretest	19	18	2.895	0.658	-7.324^{*}
	Posttest	13	12	3.077	0.862	-3.860^{*}
Students enrolled in the secondary mathematics education program and taking the 400-level geometry course	Pretest	8	7	2.5	0.535	-7.937^{*}
	Posttest	8	7	3.125	0.641	-3.862^{*}

Note: $^{*} = p < .01$

From this information it is determined that H_0 should be rejected. This indicates that the M4 van Hiele Level of Understanding Geometry of students in the 400-level course, regardless of their program of study, prior to instruction, is statistically significantly lower than Level 4, *deduction*, expected of students completing grade 12. This also indicates that the M4 van Hiele Level of Understanding Geometry of students in the 400-level course, after instruction, is statistically significantly lower Level 4, *deduction*, expected of students completing grade 12.

The Change in the Measured van Hiele Level (Pairs)

From the previous findings it was determined that after completing instruction in the Elementary Descriptive Geometry and Higher Geometry courses, students were not attaining a van Hiele Level of Understanding Geometry expected of students completing grade 8 and grade 12. This led to an investigation into: a) whether students in either class attained a higher van Hiele level after taking the geometry course, and b) whether students needed to begin the course at any particular level in order to advance at least one level as a result of the course.

The One-Sample t Test of the Change in Pretest van Hiele Level

One question was developed for this analysis:

Is completion of a course a factor in a student attaining a higher van Hiele Level of Understanding Geometry?

The null hypothesis and corresponding alternate hypothesis developed to answer this question are:

$H_0 : \mu = 1$ Students who complete a course in geometry will realize an increase of at least one level in their van Hiele Level of Understanding Geometry.

$H_1 : \mu < 1$, Students completing a course in geometry do not realize an increase by one level in their van Hiele Level of Understanding Geometry.

The hypotheses were tested for significance using the four samples. The three previously described in the investigation and a fourth sample consisting of students enrolled in an education program and completing the elementary descriptive geometry course who realized a pre-test level of less than 3. This is consistent with Byrne (2001) whose research of Vygotsky's Theory of Cognitive Development indicates that students will increase their understanding when presented with material in the upper half of their zone of proximal development. Since the elementary descriptive geometry course is geared to prepare elementary teachers it is expected that students already at a van Hiele Level 3 would not move on because the material presented is not beyond a van Hiele Level 3. The results of the One-tailed Sample t-Test of the Change in Pretest van Hiele Level are presented in Table 3.

Table 3

One-Sample *t* Test of Change in Pretest van Hiele Levels

	n	*df*	$\overline{X}$	*s*	*t*
All students completing the 400-level geometry course	11	10	0.182	0.982	-2.764^{**}
All students enrolled in the secondary mathematics education program and completing the 400-level geometry course	7	6	0.286	1.113	-1.698
All students enrolled in an education program and completing the 100-level geometry course	20	19	0.350	0.933	-3.115^{*}
Students enrolled in an education program and completing the 100-level geometry course with pretest score less than 3	13	12	0.620	1.080	-1.284

Note: $^{*} = p < .01$, $^{**} = p < .05$

This information shows that for the sample consisting of: a) students completing a 400-level course in geometry who are also enrolled in an education program, and b) students enrolled in an education program completing the elementary descriptive geometry course with a pretest Van Hiele level less than 3, the H_0 should be retained. For the remaining samples, the One-Sample t Test directs that H_0 should be rejected. These findings indicates that the change in the mean of students who complete the 400-level geometry course, irrespective of program of study, and students enrolled in an education program and taking elementary descriptive geometry, irrespective of pre-test level, is statistically significantly less than one and that these students do not attain the next higher van Hiele Level of Understanding Geometry from their Pretest level.

However, the test does indicate that the mean gain of students who complete the 400-level course in geometry and who are enrolled in an education program, and students completing the elementary geometry course who are enrolled in an education program and have a pre-test van Hiele Level of less than 3, have a mean gain that is statistically significantly equal to or greater than one, meaning these students do attain the next higher van Hiele Level of Understanding Geometry. These findings suggest that even though the subjects are not at the level of understanding expected of their prospective students, the courses required for each of the education programs are beneficial to these students. One cautionary note is that, given the small sample size of 7, care should be taken when drawing conclusions regarding the secondary educations in the 400-level course.

As a result of these investigations additional inquiry was made into possible correlation between the student's gain, their pre-test van Hiele Level of Understanding Geometry, and their program of study. These findings are detailed in fuller work by Author (2006).

Discussion

The study investigated the van Hiele Level of Understanding Geometry of Preservice Elementary and Secondary Mathematics teachers. The expectation was that subjects would have a level of understanding at or above at least a Level 3. It was also expected that these subjects, after completing the geometry course required by their respective program, would attain a van Hiele Level of Understanding Geometry at or above the level of the students they expect to teach. The need for the teachers to attain this level is important in order to help students achieve the expectations of their their current academic setting as well as their post academic careers.

The initial investigations found that the van Hiele Level of Understanding Geometry for Preservice Elementary and Secondary Mathematics teachers, both before and after completing their program of study is statistically significantly lower than the levels expected of their target audience. These findings were interesting and disconcerting. The findings parallel the earlier notion by Usiskin (1987) that students are not achieving a level of understanding at the elementary level that will enable them to be successful in secondary school geometry, only in this case we are looking at the understanding of the people who will eventually be the teachers, which raises concern about how to break the cycle of limited geometric understanding.

Attention then turned to investigating if the subjects are advancing their understanding to at least the next higher level. In other words, do the courses being offered provide students with the scaffolding they need to increase their M4 van Hiele Level to the next van Hiele Level? This investigation found that the mean gain of students who complete the 400 level geometry course, regardless of major program of study, is statistically significantly less than one and that these students do not progress to the next higher van Hiele Level of Understanding Geometry. However, the test does indicate that the mean gain of students who complete the 400-level course in geometry and who are enrolled in an education program, or students who complete the elementary descriptive geometry course with a pre-test van Hiele Level less than 3, have a mean gain that is statistically significantly equal to or greater than one. This means that these students may attain understanding at the next higher van Hiele Level but the new level is not the level at or above the level of their expected audience.

These findings suggest (and corroborate earlier findings) that additional instruction is needed for the preservice mathematics teachers. This additional instruction should provide these future teachers with the opportunity, through additional content courses or seminars, to attain understanding at these levels and therefore help them to be successful in the current courses that are offered.

One of the limitations of this study is the number of subjects. Traditionally only four sections of the elementary descriptive geometry are offered yearly, two in the Fall Semester and two in the Spring Semester, and one section of the 400 level geometry course is offered yearly. Due to the time constraints in completing this study only two sections of the elementary descriptive geometry were for testing. This data should be collected for several years and a reinvestigation into the findings with a larger sample size should be considered.

Another study could investigate the van Hiele Level of Understanding Geometry using subjects from multiple state-supported schools. This study would broaden the population to the entire region. Another study, investigating the van Hiele Level of Understanding Geometry of inservice Elementary and Secondary Mathematics teachers, could be compared to these studies to see if there is any correlation to the number of years certified and if number of years teaching this content area is a factor in the van Hiele Level of Understanding Geometry for this population. Finally, another avenue for following implications of this research project is to undertake curriculum design, with the intent being to determine whether it is possible to create learning experiences that specifically enhance the sorts of geometric reasoning investigated in this study.

Knowing that the reasoning and logical thinking embedded within school geometry are not only basic skills but can lead to successfully earning technical and advanced degrees, it is imperative that the educators preparing today's elementary and secondary students have themselves obtained the experience and understanding they need as teachers and mentors.

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