# **Mathematics Graduate Teaching Assistants' Question Strategies**

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Abstract: As part of an earlier pilot study, we developed a framework for structuring, identifying, and discussing questions mathematics graduate teaching assistants (MTAs) ask while teaching. We continued this research through an exploratory study of classroom video of five university calculus instructors with differing levels of experience. We reviewed existing literature for connections to our question categories, techniques, and strategies to establish an inductive hypothesis for comparative analysis of the five instructors. This qualitative study focused on refining our framework to create a tool for identifying and discussing the questions MTAs ask while teaching an undergraduate calculus course.

Key words: Graduate teaching assistants, calculus, professional development, questions.

Tell me and I forget. Teach me and I remember. Involve me and I learn.

-- Benjamin Franklin, 1706-1790

Mathematics and science graduate students often welcome guidance in learning to teach, though few actually receive it (Austin, 2002; Seymour, Melton, Wiese, & Pedersen-Gallegos, 2005). For many graduate teaching assistants, college classroom knowledge comes from their experience as students of lecture-based instruction (National Center for Education Statistics, 2002; Sofronas & DeFranco, 2009). Initial experiences as an instructor, particularly in learning about how students think, likely will influence mathematics teaching assistants (MTAs) in their subsequent work as teachers (Kung, 2009). The expectations these future faculty members face when they enter the professoriate include engaging students deeply in learning (Holton, et al., 2001). In order to design effective professional development for novice college mathematics instructors, we need an understanding of the complex influences on MTAs and their teaching. Several have addressed mathematics graduate student experience in general (e.g., Golde & Walker, 2006; Herzig, 2004) and emerging work is examining the nature of MTA teaching experiences (Belnap & Giullian, 2008; Hauk, Mendoza-Spencer, & Toney, 2009; Speer & Hald, 2008) and practices (DeFranco & McGiveney-Burelle, 2001; Kung, 2009). Building on this foundation, in Spring 2008 we did a pilot study examining how watching and discussing a video of a college mathematics instructor use a variety of questioning techniques might influence questions MTAs asked in class. We found that the types of questions asked could be influenced by such video case work. In particular, we developed an initial framework for the structure, identification, and discussion of questions MTAs ask, categorized by interaction group size (whole class, small group, individual) and instructor intention (e.g., comprehension checks, finding out about student thinking processes). We report here on our work extending that pilot. Our research question for this report was: How do we refine our framework to create a tool for identifying and discussing GTA classroom questions in a calculus course?

## **Theoretical Perspective**

The foundations of our framework come from combining ideas from K-12 research on classroom "math talk" (Hufferd-Ackles et al., 2004) and task analysis (Stein, Smith, Henningsen, & Silver 2000) along with college-level explorations about "good questions" in mathematics instruction (Miller, Santana-Vega, & Terrell, 2006). In particular, we address teacher-generated questions in the context of undergraduate first-semester calculus.

Our first stop in the K-12 literature looked at levels of classroom discourse. Hufferd-Ackles and colleagues (2004) reported on a case study of a particular novice third grade teacher (from a larger study) and the evolution of classroom discourse over a year as the teacher implemented a reform-based curriculum for the first time. The authors' defined "math talk" as discourse that supports the learning of mathematics of *all* in the classroom. Their framework for identifying trajectories in the discourse, for both teacher and student, had four categories: questioning, explaining thinking, source of mathematical ideas, and responsibility for learning. For our work, we focused on the *questioning* category along with the coding scheme they identified for Levels of Discourse. Hufferd-Ackles et al. discussed four levels of questioning. Level 0 was considered to be a "traditional classroom" in which the teacher directs discourse with brief answers or responses required from students. The teacher is the only one who asks questions and the questions mostly require a yes or no response. In Level 1, the teacher attends some to students' mathematical thinking and focuses less on correct answers; however, the teacher is still the center through which communication occurs. The teacher is the only one who asks questions and there are more follow up questions about procedures. In Level 2, a teacher expects and supports students to build new, inquiry-rich roles as and the students may even be "co-teaching." In this sense, a teacher is modeling "math talk." The teacher asks probing questions and facilitates the students talking to each other by asking the students to explain to each other their reasoning. In the upper level, Level 3, the teacher is co-teacher and co-learner. While the teacher observes and monitors everything that is going on, students are expected to ask each other about their work and explain their thinking to one another without prompting. At Level 3, many questions are "Why?" questions that require justification (in addition to the kind of explanation seen at Level 2). The authors reported that in the case study classroom the

community of learners moved from a Level 0 to mostly Level 3 discourse over the course of the year. When the teacher introduced a new topic, she would fold back to a Level 0 or 1 and then rapidly push the discourse higher by eliciting more complex explanations from students with "how" and "why" questions.

Similar to the "math-talk" levels, Stein and Smith developed a Mathematical Task Analysis Guide to offer a framework to help identify and discuss the cognitive demand of a given mathematical activity (Stein et al., 2000). In their work about implementing standardsbased curriculum in the context of middle school mathematics, these authors investigated the kinds of mathematical activities used in classrooms and found that often the activities that required a higher cognitive demand were more difficult to implement well and that teachers might funnel information and transform a "procedures with connections" task into a less demanding recall or "memorization" task. In other work that looked at middle school mathematics teaching, Sorto, Mccabe, Warshauer, and Warshauer (2009) compared the nature of interpretation of teacher questions and student responses in isolation and in discourse neighborhoods. The authors explained that without an awareness of the context and setting of an instructor's question, it might be difficult to comprehend the appropriateness or depth of the question. That is, sometimes a teacher may ask a "good question" but it may be contextually inappropriate or inaccessible to students. With this awareness in mind, we also referred to the Good Questions Project work of Miller, Santana-Vega, and Terrell (2006).

Though research and development is sparse around questions people ask in teaching calculus in college (the specific focus of our study), Miller, Santana-Vega, and Terrell (2006) explored "good questions" in college calculus with novice instructors. While largely theoretical, their initial work came from many years of classroom experience and mentoring of MTAs.

Miller and colleagues state that good questions will spark classroom discussion and allow the instructor to assess the understanding of the students. These questions may not have one correct answer, but can be used to illustrate the larger concepts of calculus.

We take from these works that questions in the classroom can promote an engaging learning environment. Questions are often a gateway for teachers to guide students to become active participants in the mathematics classroom. Using this body of work, we are developing a framework to help MTAs identify and discuss the types of questions they ask (or want to ask) in a calculus classroom.

## Methods

The first stage of project methods development was to compare our previous framework for identifying and categorizing questions to the existing literature through analytic induction. *Analytic Induction* is a comparative analysis method that starts with a theory-supported hypothesis about how two or more aspects of something are linked (either causally or relationally). Theoretical hypotheses are formed and negative cases are sought out with the intent to challenge and refine the initial hypotheses. Like selective coding in grounded theory, analytic induction relies on comparisons among multiple cases to seek disconfirming evidence (Strauss & Corbin, 1998). The goal is to see how the original hypothesis must be altered (if at all) in order to be consistent with coded categories. Like grounded theory, analytic induction is never "complete" in that we might examine another data point (e.g., watch another calculus class) and look for disconfirming evidence. Also, both approaches depend on the idea of saturation: one collects data that varies across several possible aspects of the concept of interest to ensure likely sources of disruption to the hypothesis have been checked.

To establish our inductive hypotheses, we reviewed existing literature for connections to our question categories, questioning techniques, and what makes a "good" question. For example, Miller, Santana-Vega, and Terrell (2006) defined a good question to include engaging students' interest, promoting students' self-awareness, making connections to prior knowledge, and encouraging an active learning environment. We delved into theory to find things related to the limited scope of the space defined by our (group size) X (intention) framework, and then expand the details of the framework into a hypothesis about the categories and patterns of questions MTAs use. Throughout, we kept in mind the lessons learned from Sorto, Mccabe, Warshauer, and Warshauer (2009): the context of a question is critical to analyzing questions and using questions effectively in the classroom.

### Settings

This exploratory study was conducted by examining existing video of five college instructors teaching Calculus I. The classes were recorded mid-semester in 2007 and 2008. The video was gathered from two universities and research data was comprised of at least two hours of classroom interactions for each of the five instructors: one experienced professor, and four MTAs. We included the video of one experienced instructor for two reasons: (1) as a potential point of dissonance for ways the framework may be valid for only novice instructors and (2) as a potential source of ideas for generating a video case with examples that MTAs might find useful.

The instructors were Dr. Phelps, an experienced professor in his seventh year of teaching at the university and "coach" of three first year MTAs: Bethany, Daniel, and Jennifer. Each of Bethany, Daniel, and Jennifer were also videoed the following semester when they were the instructors of record for their own calculus classes. A fourth MTA, Lauren, was the instructor of record for a calculus class and had not been coached by anyone else. For each of the MTAs, the video came from their first experience as instructor of record for calculus.

## Data Analysis

We conducted constant-comparative coding of at least two hours of instruction for each person. Pairs of researchers watched each video recording, analyzing the instructors' teaching by focusing on the questions asked over the course of a class period, while also attending to the context in which the question appeared. Before we could begin coding instructors' questions, we first had to come to agreement on what it meant to be a question, identifying features of a question. Using this understanding, the researcher pairs transcribed all the questions asked by the instructors. During this transcription process we included contextual descriptions of each question to ensure the appropriateness and depth of the instructors' questions could be comprehended without the aid of the video recordings (Sorto, et al., 2009). The inclusion of the context in which a question was asked proved to be beneficial in our coding: it gave information that helped in estimating an instructor's intended purpose and audience for a question, as well as the cognitive demand required in an instructor's expectations for response to a question.

In our coding we first categorized an instructor's intention for a question by deciding whether the instructor was attempting to assess a student's understanding (*comprehension check*), make explicit what a student was thinking (*elicit student thinking*), or gain insight into the reasoning behind a student's thought or thought process (*probe student thinking*). These categories have two dimensions: (1) the audience a question is directed toward and (2) the cognitive demand placed upon the members of that audience by a question. We noted that an instructor might pose a question to an individual student, a group of students, or the class as a whole. We took into account the mathematical concepts covered up through the classes of focus,

and followed Stein and colleagues' (2000) levels of mathematical activity to determine the cognitive demand a question seemed to place upon the students. Pairs of researchers coded the questions together and, through discussions and reasoned argument, came to a consensus. After this coding, a fourth researcher reviewed the codes and then we discussed and recoded any discrepancies until all the researchers were in agreement.

#### Results

During our initial coding process, we found that teachers utilized questions to explore students' understanding at varying levels. While we were able to use our previously established framework to describe teachers' intention (check, elicit, probe) when posing questions to their students, we were not satisfied that it effectively conveyed teachers' attempts to explore and survey the depths of their students' conceptual development of a topic. We addressed this issue by establishing a four point scale for describing the level of mathematical discourse from a teacher's question as described by Hufferd-Ackles, et al. (2004). This notion also took into account the underlying level of cognitive engagement such as we saw in the contextual analysis of Sorto et al. (2009). There, the authors found the teacher in their study used a sequence of questions in the presentation and instruction of the mathematical concept of measuring angles. This sequence required students to construct connections from the previous knowledge of angles to more abstract ideas before delving into the purely procedural aspects of measuring angles. We found that our instructors tended to follow a similar progression of questions during sustained interactions with students, where students either communicated that they were having difficulty grasping the question or did not respond to the question. In the latter case, instructors generally would follow up with either (1) a question that was not as cognitively demanding, (2) a question that established some conceptual connection for the students before repeating the previous

question, or (3) by telling students what to do next. In most cases, for the MTAs, these followups had the effect of funneling (to lower cognitive levels) without further scaffolding back to higher levels. In the case of Dr. Phelps, he regularly scaffolded discourse back to a higher cognitive demand.

As a way to record the analysis of question-related discourse, we used time series representations of the levels of discourse. Figure 1 gives examples of some of these diagrams.



Figure 1. Examples of coding representation for levels of discourse over time.

In our analysis, at Level 0 the instructor often asked a question and did not wait for a response. This was most common with Daniel, Lauren, and Jennifer. However, the more experienced instructor (Phelps) waited for a response and then asked questions that pushed

students to discuss possible solutions or ways to solve the problems. Common to all the MTA instructors during class group work was a habit of telling students the procedure needed to work the problem and then walking away (e.g., this occurred at the end of the interactions for Daniel and Jennifer pictured in Figure 1). Students then applied (or attempted to apply) the suggestion to finish the problem and move on to the next task. In some instances, MTA instructors would ask questions about how things work with some suggestion about what to think about, then walk away from the group. In these instances, the questions asked would be on a higher level usually a Level 1 or, rarely, Level 2. And, after the MTA left the group, student discussion and efforts at discovering a solution method continued. For example, students left with a "why" or "how" question by an instructor, particularly in Dr. Phelps class, would regularly excitedly debate the problem until they solved it (e.g., after the interaction with Dr. Phelps pictured in Figure 1). Only Dr. Phelps was observed asking Level 3 questions and, unlike the MTAs, he also asked probing questions that required justifications from students. Dr. Phelps rarely asked Level 0 questions.

Huffard-Ackles et al. (2004) have said that in order to get students to become more capable at math-talk, teachers need to ask probing questions to try to understand the students' thinking and to get students to articulate their thinking. In their study, as students became more comfortable with escalating cognitive demands in lines of questioning, they became more supportive and encouraging of each other. This was also evident in Dr. Phelps interactions with students. By mid-semester – at the time of the videos we watched – when students had interactions with Dr. Phelps (particularly in small groups or whole class discussion), the classroom context had been established that sense-making with clear explanation and justification were the responsibility of the students. The students were the primary "teachers" and it was their job to question each other. While the MTA instructors we observed made statements in class that they expected such behavior from their students, the MTAs' interactions with students (particularly in group work) were more often focused on helping students to identify and write down a correct answer through use of Level 0 and 1 questions and through assertions, than in scaffolding students to ask their own questions.

## Conclusion

How to initiate and sustain the socio-mathematical norm of sense-making, particularly a norm for students to ask questions at all levels we identified, is the aim of our work. Discussion during our presentation of this work at the RUME conference in February 2010 surfaced several important factors for us to consider in moving forward. The present report offered details on the nature and depth of questions college calculus instructors ask. This work diverges from the existing K-12 research on questions in the classroom in two important ways. First, the discussants are all adults and, particularly in the case of MTAs, may be near-peers in age. Second, the knowledge of MTAs about tackling the highest level of task, "doing math" (Stein et al.'s term), is densely packed and routinely used as implicit knowledge in their own graduate courses. That is, helping instructors learn to "unpack" their knowledge and ways of knowing is a different challenge in the professional development of MTAs than for in-service teachers school teachers are unlikely to need to use well-packed mathematical understandings within an hour of also trying to use unpacked versions. Additionally, future work will examine patterns of types of questions (the reader may notice there are different shades of gray in Figure 1 – each shade actually represents one of comprehension check, elicit, or probe). We plan to explore more fully the nature of discourse regarding these types of questions after analysis of at least 20 more hours of classroom video for at least five other instructors. Finally, in developing materials for the preparation of MTAs for teaching, we are in the beginning stages of designing a protocol for

MTAs to use in watching and identifying questions in video of instruction – both to support awareness of questions as well as develop awareness of the potential conflicting messages about classroom socio-mathematical norms that can be generated (e.g., when instructor statements explicitly assert that sense-making by students is most important and yet instructor actions, possibly driven by a desire to witness students getting the right answer, seem to undermine sense-making by students).

The focus of continuing work is helping college teachers achieve what Hufferd-Ackles and colleagues reported in their case classroom: "As the math-talk learning community developed, responsibility for learning shifted as students became increasingly invested in their own and one another's learning of mathematics... responsibility for their own learning was indicated by their desire to ask question in class, their eagerness to go the board to demonstrate their understanding of problems, and their volunteering to engage in the work of and to assist struggling students at the board" (Hufferd-Ackles, et al., 2004, p106).

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