

From Intuition to Rigor: Calculus Students' Reinvention of the Definition of Sequence Convergence

Abstract

Little research exists on the ways in which students may develop an understanding of formal limit definitions. We conducted a study to i) generate insights into how students might leverage their intuitive understandings of sequence convergence to construct a formal definition and ii) assess the extent to which a previously established approximation scheme may support students in constructing their definition. Our research is rooted in the theory of Realistic Mathematics Education and employed the methodology of guided reinvention in a teaching experiment. In three 90-minute sessions, two students, neither of whom had previously seen a formal definition of sequence convergence, constructed a rigorous definition using formal mathematical notation and quantification nearly identical to the conventional definition. The students' use of an approximation scheme and concrete examples were both central to their progress, and each portion of their definition emerged in response to overcoming specific cognitive challenges.

Keywords: Limits, Definition, Guided Reinvention, Approximation, Examples