Teaching Mathematics Reading Strategies to First Year College Students and the Effect on Reading Comprehension

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Abstract: There have been calls by many teachers to teach specific reading strategies for reading mathematical text as found in textbooks. General reading strategies do not appear to be as effective when mathematical text contains symbols along with text. Since this is the type of text that appears in many first-year college mathematics textbooks, it is little wonder that students find reading their mathematics textbooks difficult and often avoided. This preliminary report will present the results of an experiment where precalculus students are assessed on what they understand from reading two different passages from their regular textbook. One passage is read without any special instruction in how to read mathematics, one with guidance in some potentially useful reading strategies related specifically to mathematics textbook reading.

Introduction: Many mathematics faculty want and tell their students to read their textbook. A brief survey of first year college students might discourage these teachers because it appears through general questioning that this does not happen as frequently as faculty would like, if at all. Informal surveys of students seems to indicate that they do not read large parts of their mathematics textbooks at all, much less effectively. There might be several reasons for this, but the most likely is that students do not find reading their mathematics textbooks to be useful to them. A couple of reasons for this might be that either they don't understand the material they read (because of not reading it effectively), or they know that the teacher will cover it anyway whether they read it or not. It does appear that as mathematics students progress through coursework, the ability to read mathematics in any form, textbook or journal, does increase. The question that I have pondered is whether first-year college students can read their textbooks with some amount of understanding.

My initial interest in this area came when I moved from SUNY-Potsdam (my first teaching job post-PhD) to Northwest Missouri State University in 2001. At SUNY-Potsdam I had been exposed to the teaching philosophy of Clarence Stephens, and his third guiding principle was:

> YOU CANNOT PUSH THE STUDENTS FROM ONLY THE BOTTOM; YOU MUST ALSO RAISE THEM UP FROM ABOVE—You can best achieve your goal as a teacher by helping the students to learn to think for themselves, ***to read mathematical literature independently with understanding and enjoyment***, and to become free from the need of a teacher.

I became interested in trying to apply the reading part of this principle in my teaching, and have tried some different teaching strategies to get students to read their mathematics textbook. But when I began to actually observe students reading their mathematics textbooks, I wondered who is teaching students to read them and why are they not getting from the reading what I and, I assume, other teachers want and expect them to "get".

Background: From personal teaching experience and in talking with colleagues, this researcher (and others) have come to believe that many, perhaps most, first-year university students do not read large parts of their mathematics textbooks effectively. Research (Osterholm, 2006; Shepherd, Selden & Selden, 2009) would indicate that students are not effective mathematics textbook readers because general reading skills are not sufficient for mathematical text, especially text that has many symbols in it. There have been a number of calls for teachers to instruct students on how to read mathematics (Bratina & Lipkin, 2003; Cowen, 1991; Datta, 1993; DeLong & Winter, 2002; Draper, 2002; Fuentes, 1998; Pimm, 1987; Shuard & Rothery, 1988). Also, the textbooks for many first-year university courses, such as college algebra, precalculus, and calculus seem to be written with the assumption that they will be read thoroughly and precisely. A brief, informal survey of about 20 new precalculus and calculus textbooks shows about half explicitly state (in the preface or "To the Student" section) an expectation that students could or should be able to read the textbook, and about another quarter seem to imply the textbook should be readable (e.g. "… an effort has been made to write the text in language accessible to students"). A brief survey of calculus one students at a good Eastern liberal arts school a couple of years ago, though, found that few read the textbook at all (Exner & Shepherd, 2008). For example, this is a typical student comment from these interviews: "When I think there is a formula I need, I'll go back and look if there is a formula, otherwise… there is very little chance, that I'm going to read any of it."

I have been observing and recording observations about students reading for several years. Initially, in 2002, I gave students some propaganda that I hoped would encourage them to read their mathematics textbooks. All I did was "tell" students to read the sections before class and then give a brief reading quiz to start the class. Even this small intervention seemed to have some benefit. Student test scores were better when I did this compared to semesters when I didn't (Shepherd, 2005).

Encouraged by these results, and with some funding from a SIGMAA-RUME mini mentoring grant in 2003, Annie and John Selden helped me design a project where students were asked to read from their textbooks and try routine exercises based on their reading. For example, some were asked to read the definition of a relative maximum and minimum and then attempt the routine exercise that was to look at graphs of various functions and identify the relative maxima and minima. The results were that most students could do less than half the routine exercises, and none did all correctly (Shepherd, Selden & Selden, 2009).

Reading researchers view reading as an active process of meaning-making in which readers use their knowledge of language and the world to construct and negotiate interpretations of texts in light of the particular situations within which they are read. (Flood & Lapp, 1990; Kintsch, 1998; Palincsar & Brown; 1984; Pressley & Afflerbach, 1995; Rosenblatt, 1994). These conceptual shifts have expanded the notion of reading from that of simply moving one’s eyes across a page of written symbols and translating these symbols into verbalized words, into the idea of reading as a mode of thinking and learning (Draper, 2002). Based on research related to reading comprehension (Flood & Lapp, 1990; Pressley & Afflerbach, 1995; Rosenblatt, 1994), it appeared the students in our 2003 study were using good general reading strategies but not getting enough of the material to successfully complete routine exercises. Thus, it would appear that there is something different about reading mathematics textbooks.

But what reading strategies should one teach for reading mathematical textbooks? For several semesters, this researcher has been observing students as they read, encouraging different strategies, discussing with students what seems to work to help students learn to read mathematical text for understanding. The theoretical framework underlying this research is based largely on the Constructively Responsive Reading framework (Pressley & Afflerbach, 1995) with a strong influence from the Reciprocal Teaching strategies of Palincsar and Brown (1984). I tried to encourage strategies that I thought might help them. After they read, I quizzed them about what strategies they thought helped the most. The main strategies I thought worked well (and most students picked at least one of these also) included:

(1) Doing some warm-up activities before reading to bring to mind algebra or concepts needed in the reading.
(2) Stopping frequently to ask if “that” made sense, and not going on until the reader felt it did make sense.
(3) Directing students or giving “permission” to go back in the text to check understanding or re-read material.
(4) Encouraging students to read notation correctly, and correcting them when they did not.
(5) Encouraging interaction between the written text and figures in the text.
(6) Trying to work examples (on paper) with the solution covered, and uncovering only parts of the solution if they needed help getting started.
(7) Creating examples or figures to demonstrate definitions or theorems.
(8) Working a similar problem in assigned homework right after finishing an example.

The next step in my observations was to ask one student to come in for reading guidance for several sessions. All my previous observations had been for a single one-on-one reading session. I began to doubt whether “habits” of reading had been formed. I began wondering if it was possible to teach a student to read a mathematics textbook with some level of understanding. Although this method was time consuming, it would appear so from this initial case study (Shepherd, 2008). The student in this study was a GED student who read with me in precalculus and who eventually completed Calculus 2.

From the above study and other observations of students reading their mathematics textbooks, all the strategies above appear to help students gain understanding while reading—at least to the point of being able to do routine problems.

The purpose of this study is to see if, even in a single “lesson” on reading a mathematics textbook, there is measurable increase in ability to work routine problems. The effect on basic student understanding will be measured looking at their success on routine exercises. The specific research question addressed is this: Is there a difference in ability to work problems when given specific mathematics reading strategy guidance while reading over reading with no guidance?

The study: During the Fall, 2009 semester, 16 students volunteered to participate in this study. The students attended a U.S. mid-western comprehensive state university at which they took all their coursework. The university has a student body of 7,100 students of which 6,100 are undergraduates. It has a moderately selective admissions standard. All the students in the study were students in a mathematics/science magnet secondary school located on the campus of the university. Nine of the students were female. Four of the female participants were in the pilot study. Four of the students, all male, had English as a second language, but were quite fluent in

English. Five of the students were minorities, either Asian or Hispanic. The mathematics course for these students was precalculus, carrying regular university credit and taught by the author. Four of the sixteen students were considered to be a pilot study to the main study to confirm/modify data collected. All the students were volunteers.

Study design: The students signed up for 90 minute time slots over a 10 day period. Except for the pilot study students, each student was given a pretest of 10 questions, 5 each on vectors and sequences. This pretest was on paper (see Appendix 1) . All the students were then randomly assigned one of two groups. The groups were based on which passage (Vectors or Sequences) would be read first and which would be read second. The first passage was to be read using their normal reading practices, the second passage was to be read with the researcher giving reading strategy guidance during the reading. After reading the first passage, students were given a 5 question test over the material. The questions were the same as in the respective pretest with numbers changed. Then they were given three slightly more conceptual problems to work. The students were then asked to read the second section. While reading the second passage, students were asked to read out loud and were given mathematics reading strategy guidance while reading. They were also asked to complete another 5 question test, but these were interspersed with the actual reading, as they might find homework problems. At the end of the reading they were also given three slightly more conceptual problems to work. The sessions with the reading guidance were recorded. During both sessions, the researcher took notes, and during most of them a second observer (a master's level graduate student) also took notes. It is from these notes (of both observers) that this preliminary report is being given.

The mathematics reading strategies encouraged included:

(1) Stopping frequently to ask if “that” made sense, encouraging re-reading or looking back in text, and not going on until the reader felt it did make sense.

(2) Encouraging students to read notation correctly, and correcting them when they did not.

(3) Encouraging interaction between the written text and figures in the text.

(4) Trying to work examples (on paper) with the solution covered, and uncovering only parts of the solution if they needed help getting started.

(5) Working a similar problem in assigned homework right after finishing an example.

(6) Encouraging students to use notation correctly as they worked the examples or problems.

Results and Observations: Data collected on each of the 12 students in the study included:

(1) Score on pretest (split by number correct on the vector questions and number correct on the sequence questions—each out of 5 possible)

(2) Passage read first (vector or sequences)

(3) Time spent reading the first passage

(4) Number correct out of the first passage post-test (out of 5)

(5) Time spent working on the first 5 post-test questions

(6) Number correct out of the other “3” post-test problems, first passage.

(7) Time spent on these 3 problems

(8) Time spent reading the second passage including time on the 5 post-test questions

(9) Number correct out of the second passage post-test (out of 5)

(10) Number correct out of the other “3” post-test problems, second passage.

(11) Time spent on these 3 problems.

Only the results of 11 students are presented. One student exhibited such strong mathematical reading strategies on the first passage, that there were few new strategies to introduce her to on the second reading, and so her "results" are not included in the data below.

Results are summarized below.

Description	**Statistic**
Pre-test	
Vectors—average number correct out of 5	0.77
Sequences—average number correct out of 5	1.23
Number of students	
First reading of sequences	4
First reading of vectors	7
Average time spent reading first passage	18.5 minutes
Average time spent on 5 question post-test Passage 1	12 minutes
Average number correct on 5Q post-test Passage 1	3.286 (or 65.7 %)
Average time spent on "3" Q post-test Passage 1	11.8 minutes
Average % correct on "3"Q post-test Passage 1*	60.5%
Average time spent reading second passage (including 5Q test)	46.9 minutes
Average number correct on 5Q post-test Passage 2	4.084 (or 81.7%)
Average time spent on "3"Q post-test Passage 2	8.6 minutes

Average % correct on "3"Q post-test Passage 2*	64.2%
Difference in 5Q post-tests Passage 2 over Passage 1 (correct)	.799

*As the data was being analyzed, it was observed that no one completed correctly one of the post-test problems in the group of three on sequences. The most credit given was ½ point. Thus, the 3 question post-test for the sequence passage really only had 2.5 points available. These percentages were calculated using 3 as the base for the vector test and 2.5 as the base for the sequence test. If no adjustment between the two passages had been made, the percentages would have been 56.3% and 57.1% respectively.

To test the claim that reading instruction guidance did indeed make a difference in the ability of students to work routine problems, a *t*-test on the last statistic above was performed. This was a test on paired data with an alternate hypothesis that there is an increase in these scores as a result of being given some mathematics reading instruction. A *p*-value of 0.087 was obtained. Thus it appears that there is some increase in the ability of the students to answer routine questions when given specific mathematics textbook reading instruction.

Although, it may not be a typical practice to look at the actual background of the students participating in the study, the researcher/teacher found this to be a very different group of students from those she has encountered in the past. The group of magnet school students in the two precalculus classes from which the students for the study came had several students who came, before the study was even announced, and asked for instruction or help in reading from their textbook. This was the first group of students in several years who have not been given the "propaganda on reading" handouts to read as classes start, but several found the handouts I prepare on the class website and read them. And I believe students at this magnet school who were one year ahead and had had me for precalculus a year ago strongly encouraged these new students to learn to read and practice reading their mathematics textbooks. I had never had this happen before, and this may have actually increased the *p*-value showing less of an effect from the reading instruction (since some clearly were trying successfully to read with more understanding) than might occur with more typical first year college students.

This was particularly obvious with the student whose data was not included above. Her reading time for the first passage (which was on sequences) was 51 minutes, more than 2.5 times the average time. On the second passage it was 50 minutes, much closer to the average time. Her scores on both the 5 point post-tests were each a perfect 5. In addition, a little after the reading interview, she sat for the ACT exam and raised her Math subscore by 5 points from a 24 to a 29 and her Science subscore by 6 points from a 26 to a 32. Another student whose data is included raised her Math subscore by 9 points from a 25 to a 34 in her next attempt at the ACT. Other students exhibited some of the encouraged reading strategies while reading the first passage, even before any instruction. Two students routinely covered the solutions and tried to work the problems without seeing the solution first. This has not been a commonly observed reading strategy in previous observations.

Other observations from the data collected include:

(1) There was not a significant increase in the ability of the students to perform more conceptual problems with the reading guidance. These were the "3" other problems given in each post-test.

(2) The reading guided sections did take longer by about 50%. To see this, one needs to include the average reading time for the first passage and the average time to complete the first 5 question post-test since the 5 question part was done while reading the second passage—30.5 minutes for Passage 1 versus 46.9 minutes for Passage 2.

(3) Although the results are not significant, it took the students less time on the "3" question post test for the second passage (on average about 3 minutes less), and they scored slightly higher (about 4% higher) on it in the shorter time.

(4) Although the researcher tried to get passages of equal difficulty and exercises of similar quality, whether this indeed was the case seems to be difficult to quantify, and might only be discernable in a larger scale study.

Conclusion: It appears that encouraging students to read mathematics with the specific reading strategies given above can improve their ability in the short term to perform routine exercises that are based quite closely on the reading. It is unclear whether these strategies are enough to affect performance on less routine exercises, or over a longer period. Since these students seemed to be unlike other students in their desire to read their textbooks with understanding, it would be interesting to repeat this experiment with more typical first-year college students. Additional questions that arise include:

(1) Are there other reading strategies that could be researched in relation to reading mathematics?

(2) Is there another level at which to look at teaching mathematics reading strategies? (Should it be first-year college or high school, middle school, after first-year college?)

(3) Most first-year college mathematics students are not mathematics majors. What reading strategies should they carry with them, particularly if there is only one college mathematics course they take?

(4) Are more general reading strategies enough for other first-year college mathematical textbooks, such as general statistics, where the symbols are fewer than in, say, calculus textbooks?

(5) Is it even worthwhile to teach students how to read a mathematics textbook with understanding?

(6) As a framework for reading mathematical text develops, what is it that experienced mathematicians "do" as they read, or how do they respond to text that is unfamiliar?

(7) Do the strategies that work for reading a mathematics textbook transfer to other textbooks such as chemistry or physics?

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Appendix 1: Pre-test questions (and post-test questions were like these with the numbers changed):

1. Find the first 4 terms, the 10th term, and the 15th term of the sequence given below. Then graph the sequence (two graphs given for the student to choose between).

$$a_n = n + \frac{1}{n}$$

2. Predict the general term, or nth term, a_n, of the sequence.

$$-10, 50, -250, 1250, -6250, \ldots$$

3. Find the indicated partial sum for the sequence.

$$1, 2, 3, 4, 5, 6 \ldots ; S_5$$

S_5=

4. Evaluate the sum.

$$\sum_{k=2}^{5} \frac{1}{2k}$$

5. Rewrite the sum using sigma notation.

$$3 - 9 + 27 - 81 + 243 - 729$$

6. For the points $A = (2, -2), B = (-2, -10), C = (-3,3)$, and $D = (-11, -13)$, are the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ equivalent?

7. Find the component form and magnitude of $\overrightarrow{PQ}$. (a graph of the line segment PQ given and below it the coordinates of P and Q). $P(3, -5), Q(-2,7)$

The vector in component form is <___,___>

The magnitude is_____

8. Perform the calculations given that $\mathbf{u} = \langle 5,10 \rangle$ and $\mathbf{v}\ \langle 7,7 \rangle$. Find $|7\mathbf{u} - 4\mathbf{v}|$.

9. Perform the calculations given that $\mathbf{u} = \langle 8,4 \rangle$ and $\mathbf{v}\ \langle 8,2 \rangle$. Find $4\mathbf{u} + 6\mathbf{v}$.

10. Find the unit vector that has the same direction as the given vector.

$$\mathbf{v} = \langle 5,12 \rangle$$

A unit vector in the same direction as $\mathbf{v}$ is <___,___>.