

Students' Reinvention of Formal Definitions of Series and Pointwise Convergence

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The purpose of this research was to gain insights into how calculus students might come to understand the formal definitions of sequence, series, and pointwise convergence. In this paper we discuss how one pair of students constructed a formal ε - N definition of series convergence following their prior reinvention of the formal definition of convergence for sequences. Their prior reinvention experience with sequences supported them to construct a series convergence definition and unpack its meaning. We then detail how their reinvention of a formal definition of series convergence aided them in the reinvention of pointwise convergence in the context of Taylor series. Focusing on particular x -values and describing the details of series convergence on vertical number lines helped students to transition to a definition of pointwise convergence. We claim that the instructional guidance provided to the students during the teaching experiment successfully supported them in meaningful reinvention of these definitions.

Keywords: Reinvention of Definitions, Series Convergence, Pointwise Convergence, Taylor Series