How Mathematicians Use Diagrams to Construct Proofs

Contributed Research Report

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Abstract:

Although some researchers argue that diagrams can aid undergraduates' proof constructions, most undergraduates have difficulty translating a visual argument to a formal one. The processes by which undergraduates construct proofs based on visual arguments are poorly understood. We investigate this issue by presenting ten mathematicians with a mathematical task that invites the construction of a diagram and examine how they used this diagram to produce a formal proof. The talk focuses on the extent to which mathematicians based their proofs on a diagram, the ways in which they used the diagram, and the skills and strategies they used to translate an intuitive argument into a formal one. We observed that mathematicians used diagrams to notice mathematical properties, to verify logical deductions, and to justify assertions. However their use of diagrams relied on sophisticated proving strategies and a range of logical skills, such as the ability to strategically reformulate logical statements.

Keywords:

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Proof construction